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THE ELLIPSOIDAL VARIABLE STAR, *b* PERSEI

By JOEL STEBBINS

ABSTRACT

Variable star b Persei; photometric study.—During 1920–1922, 140 observations were made with a photo-electric photometer. The *light-curve* plotted from the normals, Figure 1, shows this star (magnitude 4.57; spectrum A2) to be an *ellipsoidal variable*, with a period of 1.52732 days and a total range of variation of $0^{M}.06$, of which about $0^{M}.05$ is due to the ellipsoidal figure of the components of the binary system, and the rest may be ascribed to a periastron effect in the elliptical orbit. If there is any eclipse it is slight. The *precision of the measurements* is such that the probable error of a normal is $\pm 0^{M}.0027$.

Comparison star λ Persei is probably *constant* in light.

The variability of *b* Persei was discovered at this observatory in 1920, and it was very soon found that the variation is due to the ellipsoidal figure of the components of this binary system, period 1.52732 days. Series of observations have been taken with the photo-electric photometer during the winters of 1920–1921 and 1921–1922.

Following are the Harvard data for the variable and the comparison star:

H.R. 1324, *b* Persei, magnitude 4.57, spectrum A2
H.R. 1261, λ Persei, magnitude 4.33, spectrum A0

It was not convenient to take another comparison star, and in the long series of observations there has been no evidence of variability of λ Persei.

In Table I are the observations. The first column gives the Julian date. The phase is computed from the final elements, the reduction of the times to the sun being neglected. Each difference of magnitude is derived from the number of sets of measures indicated, each set being the mean of four measures on each star taken symmetrically. The few measures which were rejected are placed in parentheses. Where haze or smoke is indicated, it was only suspected that the poor conditions may have affected the measures, though the readings themselves did not show it.

The observations in Table I were combined into the normal magnitudes in Table II in the usual way, each normal being the mean of some twenty to twenty-four sets of measures, and after once formed the normals were all considered to be of equal weight.

For a comparison with the light-curve we have the spectroscopic elements by Cannon.[1]

$$
\begin{aligned}
P &= 1.52732 \text{ days} \\
e &= 0.22 \pm 0.021 \\
\omega &= 151^\circ.75 \pm 2^\circ.08 \\
K &= 41.89 \text{ km/sec} \pm 0.97 \text{ km/sec} \\
\gamma &= 23.09 \text{ km/sec} \pm 0.57 \text{ km/sec} \\
T &= \text{J.D. } 2418956.166 \pm 0.02 \text{ day} \\
a \sin i &= 837{,}600 \text{ km}
\end{aligned}
$$

The faint secondary spectrum was measured and gives

$$m_1 \sin^3 i = 0.85\odot, \text{ and } m_2 \sin^3 i = 0.23\odot.$$

From a graph of the normals there was derived the epoch of primary minimum which was found to occur 0.033 day later than the prediction from the spectroscopic elements. Cannon's period was derived from observations extending from 1903 to 1910, and the discordance between the light-curve and the prediction is accounted for by a correction of only +0.000013 day to his period. However, I have adopted the spectroscopic period unchanged for the present purpose, and the adopted light elements of *b* Persei are as follows:

$$\text{Minimum} = \text{J.D. } 2422780.433 + 1^d.52732 \cdot E$$

The phases in Tables I and II are computed from these elements.

[1] *Publications of the Dominion Observatory*, 1, 298, 1914.

TABLE I

Observations of *b* Persei

Date, G.M.T.	Phase	$b-\lambda$	Sets	Remarks	Date, G.M.T.	Phase	$b-\lambda$	Sets	Remarks
2422584.818...	1^{d}409	0^{M}394	3		2422716.738...	0^{d}452	(0^{M}376)	6	Rejected
2585.809...	0.873	.392	3		2718.594...	0.781	.392	6	
2586.781...	0.318	.350	3		.638...	0.825	.390	6	
2591.767...	0.722	.387	3		2723.600...	1.205	.355	5	Haze
2592.760...	0.187	.369	3		2735.566...	0.953	.369	6	
2599.749...	1.067	.331	3		.592...	0.979	.367	6	
2601.731...	1.522	.417	3		.617...	1.004	.371	6	
2603.738...	0.474	.336	6		.640...	1.027	.356	6	
2605.703...	0.912	.386	6		.667...	1.054	.360	6	
2606.756...	0.437	.330	3		.731...	1.118	(.354)	2	Test only
2607.742...	1.423	.392	6		2737.592...	1.451	.396	6	
2609.685...	0.312	.335	3		.611...	1.470	.404	3	
2613.708...	1.280	.365	6		2738.566...	0.898	.368	6	Poor
.766...	1.338	.376	6		.606...	0.938	.378	6	Poor
2618.676...	0.139	.383	6		2742.629...	0.379	.340	6	
753...	0.216	.361	3		2744.642...	0.865	.389	2	
2619.704...	1.167	.345	6		2751.631...	0.217	.383	3	
2627.683...	1.509	.398	6		2752.596...	1.182	.366	6	
2632.642...	0.359	.339	6		2758.572...	1.049	.379	6	
.723...	0.440	.316	3		.598...	1.075	.361	6	
2633.653...	1.370	.378	6		2759.576...	0.525	.349	2	
2639.724...	1.332	.386	3		2766.573...	1.413	.388	4	
2641.675...	0.228	.360	6	Smoke	2778.572...	1.194	.344	6	
2642.662...	1.215	.350	6		2780.583...	0.150	.415	3	
2647.619...	0.063	.399	6		2954.793...	0.246	.368	6	
.704...	0.148	.397	6		.811...	0.264	.354	3	
2648.668...	1.112	.364	6		2955.815...	1.268	.369	6	
.738...	1.182	.361	3		2958.842...	1.240	.362	6	
2649.636...	0.553	.354	6		2967.790...	1.024	.364	6	
.703...	0.620	.381	3		2970.749...	0.928	.366	4	
2656.651...	1.458	.407	6	Smoke	2975.728...	1.325	.365	3	
2669.590...	0.651	.383	6		2976.740...	0.810	.392	6	
.675...	0.736	.394	6		2977.706...	0.249	.358	6	
2670.684...	0.218	.357	3		.754...	0.297	.360	6	
2674.690...	1.169	.358	3		2983.786...	0.219	.382	6	Smoke
2675.580...	0.532	.358	6		2984.693...	1.126	.355	6	
.628...	0.580	.382	6		.746...	1.179	.348	6	
.649...	0.601	.363	3		2986.694...	0.073	.396	3	
2682.634...	1.477	.400	6		.740...	0.119	.392	5	
.672...	1.515	.404	6		3015.705...	0.065	.386	5	
.702...	0.017	.409	6		3033.707...	1.266	.362	6	Haze
.750...	0.065	.406	6		3037.728...	0.705	.373	3	
.772...	0.087	.388	6		3043.655...	0.523	.359	6	
2683.764...	1.079	.359	6		3049.689...	0.448	.345	6	
.783...	1.098	.343	3		3054.704...	0.881	.388	6	
2692.690...	0.842	.374	3		.740...	0.917	.381	6	
2695.575...	0.672	.384	6	Smoke	3056.754...	1.403	.392	6	
.608...	0.705	.388	6	Smoke	3062.646...	1.186	.348	6	
.705...	0.802	.387	6		.719...	1.259	.363	3	
.754...	0.851	.392	6		3063.670...	0.683	.379	6	
.778...	0.875	.385	6		3068.669...	1.100	.362	6	
.795...	0.892	.398	3		.758...	1.189	.364	6	
2698.550...	0.592	.390	6	Smoke	3069.783...	0.687	.396	6	
.644...	0.686	.395	6	Smoke	3071.628...	1.004	.368	6	
.675...	0.717	.397	6		3076.666...	1.460	.413	6	
.706...	0.748	.398	6		.730...	1.524	.405	6	
.722...	0.764	.385	3		3087.614...	0.190	.384	6	
2699.642...	0.157	.385	6		3089.590...	0.638	.370	6	
.687...	0.202	.379	6		.620...	0.668	.393	3	
.742...	0.257	.354	6		.692...	0.740	.406	6	
2706.698...	1.104	.366	4	Poor	3101.643...	0.473	.340	6	Smoke
2716.528...	0.242	.363	6		3103.576...	0.878	.400	3	
.554...	0.268	.358	6		3105.642...	1.417	.398	2	Smoke
.584...	0.298	.358	6		3116.578...	0.135	.396	6	Smoke
.656...	0.370	.348	6		3117.575...	1.132	.359	6	
.678...	0.392	.352	6		3125.578...	1.498	.397	6	
.702...	0.416	.354	6		....	......	......	.....	

The normals are shown in Figure 1, and it is evident that in addition to ellipsoidal variation there is a slight effect possibly

TABLE II

NORMAL MAGNITUDES

Phase	$b-\lambda$	Residual	Phase	$b-\lambda$	Residual
$0^{d}.034$	$0^{M}.400$	$-0^{M}.003$	$0^{d}.867$	$0^{M}.391$	$+0^{M}.003$
0.087	.396	.000	0.896	.383	.000
0.142	.395	+ .009	0.935	.374	− .003
0.184	.381	+ .004	1.003	.368	+ .003
0.221	.369	+ .001	1.047	.360	+ .001
0.249	.361	− .001	1.087	.358	+ .003
0.284	.358	+ .003	1.120	.361	+ .008
0.356	.342	− .004	1.175	.353	− .001
0.424	.344	− .002	1.189	.354	− .002
0.494	.345	− .008	1.227	.357	− .003
0.564	.371	+ .005	1.279	.365	− .004
0.638	.378	− .002	1.365	.383	− .003
0.682	.388	+ .001	1.427	.393	− .004
0.718	.390	− .001	1.466	.406	+ .004
0.757	.397	+ .003	1.509	0.402	− .002
0.817	0.387	− .005	....	....	....

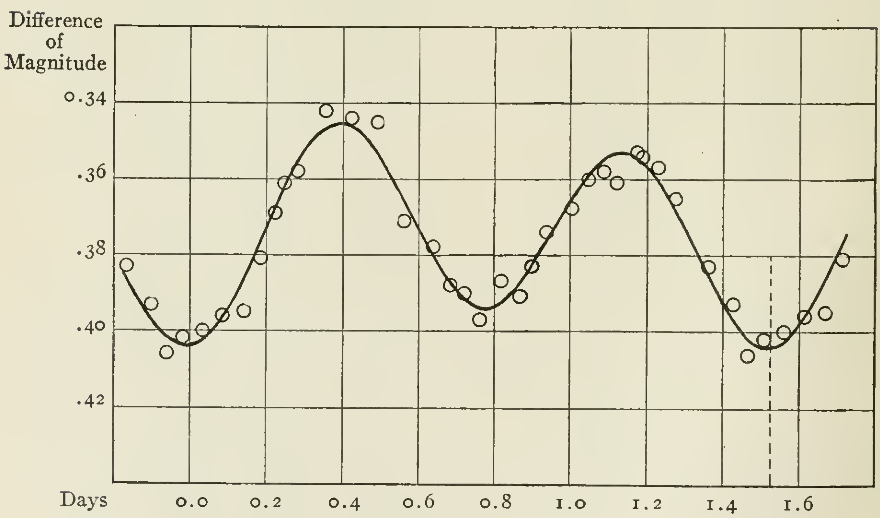

FIG. 1.—The light-curve of *b* Persei

due to the close approach of the bodies at periastron, the spectroscopic eccentricity of the orbit being 0.22. It seemed best to assume that there are no eclipses, and to represent the variation

with as few periodic terms as can be given a simple physical interpretation. From a least-squares solution of the equations set up from the thirty-one normals, I find the following:

$$\begin{array}{l} m=+0^{M}.3492-0^{M}.0040\sin\theta+0^{M}.0052\cos\theta+0^{M}.0498\cos^2\theta \\ \quad\;\;\pm 8 \qquad\quad \pm 7 \qquad\qquad\quad \pm 7 \qquad\qquad\quad \pm 14 \end{array} \tag{1}$$

Here m is the magnitude referred to λ Persei, and the probable errors of the different terms are each given below the corresponding quantity. The terms in $\sin\theta$ and $\cos\theta$ take care of the combination of the periastron effect and a possible reflection of light from the adjacent sides of the two bodies, while the term in $\cos^2\theta$ may be considered as due to the ellipsoidal figure.

The probable error of one normal magnitude is $\pm 0^{M}.0027$, and the probable errors of the different terms in (1) are all of the order of a thousandth of a magnitude. As far as accidental error is concerned, the results may be considered satisfactory, but there is always a possibility of systematic error in the correction for the atmospheric extinction, which, however, was usually less than $0^{M}.02$.

Equation (1) may also be written:

$$\begin{array}{l} m=+0^{M}.3492+0^{M}.0066\sin(\theta-232^{\circ}.4)+0^{M}.0498\cos^2\theta \\ \quad\;\;\pm 8 \qquad\quad \pm 7 \qquad\qquad\quad \pm 6^{\circ}.1 \qquad\quad \pm 14 \end{array} \tag{2}$$

The second term in (2) gives a maximum of light at phase 0.604 day or 0.429 day after periastron, but with the other complications there is no certainty that this difference is of any significance. There is the possibility that the apparent difference of about $0^{M}.01$ between the two minima is due to an eclipse at the predicted primary and not at secondary. The relative distances between centers of the bodies at these times are as 0.86 to 1.06. It is sometimes difficult enough to separate the exact ellipsoidal variation from the eclipses in stars like β Lyrae, but in the case of b Persei there is no hope of separating any minute eclipse effect from the small continuous variation. One thing is obvious from the curve, however, and that is that the component bodies rotate with uniform motion, although the eccentricity of the orbit gives rise to large librations.

It may be noted that (1) or (2) gives the light minimum of the system at $\theta = -2^\circ\!.2$, or phase $-0^d\!.009$, on the basis of the adopted elements. Also the single term in $\cos^2 \theta$ arbitrarily fixes the direction of the axes of the hypothetical rotating ellipsoids, whereas the use of terms in both $\sin 2\theta$ and $\cos 2\theta$ would leave this orientation to be determined by the solution. However, I have limited the number of terms to the fewest possible, and with the observations satisfied with a probable error of less than $\pm 0^M\!.003$ for a normal magnitude, any further refinement seems superfluous. It might be that new spectroscopic measures would indicate a rotation of the line of apsides, and also verify the correction to the period.

A rough summary of the results for this system is that the light variation is due to the bodies being elongated at least 5 per cent, while there is possibly a slight periastron effect of about a hundredth of a magnitude.

I am indebted to Mr. C. C. Wylie for sharing the observations on this star. The work is part of that accomplished under grants from the Draper fund of the National Academy of Sciences.

University of Illinois Observatory
June 1922

AN EXTENSION OF THE FUNDAMENTAL INFRA-RED ABSORPTION BAND OF HYDROGEN CHLORIDE

By W. F. Colby, C. F. Meyer, and D. W. Bronk

ABSTRACT

Fundamental absorption band of HCl at 3.4 μ.—With the help of a new grating of the echelette type, which has 2800 lines to the inch and throws most of the energy into the first order in the region 3.5 μ, this band has been extended from 3.9 to 4.2 μ. The *seven new principal lines* measured bring the total number to 39 and enable the constants of the empirical formula to be determined with considerable accuracy. Revised constants for Kratzer's theoretical formula are also given, and a table of all wave-numbers, observed and calculated. A *new group of faint lines* was found in the new region lying between the principal lines $m=-12$ to -17, but with not quite the same spacing. The tests made prove the new lines to be real and not due to ghosts or reflections. They appear only when the gas is heated (500° C.), and they may correspond to a change of radial quantum from 1 to 2 although the frequencies do not agree with those predicted by Kratzer.

In a previous communication[1] two of us reported the results of measurements made upon the fundamental spectral absorption region of hydrogen chloride. The measurements were made for the most part on the side of the center toward short wave-lengths (positive side). This side of the band appeared at the time to be the most interesting, and we were under the impression that atmospheric absorption would set in upon the other side after the twelfth line.

Certain speculations of Kratzer[2] have lent interest to the investigation of the side of the band toward long wave-lengths (negative side), and the present work has to do mainly with measurements on this side. Upon actual examination it develops that atmospheric absorption does not set in until the eighteenth line, and even the nineteenth has been measured. The atmospheric absorption is due to carbon dioxide, and not to water-vapor as was erroneously stated.

We have used in this work a diffraction grating ruled by Dr. Barker of this laboratory upon the first Rowland ruling engine. The grating has a ruling of 2800 lines to the inch and throws most

[1] *Astrophysical Journal*, 53, 300, 1921.

[2] *Zeitschrift f. Physik*, 3, 289, 1920.

of the energy into the first order in the region 3.5 μ. The amount of energy obtained from it is many times greater than with the 7500 line grating previously used, and it is possible to detect much smaller percentages of absorption. This additional power has led to the discovery of a series of faint lines between the principal lines in the region $m=-12$ to $m=-17$.

Figure 1 represents schematically the entire known hydrogen chloride spectrum, including what is new in the present investigation. The lengths of the vertical lines are roughly proportional to absorption percentage. In Figure 1*b* the principal lines of the fundamental are shown. Of these numbers -13 to -19 are now published for the first time. The faint lines in the fundamental are also shown. In Figure 1*a* the principal lines of the harmonic are shown, and the faint lines in the harmonic ascribed by Loomis and Kratzer to the isotope of chlorine.

The wave-numbers of the new lines of the fundamental band are given in Table I with the wave-numbers of the old lines subjoined to make the table complete. The wave-numbers are waves per decimeter. Table I thus includes 39 lines of the fundamental band, a sufficient number to make the determination of the constants in the empirical formula more reliable. The following formula has been determined by the method of least squares:

$$\nu=28860.7+205.9831n-3.010228n^2-0.0205683n^3.$$

Wave-numbers computed from this equation are given in Table I and also the differences between them and the observed values.

Kratzer has extended the general theory of band spectra in this region to cover, in particular, bands of the present type which are due to molecules in which the vibration is not harmonic. His formula for this band may be written with slightly different notation as follows:

$$\nu=(W_0/h+h/8\pi^2I-a_1-hu^2/8\pi^2I)+(h/4\pi^2I-2a_1-hu^2/2\pi^2I)m \\ -(a_1+3hu^2/4\pi^2I)m^2-(hu^2/8\pi^2I)m^3.$$

In this formula m indicates the number of the line counting from the center. It is positive on the high frequency side and negative on the low frequency side. This usage is very convenient and has

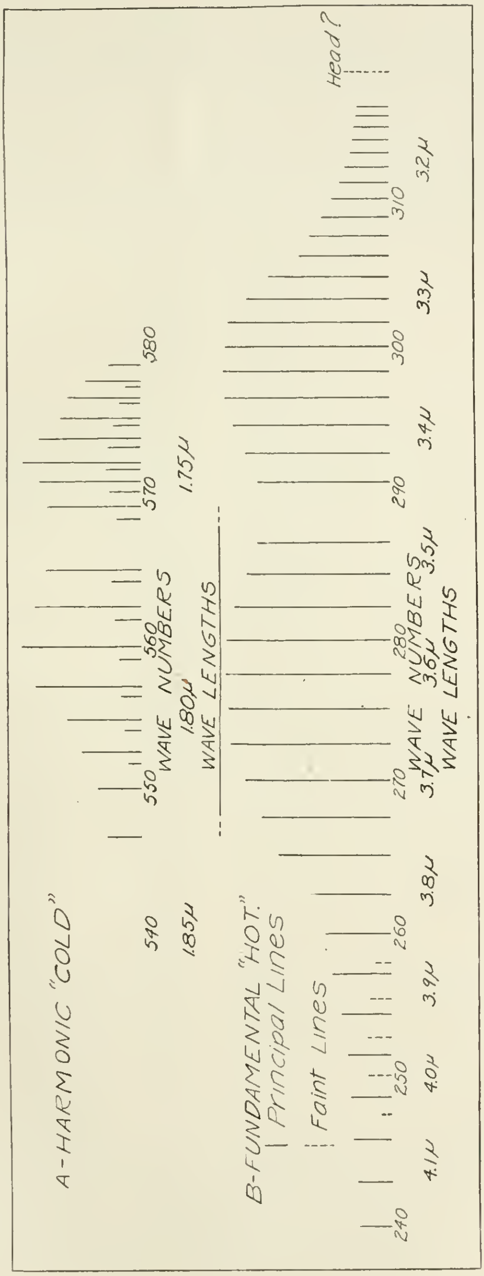
A-HARMONIC "COLD"
540
1.85μ
550
560
570
580
WAVE NUMBERS
1.80μ
1.75μ
WAVE LENGTHS
B-FUNDAMENTAL "HOT."
Principal Lines
Faint Lines
240
250
260
270
280
290
300
310
WAVE NUMBERS
4.1μ
4.0μ
3.9μ
3.8μ
3.7μ
3.6μ
3.5μ
3.4μ
3.3μ
3.2μ
WAVE LENGTHS
Head?

Fig. 1

been customary in previous experimental work. It has the additional advantage of allowing one to represent both sides of the band by one equation. For the phenomenon of absorption, m thus indi-

TABLE I

n	ν (obs.)	ν (calc.)	Diff.
−19	24012	24001	11
−18	24299	24298	1
−17	24585	24590	−5
−16	24879	24879	0
−15	25163	25163	0
−14	25443	25443	0
−13	25717	25719	−2
−12	25987	25991	−4
−11	26252	26258	−6
−10	26520	26520	0
− 9	26775	26778	−3
− 8	27031	27031	0
− 7	27278	27278	0
− 6	27520	27521	−1
− 5	27760	27758	2
− 4	27991	27990	1
− 3	28217	28216	1
− 2	28437	28437	0
− 1	28652	28652	0
1	29065	29064	1
2	29265	29260	5
3	29456	29451	5
4	29636	29635	1
5	29816	29813	3
6	29986	29984	2
7	30148	30148	0
8	30309	30305	4
9	30453	30456	−3
10	30597	30599	−2
11	30732	30735	−3
12	30858	30863	−5
13	30984	30985	−1
14	31095	31098	−3
15	31198	31204	−6
16	31298	31301	−3
17	31388	31391	−3
18	31481	31473	8
19	31554	31547	7
20	31615	31612	3

cates the original azimuthal quantum number and its sign indicates whether that quantum number has been increased or decreased during the absorption. Kratzer was limited in the application of the formula to hydrogen chloride to about half the number of lines

now available. The highest value of m which he could use was too low to determine the coefficient of the cubic term with any accuracy. This coefficient appears as a lower order term in the other coefficients and he has thus been able to neglect it. He did, however, determine the other three constants from the fundamental band and was able to evaluate the coefficient of the cubic term by relations between the fundamental and the first harmonic at 1.7 μ. The values thus determined are

$$W_0^1/h = 28767.6$$
$$a_1 = 2.975$$
$$h/4\pi^2 I = 213.0$$
$$u = 0.728 \; 10^{-2}$$

The present measurements seem to us to have sufficient precision to warrant considerable confidence in the empirical equation. If one calculates these four constants from our coefficients, the result is

$$W_0^1/h = 28757.7$$
$$a_1 = 2.979$$
$$h/4\pi^2 I = 211.96$$
$$u = 0.6965 \; 10^{-2}$$

It must be remembered that these constants are computed from an entirely new set of experimental data which differs somewhat from that used by Kratzer. All the constants are therefore slightly altered. It is, however, very gratifying to find that the value of u determined from data in this band alone, agrees as closely as it does with the Kratzer value, which involves also the frequency of the harmonic.

The hypothesis of Kratzer, regarding the faint band, had as its only experimental basis at the time it was put forward, three faint absorption maxima shown in Imes's idealized curve, one between -9 and -10, one between -10 and -11, and the other between -11 and -12. The present work was begun by attempting to establish or disestablish the existence of these faint maxima. While faint maxima were obtained in approximately the same positions in one trial, they have not been consistently found and consequently their existence has not been established, nor has it been definitely disestablished. To explain more clearly a few words may be in

order regarding the nature, and especially regarding the certainties and uncertainties of trials of the kind involved in this work. The spectrometer circle is set upon each minute of arc, and for the most part upon each half-minute of arc. At each setting, six or eight galvanometer deflections are taken, and these are averaged. When these averages are plotted against the circle settings the points may or may not fall upon a regular curve. If the points fall upon a regular curve, that is upon a curve of slight and more or less uniform curvature, it may be concluded that there is no absorption which is selective over the region investigated. But if the curve is irregular, and especially when it shows maxima, the question arises whether the irregularities are true, or due to accidental disturbances, chiefly of the galvanometer. If upon a second trial an irregularity appears in the same way, it is interesting; but only after it has appeared in four or five trials, always with the same outstanding characteristics, may it be supposed real, and then, while the irregularity may be safely supposed real, its cause is still uncertain. The supposed maxima between −10 and −11, and −11 and −12, which we may call −X and −XI respectively, if they exist at all appear only under some special conditions, the nature of which has not been ascertained, for while one trial showed irregularities, other trials give a smooth curve in the same place. Imes's experiments were carried out with a 15 cm absorption cell and the gas at room temperature. We have used the same absorption cell and also 60 cm cells both at room temperature and "hot," that is, heated to a faint glow which is visible in a darkened room without resting the eyes.

The faint lines of which the existence is established; namely those shown in Figure 1*b*, are obtained with the gas in the 60 cm cell "hot." It is necessary to heat the gas to bring out the absorption in the principal lines −13 to −19, and when this absorption appears the faint lines are also found. They are not found when working with a 60 cm chamber at room temperature. Employing the same notation as before the faint lines may be called −XII, −XIII, −XIV, −XV, −XVI. Table II gives the wave-numbers of these lines. Figure 2 gives a curve of deflections inverted, that is, the deflections increase down the page. The principal lines −13 to

− 16 show upon this curve of course, as well as the faint lines −XII to −XVI. The dotted line gives the energy-curve of deflections with no hydrogen chloride in the cell. The rise and fall in this curve is due to interference bands formed by the mica used as windows in the absorption cell, and more particularly before the thermopile. This same rise and fall appeared at regular intervals through the entire region explored whenever there was mica in the path, but disappeared when all mica was removed.

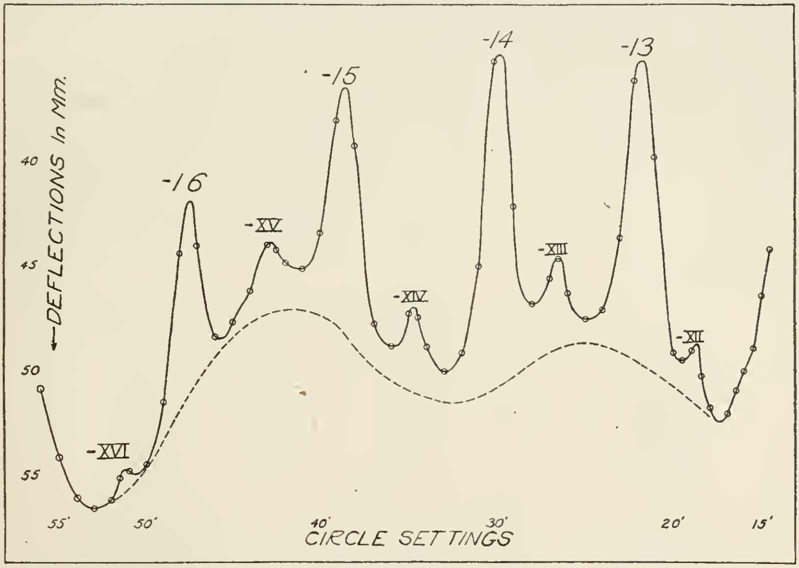

Fig. 2

Of the group −XII to −XVI the end lines are the faintest. The central one, −XIV, is probably the strongest, though it is difficult to make a definite gradation of the three lines −XIII, −XIV, and −XV. The absorption percentage for these three is about 7 per cent and for −XII and −XVI about 3 per cent, or in a galvanometer deflection of 45 mm, about 3 mm and 1.5 mm respectively. The absorption percentages of the principal lines −13 to −17 vary from 30 per cent to 12 per cent. It should further be noted that the position of the faint lines with reference to their neighbors is not uniform, but, instead, the faint lines have a characteristic spacing.

These faint lines do not fall into the band predicted by Kratzer, although their spacing does agree with the spacing which the lines of the predicted band would have in this region. This band was computed on the supposition that the absorption involved a change of the radial quantum number, n, from 1 to 2. Using Kratzer's constants the formula would read in our notation

$$\nu = 27901 + 201.30m - 2.895m^2 - 0.02259m^3.$$

Table II gives wave-numbers computed from this equation which lie in this region. It will be noticed that they differ from the observed lines by about 80 units. The first term of the equation involves the frequency of the harmonic, but a variation of this magnitude cannot be ascribed to errors in any of the experimental data used. We have, nevertheless, a group of lines which only appears when the gas is hot and which we must associate with a higher energy content than is possessed by the majority of the molecules at room temperature.

TABLE II

n	ν (obs.)	ν (calc.)	Diff.
−XII	25797	25871	−74
−XIII	25547	25621	−74
−XIV	25288	25367	−79
−XV	25021	25108	−87
−XVI	24760	24845	−85

We fortunately have at hand the original curves for this absorption band as observed by Imes. In his publication[1] he did not extend his curves to line −12 except in Figure 3, which he designates as an idealized curve. It is not a duplication of the actual observations and does not reproduce faithfully the irregularities between the lines "since they are not significant." Kratzer was forced to search for the faint lines of the series under these unfortunate circumstances and to assign wave-lengths to these faint maxima. This must have been very difficult indeed. If one turns to the original curves the difficulty is even greater. The variation of absorption percentage never rises to 5 per cent, which with Imes's small deflections could not have corresponded to a difference of galvanom-

[1] *Astrophysical Journal*, **50**, 251, 1919.

eter reading of more than 0.5 mm. Such variations can be accepted as indications of real lines only under exceptionally favorable observing conditions and after many successful trials. Moreover the points are badly scattered and no smooth curve can be drawn through them. The present writers are of the opinion that Imes's data do not give any convincing evidence of faint lines in this region. If the present new group of faint lines be extrapolated over the region -12 to -9, it is found to lie very close to, and finally to coincide with, the principal lines. Even the line

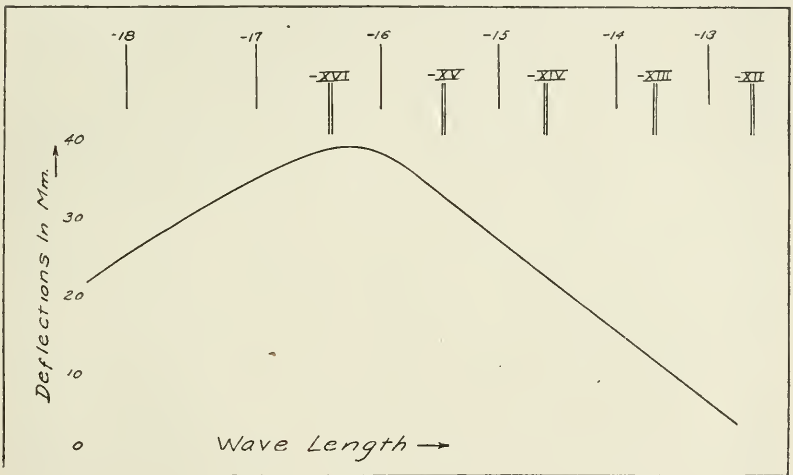

FIG. 3

near -12 lies so near as to require a resolution which our spectrometer barely furnishes.

In the use of a grating spectrometer there is under certain circumstances great danger of falling into error in the interpretation of the spectra obtained. This danger exists when faint lines are obtained in the same spectrum with strong ones, as in this investigation. The faint lines might be ghosts of the Rowland type due to periodic error in the grating, they might be Lyman ghosts, or they might be due to secondary reflections within the spectrometer. Reflections other than the principal ones intended in the design of the spectrometer sometimes occur, and may give rise to spurious lines if they happen to be in focus.

If the lines were Rowland ghosts one would expect to find them symmetrical on the two sides of each principal line, and this is not the case. The apparent wave-length of the ghosts of various orders can be calculated from the knowledge of the number of rulings of the grating per turn of the screw of the ruling engine. In this grating there are 144 rulings per turn so that a real line of wave-length 3.974 μ would give rise to first order ghosts 4.0016 μ and 3.9464 μ, second order ghosts 4.0292 μ and 3.9188 μ, etc. In the region in which the faint lines have been found, this gives for the spacing of the first order ghost about 0.6 of the spacing of the principal lines. In the region of the center of the fundamental, the ghosts have about the same spacing as the principal lines, and so might easily coincide with them except at the very center of the band, where there is a missing line. Here the first order ghost from $+1$ and -1 should nearly coincide halfway between these lines, or the second ghost of $+2$ and -2, etc. The principal lines are much stronger in this region than in the region -12 to -17, yet a number of careful trials in the region between $+1$ and -1 yielded nothing except one slight irregularity near the base of -1.

It seemed at first upon finding the faint lines that they might well be Lyman ghosts, let us say of the principal lines -8 to -13, which are the only lines of the band which have a suitable spacing. But if they were, one would expect the intensities of the faint lines to decrease continuously in passing from $-$XII to $-$XVI, since the intensities of the principal lines diminish in going from -8 to -13. However, a more critical test of this question of Lyman ghosts seemed desirable. The entrance slit of the fore-prism spectrometer is as a rule open rather wide (e.g., 2 mm), in order to obtain more energy and a more nearly flat energy-curve. Then a comparatively wide spectral region falls into the grating spectrometer, wide enough to include more than the entire fundamental band, but still amply narrow to prevent overlapping orders of diffraction. To make the critical test, the entrance slit of the fore-prism spectrometer was narrowed to 0.35 mm, thus allowing energy to enter only as shown in the curve of Figure 3. The energy in the region of the strong lines of which Lyman ghosts were at first feared thus never entered the grating spectrometer. The faint lines were still present. In

order to restrict still farther the number of strong principal lines which entered the grating spectrometer, two trials were made with the fore-prism so set that the maximum of the energy-curve was on the side of long wave-lengths from the region examined, namely the region of −XV, so that the energy of the wave-length of line −14 had been cut down by 50 per cent and that of the line −13 by 90 per cent. The faint line still appeared in the same manner.

The same tests and arguments, which seem to preclude the possibility that the faint lines are Lyman ghosts, preclude also the possibility that they are secondary reflections.

It thus seems as though the conclusion were justified that the faint lines are a true part of the observed spectrum. Question still arises as to whether they are due to hydrogen chloride or due to an impurity. On account of their general similarity, that is, magnitude of spacing and regularity of both spacing and intensity, they may well be attributed to hydrogen chloride. There is no other known spectrum of which these lines might form a part. In the fundamental of hydrogen bromide lines −5 to +2 fall into the same region, but the lines do not agree with the faint lines here given, either in position or spacing. The fundamental of hydrogen iodide has not been found. It was carefully searched for by Mr. J. P. Cooley, of this laboratory, on both sides of the 4.2 μ band of carbon dioxide, but not found. This means either that it is entirely covered by the carbon dioxide band, or else that hydrogen iodide is so highly dissociated even at room temperature that it does not yield its molecular spectrum.

As has been stated above, there may be other faint lines falling within the already explored region of the hydrogen chloride fundamental. The continuation of the new group may easily be covered by −12 to −9. Proceeding from these in the direction of higher wave-numbers, the principal lines crowd more closely together and they overlap at the base, so that the chance of finding faint lines is greatly diminished. Proceeding from −17 toward lower wave-numbers, the absorption of carbon dioxide soon sets in, and at −19 is so strong that there is scarcely hope of obtaining observations beyond this, unless carbon dioxide were entirely excluded from the path of the beam.

The irregularity between -1 and $+1$, near the base of -1, is interesting in connection with the speculation upon the possibility of existence of other faint lines. It was consistently found with a 15 cm cell with the gas at room temperature. It is probably not a ghost. On two trials with the 60 cm cell hot it failed to show up, but it may have been blotted out by continuous background (overlapping) and the galvanometer was not at its best. Extrapolation of the new group as far as this is very uncertain. Moreover, the line is very weak and is very near the strong line -1. If we assign a provisional center to the new faint group at about 27810, it is possible to call this line $+4$. Higher temperatures might strengthen it and weaken the principal line -1. Then it could be measured precisely and perhaps new faint lines would be revealed in this open region.

Some attempts have been made by Mr. H. L. Smith of the Ypsilanti Normal School, in collaboration with one of the writers, to photograph possible higher harmonics of the hydrogen chloride spectrum. This was tried both by absorption and emission. The region investigated was from 6500 A to 9500 A. The absorption in the higher harmonics would probably be faint, but it was thought that due to greater definition obtainable on the photographic film something might show. An absorption chamber about 12 cm in length was used and light from a Nernst glower passed through it. Filters were used to cut out overlapping orders. To minimize the effect of false illumination, as much of the spectrum on the side of short wave-lengths as possible was also cut out by filters, but no fore-prism spectrometer was used. In the experiments on emission, a spark was passed between monel metal electrodes in a flowing atmosphere of hydrogen chloride. Nothing was found attributable to the molecular spectrum.

The present contribution includes all lines of the 3.4 μ band which are present with measurable intensity at this temperature, about 500°. Even with this grating we were not able to extend the band beyond $+20$ on the side of shorter wave-lengths, nor to find the head which probably lies not far beyond. The present chamber does not permit higher temperatures, but it is hoped soon to have a chamber of new design which will permit not only higher temperatures but

also a control of both temperature and pressure. This is of course essential to precise intensity measurements with which one may contribute experimental data to the question of the distribution of rotational velocities.

The authors wish to acknowledge their indebtedness to the American Association for the Advancement of Science for funds which assisted this investigation very materially.

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THE THERMAL IONIZATION OF GASEOUS ELEMENTS AT HIGH TEMPERATURES: A CONFIRMATION OF THE SAHA THEORY

By ARTHUR A. NOYES and H. A. WILSON

ABSTRACT

Saha's theory of thermal ionization.—The thermodynamic equation used by Saha for calculating the thermal ionization of the neutral atoms M of gaseous elements into positive ions M^{+} and electrons E^{-} is first reviewed, especially with reference to the *assumptions* involved in the calculation. These assumptions are (1) that the ionization results from a chemical reaction $M=M^{+}+E^{-}$ whose equilibrium or ionization constant K is given by the mass-action expression $K=p_{M}^{+}p_{E}^{-}/p_{M}$; (2) that the increase in energy-content attending the ionization of each atom is equal to its ionization-potential V times the electronic charge e, and (3) that the heat-capacity and entropy of electron gas correspond to those of ordinary perfect monatomic gases. Introducing the best values of the constants involved there results the thermodynamic equation $\log_{10} K=-5048V/T+2.5 \log_{10} T-6.56$.

Calculation of ionization from the conductivities of salted flames.—The recent measurements of H. A. Wilson, as well as the earlier ones of Arrhenius, Smithells, Dawson, and Wilson, on the conductivity of flames into which salt solutions are sprayed, clearly indicate that the conductivity arises from an ionization process of the stated type; and Wilson's experiments make possible a computation of the relative magnitudes of the ionization-constants of the five elements, Li, Na, K, Rb, Cs. Absolute values of the ionization-constants are also derived from the flame conductances with the aid of previous rough determinations of the mobility of the electrons and of the number of them per cubic centimeter.

Confirmation of the Saha Theory.—The relative values of the ionization-constants of the five alkali elements computed from the flame conductances and those calculated from the ionization-potentials by the thermodynamic equation form two series of values which run closely parallel to each other; thus, though the constants vary 5700 fold, the greatest deviation between the two series is only about 40 per cent. Finally, although only the order of magnitude of the absolute values of the constants could be estimated, yet they are found to correspond well with the values calculated by the thermodynamic equation, these last being only from 1.1 to 2.3 fold larger.

I. INTRODUCTION

In a recent article Eggert[1] showed that the ordinary thermodynamic expression for the effect of temperature on the equilibrium of chemical reactions, when supplemented by assumptions as to the values of the special constants occurring in it, can be applied to the calculation of the extent to which under different conditions neutral atoms of the gaseous elements are converted into positive ions and free electrons. Saha[2] has shown that one of the most uncertain factors in

[1] *Physikalische Zeitschrift*, **20**, 570, 1919.

[2] *Philosophical Magazine*, **40**, 478, 809, 1920; **41**, 267, 1921.

Eggert's formulation can be eliminated by introducing the ionization-potential of the element. He has then calculated by this method the dissociation of the first electron from the atoms of many important elements, thus, the extent to which such reactions as $Na = Na^+ + E^-$ take place, where E^- represents electron gas. He has tabulated the values of this percentage ionization at various temperatures and pressures, and has pointed out the great significance which these considerations may have for the interpretation of the spectra of elements under solar and stellar conditions.

These principles are of great interest to astronomers, as may be illustrated by the following applications. It has long been known that the solar prominences show strongly the hydrogen and helium lines and the enhanced (H and K) lines of calcium, but not the blue line of calcium nor the familiar lines of sodium or of other alkali elements. The remarkable absence of these lines had not been explained; but the thermodynamic relations now show that the extremely small pressure combined with the high temperature of the prominences must greatly promote the ionization of all elements, and may well suffice to convert the neutral calcium atoms into the calcium ions which produce the H and K lines; and also to convert the neutral sodium atoms, to which the D lines are due, completely into sodium ions, which do not yield strong lines in the visible spectrum. A second interesting application of Saha's equations has been made by H. N. Russell,[1] who was led to the discovery of rubidium in sun-spots by the considerations that the absence of its lines in the general solar spectrum is probably due to the complete conversion by the high temperature there prevailing of the neutral rubidium atoms (which produce the characteristic lines of the element) into rubidium ions (Rb^+), and that at the lower temperature prevailing in sun-spots this conversion might well be only a partial one.

2. THE THERMODYNAMIC EXPRESSIONS

The equilibrium-constant K of a reaction $M = M^+ + E^-$ at any definite temperature in terms of the partial pressures p_M, p_M^+, p_E^-,

[1] *Astrophysical Journal*, 55, 129, 1922.

of the three substances, regarded as perfect gases, is expressed by the following equations.

$$\frac{p_M^+ p_E^-}{p_M} = K. \tag{1}$$

$$\frac{x^2 p}{1-x} = K. \tag{2}$$

The second equation, in which x represents the fraction ionized and p the sum of the pressures p_M and p_M^+, is valid only when $p_M^+ = p_E^-$; that is, only when electrons do not originate from any other source, as from the presence of another ionizing element or from thermionic causes.

The second law of thermodynamics leads to the following equations for the change of this equilibrium-constant with the temperature, in the case where the heat-content increase, ΔH, attending the reaction, can be expressed as a linear function of the absolute temperature, thus, by the formula $\Delta H = \Delta H_0 + T\Delta C_p$, where ΔC_p is the increase in the heat-capacity C_p at constant pressure that results from the occurrence of the reaction, namely:

$$d\ ln\ K = \frac{\Delta H_0 + T\Delta C_p}{RT^2}\, dT. \tag{3}$$

$$ln\ K = -\frac{\Delta H_0}{RT} + \frac{\Delta C_p}{R}\, ln\ T + \frac{I}{R}. \tag{4}$$

In this equation I is the integration constant resulting from the integration of equation (3).

Now Saha introduces the following assumptions as to the quantities occurring in this equation:

(1) That the energy increase attending the ionization of a single molecule is equal to the electrical work Ve that must be expended in order to give to an electron a kinetic energy which just suffices to ionize an atom of the element with which it collides; e being the charge on the electron and V the ionization-potential, which last may be determined by direct measurement or computed from the spectral series of the element.

(2) That the heat-capacities of the neutral atom M and its ion M^+ are equal, and that the heat-capacity of electron gas is the

same as that of any other monatomic gas, namely, 2.5 R per mol of the gas.

(3) That the constant I, which is the sum, $I_M^+ + I_E^- - I_M$, of constants[1] characteristic of the three separate substances, can be evaluated by assuming $I_M^+ = I_M$, and calculating I_E^-, the constant for electron gas, from a relation apparently applicable to ordinary monatomic gases in the way indicated in the following paragraphs.

Namely, it follows from the general principles relating to entropy, and from the fact that the molal heat-capacity at constant pressure of perfect monatomic gases at all temperatures is $\frac{5}{2}R$, that the entropy S of one mol of any such gas at any temperature and pressure is given by the following equation, in which S_1 is a constant characteristic of the gas:

$$S = S_1 + \tfrac{5}{2} R \ln T - R \ln p\,.$$

Now the considerations of Sackur,[2] Tetrode,[3] Stern,[4] Tolman,[5] and Lewis[6] have led to the conclusions that the quantity S_1 is a function only of the molecular weight M of the gas, and that the constants occurring in the functional relation can be evaluated with fairly concordant results by certain methods that need not be here described. Adopting the value obtained by Lewis from the theory of ultimate rational units,[7] and shown by him to be concordant within $\frac{1}{2}$ to 2 per cent with the experimentally determined entropies of the four best studied monatomic gases (helium, argon, cadmium, and mercury)[8] we get, in calories per degree and for the pressure in atmospheres

$$S_1 = -2.63 + R \ln M\,. \tag{5}$$

[1] These constants are 2.303 R times greater than the "chemical constants" employed by Nernst (*Theoretische Chemie*, 7th ed., p. 742).

[2] *Annalen der Physik*, **36**, 598, 1911; **40**, 67, 1913.

[3] *Ibid.*, **38**, 434; **39**, 255, 1912.

[4] *Zeitschrift für Electrochemie*, **25**, 66–80, 1919.

[5] *Journal of the American Chemical Society*, **42**, 1185, 1920; **43**, 1593, 1921.

[6] *Physical Review*, **18**, 121, 1921.

[7] Lewis and Adams, *Physical Review*, **3**, 92, 1914.

[8] Lewis, Gibson, and Latimer, *Journal of the American Chemical Society*, **44**, 1009, 1922.

Substituting for M the value 5.4×10^{-4}, we find for electron gas $S_1=-25.02$. Finally, it can be shown thermodynamically that the integration-constant I above considered is less than the entropy constant S_1 for a monatomic gas by its molal heat-capacity C; that is, $I=S_1-C=S_1-4.96$. There is thus obtained for I/R the value -15.10.

These considerations lead then to the following values of the constants occurring in equation (4):

$$\frac{\Delta H_0}{R}=\frac{NeV}{R}=\frac{96500\,V}{8.316}=11625\,V;\qquad \frac{\Delta C}{R}=2.5;\qquad \frac{I}{R}=-15.10.$$

Substituting these values in equation (4) and changing from natural to ordinary logarithms, we get finally:

$$\log K=-\frac{5048\,V}{T}+2.5\log T-6.56. \tag{6}$$

3. THE ELECTRICAL CONDUCTIVITY OF FLAMES AND ITS INTERPRETATION

It will now be shown that the investigations of Arrhenius, Smithells, Dawson, and H. A. Wilson on the conductivity of flames containing salts have led to results which clearly indicate that the conductivity arises from ionization reactions of the type considered in the previous section of this article.

The final experiments of Wilson[1] were made by spraying solutions of various salts of known concentrations into a mixture of air and gasoline, which then passed through a burner consisting of a series of parallel quartz tubes, whereby a large flame was produced. Near the sides of this flame were introduced two vertical strips of platinum, by means of which a steady current was passed through the flame. Between these electrodes were inserted in the heart of the flame two horizontal platinum wires; and the potential-difference between these two wires (as measured with a quadrant electrometer) for a given current was taken to be proportional to the resistance of the flame between them. By employing the potential-difference at these wire electrodes, instead of that between

[1] *Philosophical Transactions of the Royal Society*, A, **216**, 63–90, 1915.

the plate electrodes where the current entered the flame, the effect of the large potential-gradient around these latter electrodes was eliminated. Two such flames were always operated in series with the same current passing through them; and their conductances were compared by measuring the potential-differences between the wires in the two flames. By varying the concentration of the solution sprayed into one of the flames and keeping the other concentration unchanged, the variation of the conductance with the concentration and with the nature of the salt was obtained. The temperature in the heart of the flame, as measured with a thermocouple, registered about 1650° C. Since the couple is considerably cooler than the flame itself, we will assume the temperature to be 2000° K.

The general conclusions originally drawn from his measurements by Wilson, so far as they have a bearing on the present considerations, are summarized in the following paragraphs.

1. The conductance is independent of the acidic constituent of the salt, as was first shown by Arrhenius[1] in 1891. Thus the conductance was found by Wilson[2] to be identical when equivalent quantities of sodium chloride and carbonate, or of potassium chloride and carbonate, were sprayed into the flame. These facts show that the basic and acidic constituents of the salts must be completely separated in the flame; thus that both $NaCl$ and Na_2CO_3 are completely dissociated, with the help of the water present, into Na_2O and HCl or Na_2O and CO_2, or into Na, HCl, and O_2, or Na, CO_2, and O_2. The latter conclusion is the one to which the considerations presented below lead. Chemical equilibrium data are not available for determining whether the oxides of the alkali elements are dissociated into their elements at the temperature and partial pressure prevailing in the flame; but this conclusion is made a priori probable by the facts that the partial pressure of the salt is extremely small (of the order of 10^{-8} atmospheres), and that in the heart of the flame, where the combustion is still incomplete, reducing gases such as carbon monoxide and hydrogen are probably present and the partial pressure of free oxygen is relatively small.

[1] *Annalen der Physik*, 43, 18, 1891.

[2] *Op. cit.*, *pp.* 76, 81.

2. The conductance of a flame containing a salt of any alkali element was shown by Arrhenius to be roughly proportional to the square root of the concentration of the salt solution sprayed into it. As seen from equation (2) above, this is to be expected if the ions are produced by a unibimolecular reaction (one by which two molecules are produced from a single one), and if the fraction of the salt ionized is small.

Wilson[1] found that the relation between the conductance L and the concentration c can be expressed over the whole range of concentration by the equation:

$$10^4\, c = \frac{L^2-1}{L}\,(b+a\,L). \qquad (7)$$

In this equation b and a are constants, the values of which were determined for a number of alkali salts. When the conductance is large, this equation becomes approximately $10^4\, c = a\, L^2$, so that the conductance then varies as the square root of the concentration, as found by Arrhenius.

Equation (7) was obtained in the following way, under the assumptions that both the salt and some constituent of the flame are ionized with formation of electrons and positive ions, and that equilibrium prevails between these and the unionized parts of the two ionizing substances.

Let p_E denote the partial pressure due to the electrons in the flame, p_M^+ that due to the positive ions of the salt, p_M that due to the neutral atoms of the salt, p_F^+ that due to the positive ions of the ionizing flame-substance F, and p_F that due to the unionized atoms of this substance; also let K be the ionization-constant of the element of the salt sprayed into the flame, and K_F be that of the ionizing flame-substance. The expressions for the mass-action are then as follows:

$$K = \frac{p_M^+\, p_E}{p_M} \qquad (8)$$

$$K_F = \frac{p_F^+\, p_E}{p_F} \qquad (9)$$

[1] *Op. cit.*, pp. 78–80

Since the conductance L is proportional to the number of electrons present, the conductance of the positive ions being negligible because their mobilities are small compared with that of the electrons, we have

$$AL=p_E=p_M^+ + p_F^+ \,. \tag{10}$$

where A is a constant for all substances, equal to the ratio of the partial pressure of the electrons to the conductance of the flame.

Furthermore, the concentration c of the solutions of the various salts sprayed into the flame (when expressed in equivalents per liter) is proportional to the sum of the partial pressures of the neutral atoms and positive ions, since the conditions of spraying were always the same. Hence, denoting by B a constant that has the same value for all the substances, we have

$$Bc=p_M+p_M^+ \,. \tag{11}$$

Combining equations (8), (9), (10), and (11), we get

$$AL=\frac{KBc}{AL+K}+\frac{K_F p_F}{AL} \,.$$

Placing, as Wilson did in recording his results, the conductance of the salt-free flame equal to unity, that is, placing $L=1$ when $c=0$, we get $K_F p_F=A^2$, and therefore, assuming p_F^+ small compared with p_F so that p_F is a constant

$$c=\left(\frac{L^2-1}{L}\right)\left(\frac{A}{B}+\frac{A^2L}{KB}\right) \,. \tag{12}$$

Comparing this equation with equation (7) used by Wilson, we see that they become identical if we place

$$\frac{A}{B}=\frac{b}{10^4}\,, \qquad \text{and} \qquad \frac{A^2}{KB}=\frac{a}{10^4}\,.$$

From these equations there follows for the ionization-constant K the relation

$$K=A\,\frac{b}{a}=\frac{p_E b}{La} \,. \tag{13}$$

As an example of the satisfactory way in which equations (7) and (12) represent the variation of the conductance with the concentration, the observed conductances of cesium chloride[1] are shown in Table I, beside those obtained by plotting the curve corresponding to equation (7) when the constants b and a are taken equal to 10 and 1, respectively, and c is in grams per liter.

Further evidence that the reaction which gives rise to the flame conductivity is essentially a unibimolecular one is afforded by the fact that the conductance of flames fed with mixtures of two different alkali metals is found to be that predicted by the mass-action law. This law requires that the ionization of each element, and especially that of the less ionized one, be reduced by the electrons arising from the other; and this was shown to be the case by Arrhenius, H. A. Wilson,[2] and A. B. Bryan.[3] Thus, without entering into the details of the calculation, it may be mentioned that a solution 0.171 normal in Na_2CO_3 and 0.00238 normal in K_2CO_3 (which salts when used separately gave conductances of 21.3 and 23.3, respectively, the sum being 44.6), produced a conductance of 32.2, while the conductance calculated was 31.55.

TABLE I

VALIDITY OF THE MASS-ACTION RELATION

Grams per Liter	Conductance Observed	Conductance Calculated	Grams per Liter	Conductance Observed	Conductance Calculated
0.00	1.00	1.0	0.08	22.7	23.8
0.0032	2.88	2.9	0.16	32.8	35.4
0.008	5.72	5.5	0.8	85.2	84.5
0.016	8.9	8.7	8.0	282.	278.
0.032	13.5	13.7	80.0	883.	890.

3. The conductivity of the flame arises from the presence of positive ions and electrons; but the mobility of the positive ions is so much smaller that their conductance can be neglected in comparison with that of the electrons. Thus Wilson[4] found the mobility of the positive ions produced from the various alkali salts to be about 1 cm per second per volt per cm, while he had previously

[1] *Op. cit.*, p. 78.

[2] *Op. cit.*, p. 87.

[3] *Physical Review*, **18**, 285, 1891.

[4] *Op. cit.*, p. 71.

found,[1] by measuring the Hall effect, the mobility of the negative ions to be 2450 cm per second. These last measurements were made with flames containing different alkali salts, and the mobility was found to be very nearly the same for all salts, as it should be if the negative ions are always electrons. The fact that the negative ions have a much greater velocity than the positive ions can be explained only by supposing the negative ions to be electrons, and the positive ions to be atoms or molecules.

These various facts make it fairly certain that the conductance arises from a unibimolecular reaction by which a positive ion and an electron are produced directly from a substance whose partial pressure is proportional to the concentration of the unionized part of the salt sprayed into the flame. As there seems to be no substance other than the neutral element which could ionize in accordance with these conditions, we may conclude that the flame conductance arises from complete conversion of the chloride or carbonate into HCl or CO_2, oxygen, and the alkali element, and partial ionization of the latter in accordance with equations such as $Na = Na^+ + E^-$, where E^- represents the electron.

4. CORRESPONDENCE BETWEEN THE IONIZATION VALUES DERIVED FROM THE FLAME CONDUCTIVITIES AND FROM THE THERMODYNAMIC EQUATION

The thermodynamic equation (equation 6) applied to two different elements, whose equilibrium-constants are K_1 and K_2 and ionization-potentials V_1 and V_2, leads to the following expression:

$$\log_{10}\frac{K_2}{K_1} = \frac{5048\,(V_1 - V_2)}{T}. \tag{14}$$

Now the values of the ratio b/a of the constants of Wilson's equation are shown by equation (13) to be proportional to the ionization-constants K of the respective elements. Hence, if we assume the value of the ionization-potential for one alkali element, we can calculate it for the others by equation (14) from Wilson's values[2] of b/a. These are given in the second column of Table II. Taking the ionization-potential of sodium to be 5.111 volts and the tempera-

[1] *Physical Review*, 3, 375, 1914. [2] *Op. cit.*, p. 82.

ture of the flame to be 2000° K. the other ionization-potentials are calculated to have the values given in the third column of the table. Beside them, in the fourth column under the heading "Observed," are placed the values tabulated by Hughes,[1] as calculated from the frequencies of the spectral lines according to the quantum theory, which values agree closely for the most part with those derived from the direct measurements by Mohler, Foote, and their collaborators.

TABLE II

OBSERVED IONIZATION-POTENTIALS COMPARED WITH THOSE CALCULATED FROM FLAME CONDUCTANCES BY THE THERMODYNAMIC EQUATION

	VALUES OF b/a	IONIZATION-POTENTIAL V	
		Calculated	Observed
Cesium	10.00	4.00	3.873
Rubidium	2.25	4.26	4.154
Potassium	1.38	4.35	4.317
Sodium	0.0160	....	5.111
Lithium*	0.0023	5.46	5.362

* The value of b/a for lithium was obtained by using the results on the conductivity of flames containing lithium salts given by Smithells, Dawson, and Wilson (*Phil. Trans. Royal Soc.*, A, **193**, 108, 1899), which show that lithium salts give nearly the same conductance as sodium salts when the concentration of the lithium is about seven times that of the sodium. Hence the value of b/a for lithium is one-seventh as great as that for sodium, since the constant a is proportional to the (large) concentrations which give equal conductances.

It will be seen that the calculated and observed ionization-potentials not only change in the same order, but that the values are of the same general magnitude. Especially noteworthy is the very large change in both series in passing from sodium to potassium, and the much smaller differences between the other elements.

Instead of calculating the values of the ionization-potential by assuming its value for sodium, we may calculate relative values of all the ionization-constants by equation (14) by using the known values of the ionization-potential. The values of K so obtained should be proportional to the values of the constant b/a obtained from the flame conductivities, or the product Ka/b should be constant for the various elements. Placing the value of K for lithium equal to unity we get the results shown in Table III. The values of the product Ka/b are seen to vary to the extent of about ± 40 per cent from the mean value.

[1] *Bulletin of the National Research Council*, 2, 168, 1921.

These results confirm the validity of the term containing the ionization-potential in the Saha equation; but they do not confirm the heat-capacity and integration-constant terms, since these are

TABLE III

COMPARISON OF IONIZATION-CONSTANTS DERIVED FROM THE IONIZATION-POTENTIALS BY THE THERMODYNAMIC EQUATION WITH THOSE DERIVED FROM THE FLAME CONDUCTANCES

	V	b/a from L	K from V	Ka/b
Cesium	3.873	10.00	5731.	573
Rubidium	4.154	2.25	1119.	497
Potassium	4.317	1.38	434.	315
Sodium	5.111	0.0160	4.3	269
Lithium	5.362	0.0023	1.0	435
Mean	....	....	....	418

eliminated in the calculations. It will now be shown that the conductivity data lead also to absolute values of the ionization-constants which are at least of the same order of magnitude as those calculated by the thermodynamic equation.

Equation (13) shows that the ionization-constant K of an element is equal to $p_E b/La$ in terms of the conductances L and the constants a and b derived from them. For a flame free from salt L was taken equal to unity, so that $K = p'_E b/a$, where p'_E denotes the pressure of the electrons in a flame free from salt. Now it was found[1] that a flame into which no salt was sprayed, such as was used in the above-described experiments, has a specific conductance of 5×10^{-7} reciprocal ohms, and that the mobility of the electrons, as estimated from the Hall effect, is 2450 cm per second for one volt per centimeter. The specific conductance $\bar{L}$ is, however, substantially equal (since the positive ions have only negligible conductance) to the product of the electron mobility U_E times the charge F on one equivalent of electrons times their concentration c_E (in mols per cubic centimeter); that is,

$$\bar{L} = U_E \; F \; c_E \; ; \quad \text{or} \quad \bar{L} = \frac{U_E \; F p'_E}{RT} \, , \quad \text{since} \quad p'_E = cRT \, .$$

whence

$$p'_E = \frac{RT\bar{L}}{U_E F} = \frac{82.07 \times 2000 \times 5 \times 10^{-7}}{2450 \times 96500} = 3.5 \times 10^{-10} \text{ atm.}$$

[1] Wilson, *Philosophical Magazine* (6), **10**, 475, 1905.

An independent estimate of the partial pressure of the electrons in flames not fed with salt is afforded by the work of Wilson and Gold,[1] who from the apparent specific inductive capacity for high-frequency currents found the number n_E of electrons per cubic centimeter to be 1.1×10^9. Their partial pressure at 2000° K. is therefore given by the following equation, in which N denotes the number of molecules in one mol:

$$p'_E=\frac{n_E}{N}RT=\frac{1.1\times 10^9\times 82.07\times 2000}{6.06\times 10^{23}}=3.0\times 10^{-10}\text{ atm.}$$

Using the first of these values of this partial pressure the ionization-constants given in the following table were obtained by the equation $K=p'_E\, b/a$, derived from equation (13) by placing $L=1$. The values of K derived from the ionization-potentials by the thermodynamic equation (equation 6) for a temperature of 2000° K. are also given for comparison.

TABLE IV

Values at 2000° Absolute of the Ionization-Constant K Multiplied by 10^{12}

	Cs	Rb	K	Na	Li
From flame conductances..........	3500	780	430	5.6	0.8
From the thermodymanic equation..	8200	1600	620	6.2	1.4

It will be seen that these two sets of values do not differ by a factor of more than two or three. Since the value of the partial pressure of the electrons was derived from measurements of a flame different from that used in the conductivity measurements, the agreement is as good as could be expected. The results, therefore, afford a striking confirmation of the general validity of the calculation of the ionization of the elements from their ionization-potentials by the thermodynamic equation employed by Saha.

Pasadena, California, and Houston, Texas
October, 1922

[1] *Philosophical Magazine* (6), 11, 484, 1906; also Wilson's *Electrical Properties of Flames*, p. 110.

FILM DISTORTION AND ACCURACY OF PHOTOGRAPHIC REGISTRATION OF POSITION[1]

By F. E. ROSS

ABSTRACT

Effect of photographic film distortion on the distance apart of point images.—The results of the investigations by Schlesinger, Perrine, and Albrecht are summarized. In this investigation photographs of artificial stars were made with a precision camera reducing in the ratio of 1 to 20, on plates one inch wide pressed firmly against a rigid metal frame. In agreement with previous results, no evidence of systematic errors in distance was found, indicating that, in general, distortions are of purely local origin and extent. While the average measurements of ten exposures on a triple-coated plate are practically the same as for ten exposures on a thin emulsion, the local distortions are greater on the thick film. Yet the greatest variation from the mean was only 0.0044 mm. Faint and normal exposures gave the same average measurements within 0.0003 mm. Examination of the images showed that the effects of local variations in distribution of the silver-bromide grains and of variations in sensitiveness and in development are negligible. Intensification modifies the minor details of image structure, but does not remove the larger irregularities in the original images nor change the center of gravity appreciably. Measurements of the same plates, when wet and dry, agree accurately and indicate that on drying the images move perpendicular to the surface. However, this is not the case for images within a few millimeters of the edge, as these move outward on drying sometimes as much as 1/20 mm. This effect may be reduced by drying with alcohol or by hardening with formalin. It is concluded that the principal factor in producing the slight image displacements observed is local non-homogeneity of the gelatine.

Accuracy of measurements of the position of stellar images.—The probable error is found to be considerably less in the case of plates with a fine-grain thin emulsion (astronomical) than for those with thicker emulsions (Seed 30 and triple-coated). Intensification of weak images also decreases the error. In the foregoing measurements the probable error of a single setting was about 0.0005 mm, whereas displacement due to film distortion varied up to 0.002 mm as a maximum, but averaged only 0.0005 mm for normal exposures on the thin emulsion.

The subject of film distortion, by which is meant relative displacement of photographic images due to any one of a number of possible causes, has occupied the attention of many investigators during the past twenty years. The present paper will be confined to the particular case of images relatively far apart or beyond the sphere of mutual influence.[2] It will be of interest to outline the investigations and conclusions of a number of workers in this field. Only the most recent and important investigations will be reviewed.

[1] *Communication No. 154* from the Research Laboratory of the Eastman Kodak Company.

[2] The distortion affecting images which are very close together has been treated by the writer in previous papers. See *Astrophysical Journal*, 52, 1920; 53, 1921.

There appears to have been but very little duplication of effort, each investigator treating the subject from an original and attractive viewpoint, so that the totality of results forms a well-rounded whole. The vicissitudes to which a photographic plate is exposed in the various operations of developing, fixing, and drying, combined with the very obviously discrete structure of the photographic image, a structure subject to the vagaries of grain clumping, leave a great deal to the imagination of the investigator in a serious questioning of the faithfulness of the photographic impression.

The distortion of photographic film has been investigated by Dr. F. Schlesinger,[1] in 1906, in a manner which avoids several objectionable features of some previous investigations. Two methods were used. The distinctive feature of the first method consisted in a double development of the plate which had been exposed to a star field. The plate being measured after each development by two observers, the distortions, if present, of either an accidental or a systematic kind, can be differentiated by a simple mathematical process from the errors of bisection. No systematic distortion was found. The values of the distortion of an accidental nature and the accidental errors of measurement were as follows:

Distortion (mean error).....................	.0009 mm
Error of bisection of an image (mean error) ..	.0020 mm

from which it is concluded that the plate distortions were small and less than the errors of measurement. Nine subsequent prolonged soakings in water of the same plate failed to increase this very minute distortion. Concerning the conclusions to be derived from such a test, the author remarks: "In the preceding experiments the first measurements were made from plates that had already gone through the processes of development. There is a possibility that the film becomes set in some way in the drying after development so that thereafter it has a tenacity that it did not have before." To overcome this objection, a plate was spattered with ink, giving fine sharply outlined dots. The co-ordinates of the dots were measured, after which the plate was immersed in a developing solution in

[1] "On the Distortion of Photographic Films," *Publications of the Allegheny Observatory* 1, No. 1.

which the developing agent itself was left out, then fixed, washed, and dried in the usual manner and remeasured. The mean result from five plates was

Mean distortion error .... .0011 mm
Mean bisection error .... .0017 mm

agreeing with the result of the first experiment. The author concludes that distortion errors are not to be feared.

Important additions to the literature are contributed by C. D. Perrine.[1] Perrine remarks: "It has been the experience of practically all astronomers engaged in the measurement and reduction of photographs of stars or of spectra that the discordances found among such measures are much larger than can be accounted for by the errors of measurement alone." Perrine investigated the discordance or distortion on three kinds of plates, Seed 23, 27, and Seed Transparency, all coated on plate glass. Plates were exposed in the laboratory to a pattern plate containing small holes as a check upon stellar exposures. Distances of the order of 5 mm were measured. His results are summarized in Table I. By "range" is

TABLE I

Mean Ranges

	Laboratory Exposures	Stellar Exposures
	mm	mm
Seed 23	.0051	.0051
Seed 27	.0051	.0043
Seed Transparency	.0021	.0026

meant the difference between the greatest and least result of measurement, or the dispersion in readings. Perrine finds that the error of bisection forms but an insignificant part of the foregoing, the range of the bisection error being .0006 mm (mean of five settings, direct and reversed). The agreement between laboratory and telescope exposures is considered to place the burden of the difficulty on the photographic plate. Accepting the result of numerous investigations, that distortion of the film itself is negligible,

[1] "Some Results of a Study of the Grains and Structure of Photographic Film," *Lick Observatory Bulletin*, No. 143, 1908.

the author concludes that the difficulty is to be found in the images themselves or more specifically in the "structure of the image." Examination under high magnification of a number of images, given identical exposures, discloses such diversity of outline that no doubt remains in the author's mind but that a true cause has been found. He concludes:

> Broadly speaking, we may say that the trouble is due to a lack of homogeneity in the structure of the film. If the grains of silver thrown down were much more uniformly distributed than at present, most of the trouble would probably vanish. There are at least three possible causes for such an irregular structure as that observed. It may result from a real difference in the sensitiveness of the silver grains, from the lack of uniform transparency of the gelatine covering, or from regions deficient in silver grains. The volume of gelatine in a sensitive emulsion appears to be several times that of the silver grains.[1] Hence it would seem that there must be considerable regions which are devoid of any silver grains . . . . an increase in the amount or a further subdivision of silver ought to reduce the scale of the lane structure in the penumbra of star images, with a consequent improvement in their forms and derived positions.

In a following paper[2] Perrine continues his investigations, studying in detail the effect of an increase in richness of silver in the emulsion and of a diminution in the size of the grain. Seed X-ray plates were chosen on account of their greater richness in silver (about 25 per cent). In addition to ordinary development, the fine-grain development process of Lumière-Seyewetz was employed. Table II summarizes the results obtained. Thus no

TABLE II

RANGES

	Ordinary Development	Fine-Grain Development
	mm	mm
Seed X-Ray, ordinary glass	.0030	.0029
Seed 27, ordinary glass	(.0064)	.0040
Seed 27, plate glass	.0044	.0039
Seed 23, ordinary glass	....	.0030
Seed 23, plate glass	....	.0016
Seed Lantern, ordinary glass	.0015	.0018
Seed Transparency, plate glass	.0022	.0020

[1] About nine times in an average emulsion. F.E.R.

[2] "Results of Some Further Studies on the Structure of Photographic Films and the Effect on Measures of Star Images," *Lick Observatory Bulletin*, No. 178, 1909.

improvement in accuracy due to fine-grain development was found. It is considered that the expected advantage of an increase in richness of silver is borne out. Further conclusions of importance deduced by the author are that thin or weak images show greater discordances than dense or fully exposed ones, and that, when the thickness of the emulsion is reduced, the discordances are also reduced. In addition, "there appears to be a decided tendency . . . . for thick films and thin images to give values of the distances systematically too large." He finds:

Plate	Mean Δ in mm
Rapid (thick film)	17.4628
Slow (thin film)	17.4600
Distance on pattern plate	27.4604

The author concludes:

Such an effect could be explained by supposing a swelling action of some sort to take place after the images were impressed upon the film, and during the process of developing, fixing, washing and drying. Just why a swelling should take place is not clear. It would seem more natural to expect a shrinkage, owing to the removal of unaffected silver from the film in fixing.

The subject of film distortion has been investigated by S. Albrecht[1] who covers important points in his investigation not considered by previous workers. To quote:

The most important features of the plan upon which my work was begun were investigations of the effects of (*a*) the position of the plate during the processes of washing and drying; (*b*) the rate of drying; (*c*) abrupt changes in the rate of drying during the process; (*d*) changes in the position of the plate while drying, (*e*) hardness. Emulsions on plate glass were also tried.

The results were entirely negative. The conclusions, however, which are of importance and interest, are summarized by the author as follows:

(1) For the size of plate used (4×5 inches) it was found to be entirely indifferent whether the plate be vertical or horizontal during development, fixing, washing and drying. (2) Within the range of the observations, hardener, the rate of drying, and changes in the rate of drying and in the position of the plate during the process of drying introduced no general distortion of the

[1] "On the Distortions of the Photographic Films on Glass," *Astrophysical Journal*, 35, 349, 1907.

gelatine film. (3) Local distortions were found on artificial star plates and on spectrograms. These distortions were confined in each case to an area equal to a small fraction of a square millimeter. The largest lateral displacement found at any point in the distorted area was .020 mm while the great majority were less than one-fourth of this amount. (4) The distortions appear to be principally of two different kinds: one was due to an actual movement of a minute portion of the film, the other was an apparent shift of the image due to the peculiar arrangement of the silver grains or to local differences in the sensitiveness of the film. (5) The results obtained from one plate-glass plate showed no advantages of the plate glass over the ordinary commercial plates in the matter of distortions of the film.

The present writer[1] in 1912 incidentally found an apparently new kind of film distortion. Small plates, 27×37 mm, which were dried in a chimney type of drying-box, showed a large general expansion amounting to one part in twelve hundred. It was further found that if a plate containing star images, in this distorted condition, was soaked in water and dried in the *ordinary* way, the distortion disappeared. In this same investigation a determination was made of the probable error of a single measured distance between reseau lines on plates dried in air and in alcohol respectively. There was a decided difference in favor of the alcohol-dried plates, as the following figures show:

Probable error of a measured distance, air dried	$\pm$.0020 mm
Probable error of a measured distance, alcohol dried .	.0012 mm

Uniformity of drying, secured very effectively by immersion in alcohol, is thus seen to be an important factor in reducing film distortion at least on plates of these small dimensions.

In a former paper[2] the writer has accounted for displacements in position of images which are close together by assuming local inequalities in drying. In the case of an isolated image such a hypothesis does not commend itself unless one assumes a non-homogeneity of structure in the gelatine. This would lead to local variations in the rate of drying. Except for the observations of the writer quoted above, there are no well-authenticated cases of general

[1] *Special Publications, U.S. Coast and Geodetic Survey*, No. 27, p. 44.

[2] "Image Contraction and Distortion on Photographic Plates," *Astrophysical Journal*, **52**, 106, 1920.

expansion or contraction of film on glass. The amount of the observed displacements appears to be constant for all distances. This fact suggests that the phenomenon of apparent displacement is of purely local origin and extent. From this standpoint, namely, of local action, collecting the various suggestions which have been advanced, we have as possible causes:

1. Local drying strains.
2. Local variations in distribution of the silver-bromide grains.
3. Local variations in sensitiveness.
4. Local variations in development.
5. Grain clumping, i.e., graininess or local variations in the distribution of the developed grains.

It is quite evident that much more data must be accumulated before the phenomenon in question can be established beyond dispute. The writer has made a number of experiments and measurements, suggested by various aspects of the subject, which will now be described. In general, the distances measured were chosen small, which makes for greater accuracy, but not so small that the images might possibly be affected by mutual action. Instead of forming artificial star images by contact printing, they were projected on the plate in the precision camera. It was felt that this is much the safer procedure of the two. The only drawback is a variation of focal distance from exposure to exposure due to possible irregularities in the plate. This effect should be small, however, since the plates are only one inch wide and are pressed firmly against a metallic bed in the plate holder. In order to appraise the variation, if present, the pattern plate was made with four artificial stars distributed in a straight line. If there is a variation of focus, the distance of the two outer stars should show a greater probable error. It will be seen that this is not the case.

Experiment A.—Six plates were exposed in the precision camera (reducing 20 times) to the test object containing four artificial stars, A, B, C, D, lying on a straight line. Ten exposures were made in rapid succession on each plate. The plate holder slides smoothly and accurately in metallic grooves. The plates were as follows:

1. Fine-grained orthochromatic thin emulsion (called astronomical).

2. Seed 30, medium thickness.
3. Triple-coated orthochromatic.

These plates, of varying emulsion-thickness, were chosen to test the thickness-effect (see Perrine's result, p. 37). Two exposure times were chosen, giving weak or surface images and strong, deep images, respectively. Each plate was measured twenty times, ten direct and ten reversed, on ten separate days. "One measure" in Table III signifies the mean of a single direct and reversed measure, each image being bisected but once. The readings were all started from the same point on the screw, so that secular and periodic screw and bearing errors are eliminated. The mean results are contained in Table III. Magnification used, 75.

TABLE III

MEAN Δ AND PROBABLE ERRORS

Plates	Emulsion Thickness	Diameter of Images	$BC=\Delta_1$			$AD=\Delta_2$		
			Mean Δ_1	Probable Errors: One Distance (Photographic)	Probable Errors: One Measure (Personal)	Mean Δ_2	Probable Errors: One Distance (Photographic)	Probable Errors: One Measure (Personal)
	mm	mm	mm	mm	mm	mm	mm	mm
Weak (2^s) exposure:								
Astronomical	.014	.021	1.3321	.00040	.00043	2.6512	.00085	.00058
Seed 30	.030	.027	1.3317	.00075	.00063	2.6519	.00134	.00055
Triple C. Ortho	.060	.024	1.3318	.00084	.00065	2.6520	.00180	.00068
Means	....	....	1.3319	.00066	.00057	2.6517	.00133	.00060
Strong (60^s) exposure:								
Astronomical	.014	.061	1.3319	.00046	.00043	2.6510	.00064	.00042
Seed 30	.030	.067	1.3314	.00072	.00054	.26513	.00072	.00053
Triple C. Ortho	.060	.065	1.3324	.00075	.00062	2.6519	.00080	.00049
Means	....	....	1.3319	.00064	.00053	2.6514	.00072	.00048

Comparing in this table the means of mean Δ for weak and heavy exposures, it is found that

$$\text{Mean } \Delta \text{ (heavy exposure)} - \text{Mean } \Delta \text{ (weak exposure)} = -.00015 \text{ mm}$$

Comparing mean Δ by plates, agreement is not so marked. However, the pronounced increase in Δ for thick emulsions, found by Perrine, is not in evidence.

Comparing probable errors, the superiority of the thin fine-grain astronomical emulsion is manifest. This plate is by no means slow. It accordingly has distinct advantages in astronomical

work. The photographic probable errors are in general smaller than obtained by previous workers. In fact they are so small as to materially increase faith in the accuracy of photographic registration of position, under properly controlled conditions. However, there is no doubt that occasional, sensible, apparent displacements of images manifest themselves, amounting to .002 mm at the maximum. In order to see if there is any dependence of displacement on the character of the image, all images on the foregoing series of plates concerned in producing large displacements were examined. In every case the images of each pair were round and regular, giving no sign of disturbance. On the other hand distances measured between images which were very irregular in outline—cases where one would expect an apparent displacement—did not indicate anything abnormal. These facts tend to throw doubt on the irregularity-of-outline theory of image displacement and are accordingly more favorable to the theory of local gelatine disturbance, i.e., disturbances confined to minute volumes of gelatine such as would be produced, for example, if the gelatine were not strictly homogeneous. In this case it should be found that local distortions depend, not on emulsion, but on the gelatine and on the thickness of the coating. It would be difficult to explain the large distortions found by Albrecht (p. 38) on any other basis.

The comparatively large value of the photographic error for Δ in the case of weak exposures, shown in Table III, is difficult to explain. That it cannot be due to irregularities in focal distance mentioned above is indicated by the results given in Table IV. Each Δ is the mean of twenty measures on ten days, so that accidental measuring errors are eliminated. Comparing corresponding v's, there is no evidence of correlation.

The fidelity of the photographic plate in recording distance is primarily dependent upon the physical properties of gelatine, which are peculiar and exceedingly complex. The present form of gelatine is sensitive both to heredity, history, and immediate environment. "Any 'structure' is not inherent in the gelatin, but is an environment impress, a strain structure in the original mass."[1]

[1] Sheppard and Elliott, "The Drying and Swelling of Gelatin," *Journal American Chemical Society*, 44, 379, February, 1922.

It is to be pointed out that the form characteristics are not to be regarded as merely two-phase, the wet and dry phase, with intermediate values, for even in the dried condition two or more dimensional forms can exist in the same mass, depending on its previous history. An example of this is recorded on page 38. Numberless experiments suggest themselves which would be useful in the study of the forms of gelatine masses in general and in particular in the study of sheets of gelatine containing emulsions coated on plates. Sheppard and Elliott (*loc. cit.*) have made a partial study of the shapes which drying masses of gelatine take under various condi-

TABLE IV

Δ FOR WEAK EXPOSURES

Exposure	Triple-coated Ortho Plate			
	Δ_1	v_1	Δ_2	v_2
1	1.3331	−13	2.6492	+28
2	.3319	− 1	.6478	+42
3	.3325	− 7	.6536	−16
4	.3311	+ 7	.6549	−29
5	.3329	−11	.6553	−33
6	.3295	+23	.6527	− 7
7	.3329	−11	.6520	0
8	.3323	− 5	.6503	+17
9	.3310	+ 8	.6506	+14
10	.3309	+ 9	.6540	−20
Means	1.3318	....	2.6520	....

tions, in particular with one face constrained, which is the case of interest in photography. From their experiments it might be inferred that the upper layer of gelatine on a photographic plate *contracts* on drying, at least at the edges of the plate. It is of interest to make a special study of what happens in this particular and important case. It may not be of practical interest to the astronomer to know what becomes of the photographic image when the film is wet. Knowing that the images on hydration and dehydration move in a general direction perpendicular to the plate, it is only necessary that these opposite movements take place along the same path. This is, of course, an assumption which must be proved. The expansion of the film in development takes place

under entirely different physical and chemical conditions from its contraction on drying, due to the removal of the unexposed silver bromide and to other factors. The natural inference is that distortion would be very likely to take place. This has been the viewpoint of astronomers who have gone to considerable trouble to determine the character and amount of the distortions.

Measurements on wet and dry plates.—In order to observe the effect of swelling, moderately large distances were chosen. Since the effect, if any, should be a maximum on thick films, a triple-coated plate was used. Two groups of images 30 mm apart, symmetrically located on a plate 1×5 inches, were measured. The images were small and confined to the surface, so that any surface creep would be disclosed. The fixing bath contained no hardener. After measurement, dry and wet, the plate was bathed in a hardening (formalin) bath and the measurement repeated. Δ below is the mean of five independent distances, measures being made direct and reversed.

	Mean Δ in mm
Plate measured wet	30.1380
Plate measured dry	30.1380
After formalin bath, wet	30.1368
After formalin bath, dry	30.1389

These measures show that even under the extreme conditions chosen, the movements of the images in swelling and in drying are outward and inward along the same path, which path is without a doubt perpendicular to the plate. A very minute effect is shown to have resulted from the hardening bath, but is so small as to be doubtful. The effect of hardening is taken up in more detail on page 46.

It is well known that in the case where images are located near the edge of a plate, measures are unreliable. In order to find out the exact nature of the phenomena exhibited in this case, a very narrow strip, 4.3 mm wide, was cut from a plate containing exposures of the four-hole test object, in such a way that the two outer images, *A* and *D*, were but 0.8 mm from the edge. This plate was then subjected to a series of hydrations and dehydrations and measured at each phase. Table V contains the results. Soakings were for thirty minutes in water at 70°. Very pronounced dis-

tortions are exhibited in this table, which are seen to be considerably less in the case of the images *BC* farther from the edge. It is to be noted that after the first wetting the film always *expands* on drying.

TABLE V

Distortions of Plate

	Δ			
	AD	Increase μ	*BC*	Increase μ
1. Original, dry	2.655	....	1.333	....
2. Wet	2.665	+10	1.337	+ 4
3. Dry	2.702	+47	1.372	+39
4. Wet	2.680	+25	1.345	+12
5. Dry	2.707	+52	1.368	+35
6. Wet	2.685	+30	1.352	+19
7. Dry	2.712	+57	1.374	+41
Distances of images from edge	0.8 mm		1.5 mm	

Further data were obtained from similar measurements on a wider plate (8.1 mm wide). The plate was alternately wetted and dried seven times. The mean results are given in Table VI.

TABLE VI

	AD	*BC*
	mm	mm
Original, dry	2.653	1.331
Measured wet	2.653	1.333
Measured dry	2.659	1.336
Distances of images from edge	2.7	3.4

The same phenomena are exhibited, but to a less extent, as was to be expected, since the distances from the edge are greater.

Cause of edge distortion.—At first sight, it would appear that on drying there must be a shrinkage of the gelatine at the edge of an image taking place in the plane of the plate, following the analogy of a cube of gelatine drying with one face constrained. In the above experiments, however, the opposite was found to be the case, i.e., the gelatine *expanded* on drying. The explanation of the phenomenon appears to be the same as that proposed by the writer

in a former paper[1] in explaining the mutual action of adjacent images. It is imagined that in drying, whenever any differential action occurs such as one portion of the plate drying more quickly than another, there is a migration of gelatine with its encompassed images which takes place in the direction of the region which has dried first.[2] This principle is manifestly applicable to the edges of plates which dry more quickly than the center. The drying line accordingly creeps in from edge to center, accompanied by migration of gelatine outward. It will be convenient to make use of the term "hydration gradient" which can be defined as the maximum derivative of the specific water content with respect to any direction in the plane of the plate. On account of the gelatine setting very quickly at the extreme edge, leading to a very large hydration gradient, there is greater differential action in this region and accordingly a greater transfer of gelatine. The hydration gradient at any distance from the edge greater than 10 mm, roughly, appears to be so low that transfer of gelatine and consequent distortions are insignificant. In this case the movements are strictly up and down or normal to the plate. It is quite probable that irregularities in the thickness of the film coating lead to uneven drying and consequently to distortions.

Effect of hardener.—A small plate 16×24 mm was cut in such a way that a group of images lay near each long edge in a line perpendicular to it. After measuring dry and wet, the plate was bathed in formalin and remeasured. Table VII contains the results. In the case of Δ_1, the images are 1.3 mm from the edge of the plate, as indicated in the table.

[1] "Image Contraction and Distortion on Photographic Plates," *Astrophysical Journal*, **52**, 106, 1920.

[2] Since the foregoing was written my attention has been called to a paper by H. Stintzing in *Koll. Chem. Beihefte*, **6**, 231–96, 1914, entitled "The Influence of Light on Colloid Systems." He finds that it is a general property of colloid systems, of which gelatine is an example, that exposure to radiation produces an increase in concentration of the colloid in the portions insolated, migration taking place from the unaffected regions. The phenomenon only takes place if evaporation is permitted. The author explains the phenomenon as a change in the distribution of the colloid resulting from local inequalities in the rate of evaporation and affirms that this is a general property of colloid systems.

It will be noticed from this table that no effect results from the hardening bath until the plate has dried. The small effect appearing in the fourth line of the table is doubtless due to partial drying before the measures could be completed. After hardening, true distances can no longer be obtained on the plate by soaking in water. Distances are now the same, whether measured wet or dry, and correspond to the false distances indicated in line No. 3 of the table, and not to the true distances of line No. 1. The reason for this is not far to seek. The hardening has destroyed nearly all of the power of the gelatine to swell on immersion in water. In

TABLE VII

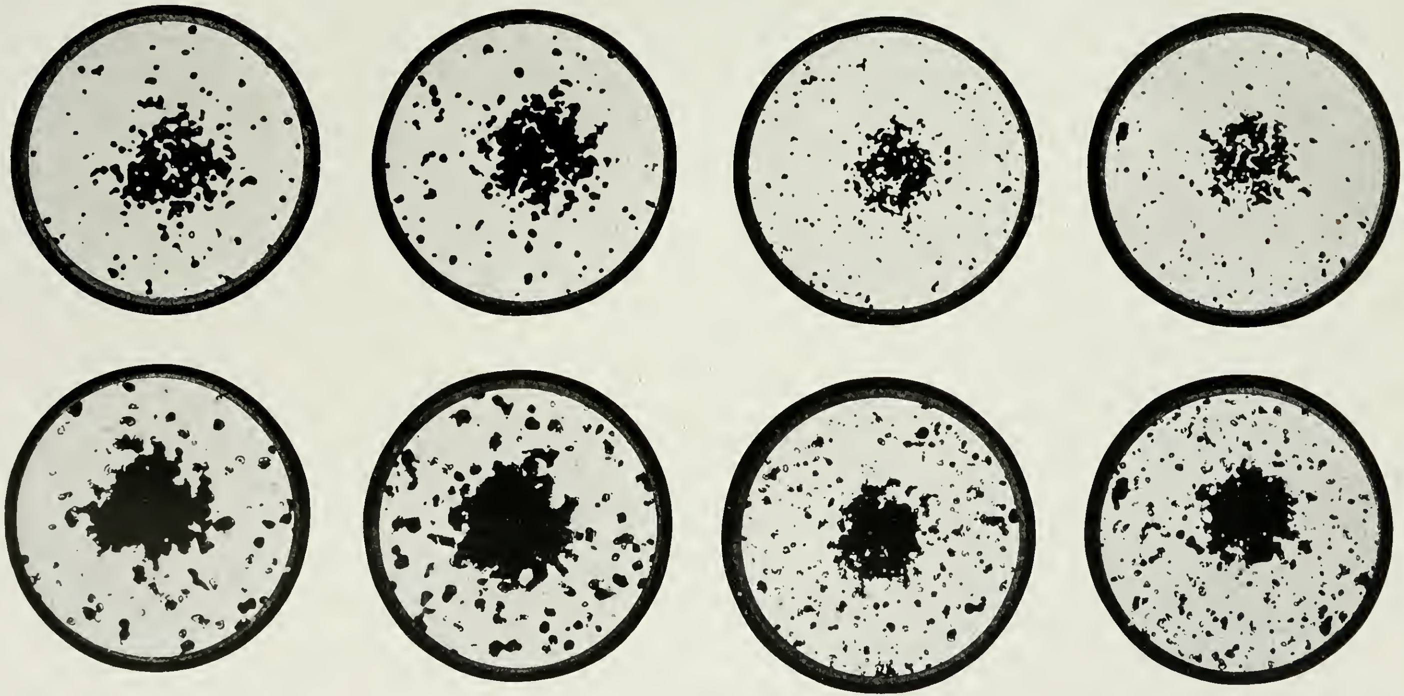
Measures on Hardened Plate

	Δ_1		Δ_2		Δ_3		Δ_4	
	mm	Exp. (μ)	mm	Exp. (μ)	mm	Exp. (μ)	mm	Exp. (μ)
Original (after cutting), dry	14.173	......	12.863	......	10.194	......	8.881	......
Soaked one hour in water, wet	14.174	......	12.863	......	10.193	......	8.877	......
Dry	14.195	+22	12.881	+18	10.204	+10	8.889	+ 8
Soaked one hr. in water, then 15 min. in formalin, wet	14.180	+ 7	12.866	+ 3	10.195	+ 1	8.880	− 1
Dry	14.194	+21	12.871	+ 8	10.199	+ 5	8.880	− 1
Soaked one hr. in water, then 15 min. in formalin, wet	14.193	+20	12.872	+ 9	10.201	+ 7	8.880	− 1
Dry	14.191	+18	12.872	+ 9	10.200	+ 6	8.881	0
Soaked one hr. in water, then 15 min in formalin, wet	14.190	+17	12.870	+ 7	10.200	+ 6	8.882	+ 1
Distance of each image from edge of plate	1.3		2.0		3.4		4.0	

the case of the foregoing plate, the swelling was found to be only 40 per cent as compared with an average of 700 per cent on unhardened plates.

Effect of intensification.—Although the present investigation has not disclosed any clear effect upon position measurements which is due to the raggedness of outline of the images measured, there is the possibility of such errors being present. The a priori reasons for the existence of such are strong. If actually present there remains the attractive probability that intensification of the images will lead to improved results. Such improvement should be especially noticeable in the case of weak images in which the defects of structure are more pronounced. Plate I shows a number of

PLATE I



photomicrograms of weak images. In the lower row of Plate I the same images are shown after intensification. Images having an unusual amount of distortion were chosen. One is of an equilateral triangular form. It is to be understood that these irregularities are not due to the optical system but are accidental configurations on the photographic plate. It will be noted that intensification has not altered the larger peculiarities. In the case of the smaller defects which include open spaces, bays, and capes, there is a decided filling in, so that there is a considerable improvement in the appearance of the image after intensification.

In order to make a numerical test of the effect of intensification, a Seed 30 plate containing weak exposures was chosen for test. The system of exposure has been described on page 40. In fact the plate chosen figures in the second line of Table III. Table VIII

TABLE VIII

EFFECT OF INTENSIFICATION

Exposure	*AD*				*BC*			
	Before Intensification		After Intensification		Before Intensification		After Intensification	
	mm	*v*	mm	*v*	mm	*v*	mm	*v*
1.........	2.6505	+14	2.6508	+ 5	1.3297	+20	1.3304	+14
2.........	.6526	− 7	.6518	− 5	.3315	+ 2	.3305	+13
3.........	.6539	−20	.6530	−17	.3308	+ 9	.3302	+16
4.........	.6539	−20	.6525	−12	.3329	−12	.3328	−10
5.........	.6524	− 5	.6521	− 8	.3314	+ 3	.3317	+ 1
6.........	.6540	−21	.6526	−13	.3318	− 1	.3326	− 8
7.........	.6502	+17	.6495	+18	.3324	− 7	.3318	0
8.........	.6483	+36	.6480	+33	.3327	−10	.3326	− 8
9.........	.6510	+ 9	.6496	+17	.3310	+ 7	.3310	+ 8
10.........	.6525	− 6	.6533	−20	.3332	−15	.3347	−29
Means..	2.6519	....	2.6513	....	1.3317	....	1.3318	....

contains the results of measurement of this plate before and after intensification. Each Δ is the mean of ten measurements made on as many days. Comparison of the *v*'s before and after intensification in this table proves that intensification has had no effect on the apparent occasional displacements of star images, which are indicated by abnormal values of *v*. On account of the improve-

ment in the appearance of intensified images noted above, it might be expected that the accidental measuring errors are reduced. That this is the case is shown by Table IX. Accordingly, the only gain

TABLE IX

PROBABLE ERROR OF A SINGLE MEASUREMENT

(Mean of Direct and Reversed)

	Δ_1	Δ_2
	mm	mm
Before intensification........	±.00055	±.00063
After intensification.........	.00044	.00050

on intensification appears to be a diminution in the error of measurement. The constant errors in position of the image are not affected.

ROCHESTER, NEW YORK
August 22, 1922

INVESTIGATIONS ON PROPER MOTION

NINTH PAPER: INTERNAL MOTION IN THE SPIRAL NEBULA MESSIER 63, N.G.C. 5055[1]

By ADRIAAN VAN MAANEN

ABSTRACT

Spiral Nebula M 63 (N.G.C. 5055).—Comparison of two plates taken in 1910 and 1922 by Ritchey and Humason, respectively, gives, with respect to 21 comparison stars, an annual *proper motion* for the nebula as a whole of $\mu_\alpha = +0\overset{''}{.}005$, $\mu_\delta = -0\overset{''}{.}015$. The motions of 98 nebular points with reference to the nebula are tabulated and also shown on the *photograph*, Plate II. The *internal motion* is found to be a stream motion of $+0\overset{''}{.}019$ outward along the arms of the spiral, combined with an outward transverse motion of $+0\overset{''}{.}004$. There is a slight increase in the motions with increasing distance from the center. The results are analogous to those found for M 33, 51, 81, 94, 101, and N.G.C. 2403, but are even more convincing because of the long time-interval between plates and the many nearly starlike points.

Curtis[2] describes this object as "a bright, beautiful spiral $8' \times 3'$ in p.a. 98°. Has an almost stellar nucleus. The whorls are narrow, very compactly arranged, and show numerous almost stellar condensations."

Two plates taken at the 25-foot focus of the 60-inch reflector were measured with the new stereocomparator: the old plate was secured by Mr. Ritchey on March 9, 1910, the new plate by Mr. Humason on May 28 and 29, 1922; both plates have an exposure of five hours and are of good quality.

The plates were measured in four positions, east, west, north, and south, respectively, in the direction of increasing readings of the micrometer screw. Twenty-one comparison stars and 98 points, the latter presumably belonging to the nebula, were measured. The measures in right ascension were combined into one set, and those in declination into another; then the measured quantities were multiplied by 0.698 to reduce the values expressed in parts of the micrometer screw to annual motions in thousandths of a second of arc. These quantities, m_α and m_δ, respectively, were used as the first members in equations of condition of the form:

$$\left.\begin{aligned} m_\alpha &= a+bx+cy+dx^2+exy+fy^2+\mu_\alpha \\ m_\delta &= a'+b'x+c'y+d'x^2+e'xy+f'y^2+\mu_\delta \end{aligned}\right\} \qquad (1)$$

[1] *Contributions from the Mount Wilson Observatory*, No. 255.

[2] *Lick Observatory Publications*, **13**, 34, 1918.

in which $a \dots f$, $a' \dots f'$ are the plate constants, x and y the co-ordinates in right ascension and declination, and μ_a and μ_δ the annual proper motions. By a least-squares solution the plate constants were determined from two sets of equations of the form (1), yielded by the 21 comparison stars. These constants were substituted into equations of the form (1) for all objects measured, thus giving μ_a and μ_δ, the components of the motion with respect to the mean of the comparison stars. These quantities are listed in the fourth and fifth columns of Table I; the second and third columns give the positions with respect to the center of the nebula, accurate to a tenth of a minute of arc.

In order to derive the internal motions of the nebula, the values of μ_a and μ_δ must be freed from the motion of the nebula as a whole. The same procedure was used as for M 33, 51, 81, 94, 101, and N.G.C. 2403.

a) The mean motion of the 98 points is

$$\mu_a = +0\rlap{.}''005\,, \quad \mu_\delta = -0\rlap{.}''015\,.$$

b) Combining the mean motions in quadrants I and III, we have

$$\mu_a = +0\rlap{.}''002^5\,, \quad \mu_\delta = -0\rlap{.}''015\,;$$

while for quadrants II and IV

$$\mu_a = +0\rlap{.}''008\,, \quad \mu_\delta = -0\rlap{.}''015\,.$$

All four quadrants combined give

$$\mu_a = +0\rlap{.}''005\,, \quad \mu_\delta = -0\rlap{.}''015\,.$$

c) Using only the 80 points closest to the center, which have a more symmetrical distribution, we find for quadrants I and III, $\mu_a = +0\rlap{.}''003$, $\mu_\delta = -0\rlap{.}''014^5$; while for quadrants II and IV, $\mu_a = +0\rlap{.}''005^5$, $\mu_\delta = -0\rlap{.}''014$. For all four quadrants $\mu_a = +0\rlap{.}''004$, $\mu_\delta = -0\rlap{.}''014$.

For the motion of the nebula as a whole, the mean result from the three methods is

$$\mu_a = +0\rlap{.}''005\,, \quad \mu_\delta = -0\rlap{.}''015\,.$$

TABLE I

Co-ordinates and Measured Annual Displacements

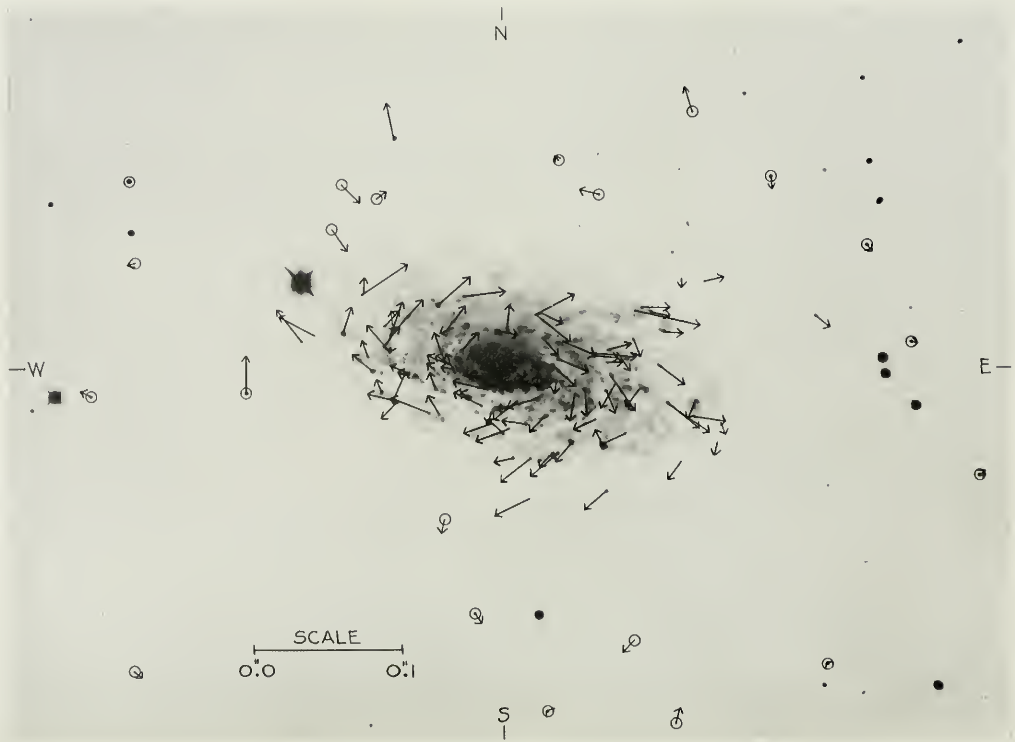
No.	x	y	μ_α	μ_δ
a	−6′.3	+3′.2	0″.000	0″.000
b	−6.2	+1.8	− 5	− 2
c	−2.9	+2.4	+ 10	− 15
d	−2.7	+3.1	+ 6	− 12
e	−2.1	+2.9	+ 6	+ 5
f	−7.0	−0.5	− 7	+ 3
g	−4.3	−0.5	0	+ 26
h	−6.2	−5.2	+ 5	− 4
i	−1.0	−2.6	− 2	− 9
j	−0.5	−4.2	+ 4	− 7
k	+0.8	−5.9	− 1	+ 1
l	+2.3	−4.7	− 8	− 8
m	+3.0	−6.1	+ 3	+ 11
n	+5.5	−5.0	− 1	+ 1
o	+8.1	−1.9	+ 3	+ 2
p	+7.0	+0.5	+ 4	− 1
q	+6.2	+2.1	+ 3	− 5
r	+4.6	+3.3	+ 1	− 9
s	+3.2	+4.3	− 5	+ 17
t	+1.7	+3.0	− 13	+ 3
u	+1.0	+3.5	− 3	+ 3
1	−3.4	+0.5	− 12	+ 7
2	−3.2	+0.6	− 19	− 2
3	−2.7	+0.6	+ 10	+ 3
4	−2.4	+1.3	+ 36	+ 6
5	−2.3	+1.3	+ 5	− 5
6	−2.3	+0.2	+ 1	− 4
7	−2.2	−0.1	− 9	− 4
8	−2.1	−0.4	+ 1	− 7
9	−2.0	+0.4	+ 15	− 1
10	−1.9	+0.5	+ 13	+ 10
11	−1.9	+0.4	+ 5	− 17
12	−1.9	+0.3	− 11	+ 3
13	−1.9	−0.5	− 12	− 12
14	−1.9	−0.6	+ 13	+ 5
15	−1.8	−0.6	− 6	− 26
16	−1.9	+0.7	+ 4	− 3
17	−1.8	+0.7	+ 23	+ 4
18	−1.6	−0.1	0	− 13
19	−1.6	−0.1	− 8	− 10
20	−1.3	−0.8	− 21	− 5
21	−1.2	0.0	+ 1	− 12
22	−1.1	−0.4	+ 1	− 3
23	−1.1	−0.8	− 3	− 2
24	−1.1	+1.1	+ 26	+ 4
25	−1.0	+0.1	− 2	+ 5
26	−1.0	+0.7	+ 18	+ 2
27	−0.9	+0.7	− 4	− 18
28	−0.9	0.0	− 2	− 14
29	−0.8	−0.1	− 1	− 2
30	−0.8	0.0	− 8	− 8
31	−0.7	0.0	− 17	− 6
32	−0.7	+0.3	+0.016	−0.002

TABLE I—*Continued*

No.	x	y	μ_a	μ_δ
33	−0′.7	+1′.2	+0″.033	−0″.012
34	−0.6	−0.5	− 1	− 11
35	−0.4	−0.7	− 14	− 6
36	−0.3	−0.2	− 6	− 13
37	−0.3	−0.4	− 18	− 10
38	−0.2	−0.8	+ 3	− 22
39	−0.2	−1.0	− 15	− 21
40	+0.2	−0.7	− 6	− 17
41	−0.1	+0.7	+ 19	− 18
42	+0.1	+0.7	+ 6	+ 1
43	0.0	−1.1	− 5	− 5
44	+0.1	−1.0	− 17	− 27
45	+0.3	−0.6	− 12	− 27
46	+0.2	−1.6	− 8	− 17
47	+0.4	−1.0	− 11	− 13
48	+0.5	−2.2	− 19	− 26
49	+0.5	−1.6	− 16	− 30
50	+0.6	−0.3	− 5	− 16
51	+0.6	−0.4	+ 4	− 5
52	+0.6	−0.6	− 15	− 20
53	+0.7	−0.9	− 5	− 25
54	+0.7	+0.5	+ 15	− 27
55	+0.6	+0.9	+ 31	− 1
56	+0.7	+1.0	+ 20	− 22
57	+0.7	+0.8	+ 23	− 30
58	+0.9	−1.5	− 4	− 24
59	+0.9	−1.5	− 12	− 29
60	+1.0	−1.5	+ 1	− 19
61	+0.9	−0.3	+ 4	− 23
62	+0.9	+0.1	+ 21	− 30
63	+1.0	+0.2	+ 25	− 24
64	+1.1	+0.5	+ 26	− 24
65	+1.2	−1.3	− 5	− 28
66	+1.3	−0.5	+ 2	− 33
67	+1.4	−0.4	+ 6	− 33
68	+1.5	+0.3	+ 36	− 18
69	+1.6	+0.2	+ 21	− 17
70	+1.6	−0.9	− 10	− 22
71	+1.8	−2.1	− 10	− 27
72	+1.7	−1.4	− 1	− 4
73	+1.8	−0.4	+ 11	− 30
74	+1.8	+0.1	+ 15	− 15
75	+1.8	+0.3	+ 21	− 6
76	+1.9	−0.2	+ 21	− 39
77	+2.0	−0.3	− 4	− 30
78	+2.0	+0.3	+ 14	− 27
79	+2.1	−1.1	− 12	− 23
80	+2.2	+0.5	+ 10	− 28
81	+2.3	+1.0	+ 48	− 25
82	+2.4	+1.0	+ 24	− 15
83	+2.5	+1.0	+ 18	− 18
84	+2.4	−0.4	− 6	− 28
85	+2.7	+0.1	+ 23	− 28
86	+2.8	+0.6	+0.018	−0.016

52a

PLATE II



INTERNAL MOTIONS IN MESSIER 63

The arrows indicate the direction and magnitude of the annual motions. Their scale (0."1) is indicated on the illustration. The scale of the nebula is 1 mm = 7."7. The comparison stars are inclosed in circles.

TABLE I—*Continued*

No.	x	y	μ_a	μ_δ
87	+2′.8	−0′.6	+0″.027	−0″.033
88	+3.0	−0.8	+ 24	− 29
89	+3.1	+1.5	+ 5	− 21
90	+3.0	−1.6	− 4	− 27
91	+3.3	−0.9	+ 26	− 18
92	+3.3	−0.6	− 2	− 28
93	+3.7	−1.3	+ 3	− 24
94	+3.7	−0.9	+ 8	− 24
95	+3.4	+1.5	+ 18	− 13
96	+5.3	+0.9	+ 14	− 23
97	−1.8	+3.9	0	+ 9
98	−2.0	+0.7	+0.015	−0.004

Subtracting these values from the annual motions given in Table I, we derive the internal motions, which are given in the second and third columns of Table II. These motions are plotted in Plate II. For the comparison stars, which are surrounded by circles, the motions of Table I are used. The scale of the motions is indicated in the lower left-hand corner. The arrows represent the motions during an interval of about 1500 years. From the results derived for the spirals previously measured, it seems that the internal motions represent a motion along the spiral arms and outward.[1] In order to derive the stream and transverse components of the motions in N.G.C. 5055, we must take into account the fact that this nebula is much inclined to the plane perpendicular to the line of sight.

It is therefore necessary to correct the motions accordingly; in order to do this Mr. Sinclair Smith has by means of two cylindrical lenses constructed a photograph of Plate II, showing the nebula as a circular object, which it presumably is. From this photograph have been read the stream and transverse components given in the fourth and fifth columns of Table II. For points Nos. 89, 95, 96, and 97, in whose neighborhood the streamers could not be traced, this was impossible and they have been omitted in the discussion. The result is: mean stream motion, +0″.019±0″.001; mean trans-

[1] *Mt. Wilson Contr.*, No. 243; *Astrophysical Journal*, 56, 208, 1922.

TABLE II

Annual Internal Motions

No.	μ_α	μ_δ	Stream	Transverse
1	−0.″017	+0.″022	+0.″042	−0.″004
2	− 24	+ 13	+ 28	+ 7
3	+ 5	+ 18	+ 38	− 2
4	+ 31	+ 21	+ 55	− 7
5	0	+ 10	+ 20	+ 8
6	− 4	+ 11	+ 22	+ 1
7	− 14	+ 11	+ 22	+ 9
8	− 4	+ 8	+ 17	− 4
9	+ 10	+ 14	+ 30	− 3
10	+ 8	+ 25	+ 44	− 9
11	0	− 2	− 5	− 1
12	− 16	+ 18	+ 26	+ 17
13	− 17	+ 3	+ 17	+ 7
14	+ 8	+ 20	+ 18	− 37
15	− 11	− 11	− 5	+ 24
16	− 1	+ 12	+ 16	+ 6
17	+ 18	+ 19	+ 41	0
18	− 5	+ 2	+ 5	+ 3
19	− 13	+ 5	+ 13	+ 5
20	− 26	+ 10	+ 30	+ 3
21	− 4	+ 3	+ 8	+ 3
22	− 4	+ 12	+ 14	− 15
23	− 8	+ 13	+ 14	− 19
24	+ 21	+ 19	+ 42	+ 19
25	− 7	+ 20	+ 36	0
26	+ 13	+ 17	+ 33	+ 8
27	− 9	− 3	− 13	+ 1
28	− 7	+ 1	+ 3	+ 7
29	− 6	+ 13	+ 24	− 2
30	− 13	+ 7	+ 18	+ 8
31	− 22	+ 9	+ 14	+ 20
32	+ 11	+ 13	+ 27	+ 4
33	+ 28	+ 3	+ 30	− 2
34	− 6	+ 4	+ 6	− 5
35	− 19	+ 9	+ 18	− 12
36	− 11	+ 2	+ 11	+ 2
37	− 23	+ 5	+ 24	− 1
38	− 2	− 7	+ 7	+ 12
39	− 20	− 6	+ 20	+ 12
40	− 11	− 2	+ 23	+ 3
41	+ 14	− 3	+ 14	− 5
42	+ 1	+ 16	+ 1	+ 32
43	− 10	+ 10	+ 9	− 16
44	− 22	− 12	+ 25	+ 16
45	− 17	− 12	+ 22	+ 22
46	− 13	− 2	+ 13	+ 6
47	− 16	+ 2	+ 16	− 4
48	− 24	− 11	+ 23	+ 24
49	− 21	− 15	+ 26	+ 27
50	− 10	− 1	+ 9	− 5
51	− 1	+ 10	− 16	− 8
52	− 20	− 5	+ 25	+ 3
53	−0.010	−0.010	+0.019	+0.015

TABLE II—*Continued*

No.	μ_α	μ_δ	Stream	Transverse
54	+0".010	−0".012	+0".018	−0".012
55	+ 26	+ 14	+ 17	+ 34
56	+ 15	− 7	+ 17	− 5
57	+ 18	− 15	+ 31	− 22
58	− 9	− 9	+ 13	+ 18
59	− 17	− 14	+ 25	+ 23
60	− 4	− 4	+ 6	+ 5
61	− 1	− 8	+ 16	+ 1
62	+ 16	− 15	+ 24	+ 16
63	+ 20	− 9	+ 18	+ 15
64	+ 21	− 9	+ 23	+ 5
65	− 10	− 13	+ 21	+ 19
66	− 3	− 18	+ 31	+ 14
67	+ 1	− 18	+ 32	+ 7
68	+ 31	− 3	+ 16	+ 27
69	+ 16	− 2	+ 6	+ 15
70	− 15	− 7	+ 20	− 5
71	− 15	− 12	+ 22	+ 19
72	− 6	+ 11	− 12	− 16
73	+ 6	− 15	+ 27	+ 8
74	+ 10	0	0	+ 11
75	+ 16	+ 9	+ 3	+ 19
76	+ 16	− 24	+ 41	+ 16
77	− 9	− 15	+ 30	− 7
78	+ 9	− 12	+ 19	− 1
79	− 17	− 8	+ 25	− 5
80	+ 5	− 13	+ 15	− 15
81	+ 43	− 10	+ 45	− 1
82	+ 19	0	+ 18	− 7
83	+ 13	− 3	+ 15	− 1
84	− 11	− 13	+ 22	− 18
85	+ 18	− 13	+ 26	+ 8
86	+ 13	− 1	+ 11	+ 9
87	+ 22	− 18	+ 31	+ 17
88	+ 19	− 14	+ 24	+ 18
89	0	− 6	....	....
90	− 9	− 12	+ 22	+ 8
91	+ 21	− 3	+ 3	+ 21
92	− 7	− 13	+ 21	− 11
93	− 2	− 9	+ 17	+ 1
94	+ 3	− 9	+ 9	+ 4
95	+ 13	+ 2	....	....
96	+ 9	− 8	....	....
97	− 5	+ 24	....	....
98	+0.010	+0.011	+0.023	−0.002

verse motion, +0".004±0".001. The change in these components with distance (r) from the center is given in Table III.

These results are analogous to those found for M 33, 51, 81, 94, 101, and N.G.C. 2403. Since N.G.C. 5055 has many nearly star-

like points and the quality of the plates is very good, and since the interval between the photographs is longer than for any of the other

TABLE III

r	Stream	Transverse	n
0′–1′	+0″016	+0″004	11
1–2	+0.018	+0.005	36
2–3	+0.020	+0.002	32
3–4	+0.022	+0.006	12
4–5	+0.021	+0.015	3

spirals measured on plates taken with the 60-inch reflector, the results give a strong corroboration of the internal motions in these objects.

MOUNT WILSON OBSERVATORY
October 1922

A PARTIAL EXPLANATION, BY WAVE-LENGTHS, OF THE K-TERM IN THE B-TYPES. II

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ABSTRACT

K-term of radial velocity for Class B stars.—In a way similar to that in which the K-term had previously been found to be in part due to erroneous normal wave-lengths for the oxygen and nitrogen lines, it is shown to be also in part due to erroneous normal wave-lengths for the silicon lines 4552, 4567, and 4575 and for the helium blend 4713. On the basis of new wave-lengths for oxygen and nitrogen, silicon, and helium 4713, the total reduction in the K-term would amount to about 2 km for classes B0 to B2, 1 km for B3, and 0.3 km for B5 to B8.

Fundamental radial velocities in Class B.—A discussion of the causes or reality of the K-term involves first of all a *discussion* of fundamental radial velocities. Radial velocities of the Class B stars contain the inherent weakness that some of the lines which were most extensively employed are double, with unequal components, and therefore not suited for fundamental work. For such work the single lines, principally those of oxygen, nitrogen, and silicon, will have to be relied upon, and these require, as a preliminary, additional laboratory study.

No one realizes better than the radial velocity observer himself the misleading nature of the precision attained in radial velocities as derived from the internal agreement in the determinations. A precision of a tenth of a kilometer, in the absolute sense, is illusory. In a note in the *Astrophysical Journal* in 1910 Professor Frost raised the question as to whether we really know the radial velocity of any star in the heavens to the nearest kilometer. In view of the great advances in this line of work made at the Lick, Yerkes, and other observatories, the above statement seemed ultra-conservative. However, we now know of large systematic errors pervading all radial velocities, amounting in classes B and M to four times the limit of 1 km postulated by Frost.

The origin of these systematic errors, now commonly known as the K-term, is not well understood, though various possible causes, including the Einstein effect, have been proposed. It has been shown[1] that they are at least in part due to inaccuracies in the laboratory values of the wave-lengths which were employed. Recent laboratory wave-lengths for twenty oxygen and nitrogen lines occurring in B-type stars, are systematically 0.063 A longer than the adopted normals. This corresponds to a change of

[1] *Astrophysical Journal*, 55, 361, 1922.

−4.2 km per second in the radial velocities derived from them. A discussion of the available portions of the data on which the K-term is based, indicated that the new wave-lengths for this one group of lines alone would reduce the K-term by about 0.3 km for the entire B class and by about 0.8 km for classes B0 to B2.

Besides showing that an additional portion of the K-term is due to inaccurate normal wave-lengths, I wish to point out what seems, at least for the present, a more or less inherent weakness in the radial velocities of the B-types and to emphasize especially the need of further laboratory study of the lines employed.

For the silicon and helium lines new laboratory wave-lengths are available. Table I gives the recent and the more important of

TABLE I

LABORATORY WAVE-LENGTHS OF SILICON AND CORRESPONDING CHANGES IN THE RADIAL VELOCITIES

EXNER & HASCHEK* (USED BY F. & A.)		FROST & BROWN†		CROOKES‡ (NEARLY PURE Si)		SAWYER & PATON§ (VACUUM SPARK)	
λ	Change in v_s	λ	Change in v_s	λ	Change in v_s	λ (Accuracy 0.1±)	Change in v_s
	km		km		km		km
4552.75	0	4552.64	+7.2	4552.841	− 6.0	4552.74	+0.7
4567.95	0	4567.90	+3.3	4568.123	−11.4	4567.82	+8.5
4574.9	0	4574.79	+7.2	4574.823	+ 5.0	4574.81	+5.9
Mean......	0	....	+5.9	....	− 4.1	....	+5.0

* *Astrophysical Journal*, **12**, 49, 1900.
† *Ibid.*, **22**, 159, 1905.
‡ *Proceedings of the Royal Society*, A, **90**, 512, 1914.
§ *Physical Review*, **19**, 256, 1922.

the earlier laboratory wave-lengths of silicon. Sawyer and Paton claim an accuracy of only about 0.1 A, which is insufficient for our purposes. Crookes, emphasizing the necessity of employing samples of silicon of the highest purity, aimed to attain a high accuracy. If we apply the wave-lengths of Crookes—with the reservation that these results are not final, as will be shown below—to the available radial velocity data[1] we obtain the changes shown in the columns "Si (Crookes)" in Table II.

For helium, only the lines at 4713 are appreciably changed in wave-length. Merrill's[2] values, reduced to the Rowland

[1] *Publications of the Yerkes Observatory*, **2**, 143, 1904.

[2] *Bureau of Standards Bulletin*, **14**, 162, 1917.

system,[1] are 4713.325, intensity 3, and 4713.548, intensity 1. Combined in the usual manner according to intensities the blend (.381) is 0.073 A longer than the normal value which was employed. Radial velocities obtained from this line are thus changed systematically by −4.6 km per second. The actual changes in stellar radial velocities are illustrated in the columns "4713 (Merrill)" in Table II.

TABLE II

CHANGES IN RADIAL VELOCITY DUE TO NEW WAVE-LENGTHS AND WEIGHTED PERCENTAGES OF LINES AFFECTED. MEASURES BY FROST AND ADAMS

Star	Type	O. & N. (Clark)		Si (Crookes)		4713 (Merrill)		Total	
		Change in Radial Velocity	Per Cent of Lines	Change in Radial Velocity	Per Cent of Lines	Change in Radial Velocity	Per Cent of Lines	Change in Radial Velocity	Per Cent of Lines
		km		km		km		km	
ε Ori......	B	−0.1	4	−1.1	13	−0.5	11	−1.7	28
ζ Ori......	B	−0.2	11	....	0	....	0	−0.2	11
κ Ori......	B	−0.2	14	−1.8	26	....	0	−2.0	40
ζ Per......	B1	−0.7	32	−1.8	33	−0.2	4	−2.7	69
β CMa.....	B1	−1.0	43	−2.1	33	....	0	−3.1	76
ε CMa.....	B1	−0.7	32	−1.9	33	....	0	−2.6	65
γ Peg......	B2	−0.2	20	−2.0	28	−0.0	1	−2.2	49
ζ Cas......	B2	−0.1	7	−1.6	26	−0.1	2	−1.8	35
γ Ori......	B2	−0.4	11	−1.7	20	−0.2	5	−2.3	36
102 Her......	B2	0.0	5	−2.0	30	....	0	−2.0	35
ι Her......	B3	....	0	−1.1	16	−0.5	10	−1.6	26
η Lyr......	B3	0.0	1	−0.6	8	....	0	−0.6	9
ε Cas......	B5	....	0	....	0	....	0	....	0
τ Her......	B5	....	0	....	0	....	0	....	0
ζ Dra......	B5	−0.1	4	....	0	....	0	−0.1	4
67 Oph.....	B5	....	0	−0.9	18	−0.3	7	−1.2	25
ε Del......	B5	0.0	6	....	0	....	0	0.0	6
β Ori......	B8	....	0	−0.2	2	−0.1	3	−0.3	5
λ Crv......	B8	....	0	....	0	....	0	....	0

The total changes in the radial velocities, due to the new wave-lengths of oxygen and nitrogen, silicon, and 4713 are given in the second last column of Table II. If these wave-lengths prove to be essentially correct systematically, they will eliminate actually about one-half of the K-term in classes B0 to B3, and less in classes B5 to B8.

The laboratory results for silicon are not at all satisfactory. They differ widely from each other, both systematically and inter-

[1] The curve in *Astrophysical Journal*, 41, 347, 1915, is satisfactory for present purposes.

nally. Table III gives a comparison of the laboratory wave-lengths (see Table I) with the corresponding stellar wave-lengths. As the systematic error of the latter cannot at present be safely approximated, this comparison is not competent to furnish a decision in regard to the absolute systematic errors of the different laboratory results. The third items in column 1 of Table III constitute the systematic adjustments of the individual sets of laboratory wave-lengths to the observed stellar wave-lengths, and thus readily yield the mutual systematic laboratory discordances. The total range of 0.153 A is equivalent to a systematic difference of 10 km per second in radial velocity. In so far as the stellar wave-lengths may serve as a guide, the wave-lengths of Exner and Haschek have

TABLE III

SHOWING THE INTERNAL AGREEMENT OF THE LABORATORY WAVE-LENGTHS OF SILICON WITH THE WAVE-LENGTHS IN STARS

$\lambda_{\text{Laboratory}} - \lambda_{\text{Stars}} - \text{Syst. Dif.}$	4552.762*	5467.967*	4574.918*
$\lambda_{\text{Exner and Haschek}} - \lambda_{\text{Stars}} + .016$..........	+.004	−.001	−.002
$\lambda_{\text{Frost and Brown}} - \lambda_{\text{Stars}} + .106$..........	−0.16	+.039	−.022
$\lambda_{\text{Crookes}} - \lambda_{\text{Stars}} - .047$..........	+.032	+.109	−.142
$\lambda_{\text{Sawyer and Paton}} - \lambda_{\text{Stars}} + .092$..........	+.070	−.055	−.016

* These are the observed wave-lengths in stars. *Publications of the Astronomical and Astrophysical Society of America*, **2**, 72, 1911.

apparently exceedingly small accidental errors. This is, however, largely fortuitous as Exner and Haschek published their wave-lengths to only the nearest tenth or hundredth of an angstrom. The accidental errors for the measures of Frost and Brown appear to be moderate, while for the measures of Sawyer and Paton and especially of Crookes they seem to be large.

The lines which are available in Class B stars in the region of spectrum generally employed with three prisms are given in Table IV. It will be noticed that of the lines which were most extensively employed, the carbon line 4267, magnesium 4481, and the helium lines 4472 and 4713 are double with unequal components, and therefore not suited for primary radial velocity work. Hγ also is not entirely satisfactory. For the helium line 4438 results in stars are somewhat erratic. For silicon the laboratory wave-lengths are erratic and mutually discordant. Except for the helium

line 4388, the oxygen and nitrogen lines alone seem free from serious objection, and these would give practically no K-term.

Speculation on the probable causes of the K-term cannot be profitable so long as we have not eliminated the possibility that this term has been artificially introduced by the use of erroneous normal wave-lengths. In other words the reality of the K-term is inseparably connected with the problem of determining fundamental

TABLE IV

LINES IN B-TYPE STARS

Carbon	Hydrogen	Magnesium	Helium	Silicon (Crookes)	Oxygen and Nitrogen (Clark)	Metallic Lines and Blends
4267.301 Double: separation 0.26 A; intensities 1 and 2	4340.634 Double; separation 0.012 A in lab. spectrum. Intensities 7 and 10. Also, enh. Bismuth at 0.7	(4352.083) Rarely measured in stars 4481.400 Double; Fowler gives: .297 2 .495 1 Blended according to intensities, .363	4388.100 inten. 3 4437.718 inten. 1. Results in stars erratic 4471.676 Double; separation 0.21 A; inten. 6 and 1 4713.308 Double; separation 0.22 A; inten. 3 and 1; Bureau of Standards blend is 0.381	4552.841 4568.123 4574.823 Lab. values are erratic. Susceptible to slight impurities. Spark terminals oxidize rapidly in air. Further laboratory study required	4317.327 19.814 45.738 48.246 49.602 51.443 67.074 4415.057 17.143 4447.205 4591.158 96.365 4621.582 30.729 39.043 42.006 49.327 51.032 61.829 4676.426	Measurable only in late B-types

radial velocities. Table IV brings out well the seemingly inherent difficulties and the urgent need for additional laboratory study to make possible determinations of fundamental radial velocity for the Class B stars. As is well known, blends of unequal components are subject to apparent shifts depending upon the quality of the spectrograms. For a few of the brighter stars the double lines of carbon, magnesium, and helium can probably be resolved and the individual components employed by the use of sufficiently high

dispersion. It would be possible to determine empirically for each spectrograph, by means of laboratory and stellar spectra of the same and higher dispersion, close approximations to the true normal wave-lengths of the blends for each of several standard conditions of intensity, contrast, and grain of the individual regions on the spectrograms. Although such methods give promise of improving our radial velocities in a systematic sense, they can give only an approximation to really fundamental results. Possible differences in the relative intensities of the components, from star to star or from type to type, would produce apparent shifts of the blends and thus introduce systematic errors in the radial velocities. For fundamental results the principal reliance must apparently be placed on the single lines, those of oxygen and nitrogen, of silicon—when the laboratory discordances shall have been removed—of the helium line 4388 (and perhaps 4437), and of the metals when available. On account of its unusual importance in these spectra, it is to be hoped that the use of $H\gamma$ may also prove feasible. Although this line is double, the separation of the components is only about 0.012 A, and the enhanced bismuth companion may be absent or inappreciable in types B5 and earlier.

For purposes of fundamental radial velocities it is highly desirable to have at least one, and preferably two, additional laboratory determinations of wave-length for the oxygen and nitrogen lines. More urgent, however, is the thorough study of the spectrum of silicon in order that the large systematic and internal discordances may be removed. The large relative discordances between the individual lines in the laboratory determinations are apparently not duplicated in stellar spectra, the internal agreement from star to star and from type to type being fairly good. The work of Crookes and of Sawyer and Paton should be repeated with silicon of high purity and with the spark terminals in air and in a vacuum. As the physical conditions in the stars and in the laboratory spark are probably very dissimilar, it would be well to vary the laboratory conditions in whatever way this may be possible.

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Reviews

Light, A Textbook for Students Who Have Had One Year of Physics. By H. M. Reese. Columbia: Missouri Book Company, 1921. Pp. 295. Figs. 139. $3.50.

Those who are responsible for giving the second courses in light in the universities have long felt the need for a more suitable text. There are many books available, but the majority present far too much material for the time which is allowed. The English texts, particularly, assume a much more general acquaintance with mathematics, even where the calculus is expressly excluded, than our junior students commonly possess. A book of the character of this one is, therefore, very welcome. In some 285 pages, it covers all the ordinary topics in sufficient detail so that the phenomena become familiar and are fitted into their places in the theory.

The book starts with a chapter on the measurement of the velocity of light, where the well-known experiments are covered in slightly more detail than in the ordinary sophomore textbook. It is to be regretted that improvements Michelson introduced in the Foucault-Fizeau method are not described even in principle, in spite of the reviewer's experience that very few junior students have understood them when they have been presented.

Chapter ii inverts the usual order to consider dispersion by a prism before discussing deviation in any detail. This is for the purpose of considering color phenomenon in a general way. It is used to lead into a discussion of the construction and theories of color perception of the eye. The reason for thus taking the eye away from the other optical instruments, and color away from considerations of wave-length, is apparently to make the early chapters more of general, diffuse discussion and therefore superficially easier. The gains seem to the reviewer to be somewhat doubtful.

The debate on "ether" vs. "space-properties" is handled reasonably, but it would seem as if the student's energy and time might be more profitably employed, since the discussion is, after all, so barren. The use of two names for the same thing only invites trouble for the beginner. The introduction of the electro-magnetic idea, early in this discussion, with frequent reference to the reaction between charges and such waves, would prepare the student for what of explanation there is in the modern view.

The subjects of formation of images and of optical instruments, generally, are treated almost entirely by the use of wave-fronts instead of rays. This serves the very desirable purpose of keeping before the reader the fact that the subject being discussed is wave-motion; but the geometry is not so simple, and in the complex cases the figures, at least, revert to the ray construction. Like many other complementary treatments, each has superior elements and any reasonably full discussion has to make use of both. There is also no doubt that here the emphasis on the wave treatment is the better introduction. The treatment in this whole section is not so complete as in many of the introductory textbooks. No suggestion is given of any more complete treatment: for instance, nodal points or nodal planes are never mentioned; the wave-front argument neglects the optical center, which is not mentioned; the geometrical construction for the formation of images is used in some of the figures, but the method receives no consideration; no emphasis is placed on the exceedingly approximate character of such formulae as are given—the whole tone of the treatment leaves an impression of finality, so that not even the brightest student would guess that whole volumes are available in which the present treatment is relegated to the scrap pile. One bright spot is the carrying through of a numerical calculation for an achromatic lens.

The discussion of simple diffraction phenomena leading up to resolving power, of simple cases of interference, of the formulation and application of the equations of simple harmonic motion, and of polarization phenomena is much more adequate than the earlier chapters and is clear and direct. It errs, if at all, on the side of being too elementary.

It is pleasant to find the electro-magnetic theory discussed here in place of the elastic-solid theory, which one still finds in some semi-elementary books. The discussion is sufficient to connect the electrical properties and light properties. X-rays must be classified now as electromagnetic waves, and, as such, their nearest relative is what has been classed loosely as light. Optical books should discuss their properties. The discussion here is adequate for the purpose as it revolves about the relation of X-rays to light.

In a general way, there is here but little more intellectual food than in many of the fuller texts on general physics. There are reasons for not desiring to use a general text for a special course on one of its subdivisions. All these general texts contain far more than the sophomore student is able to assimilate. All that can be done in a second course is to present the same material in a little different dress, with some slight extensions. Mr. Reese's book serves this purpose excellently.

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REPORT OF THE COMMITTEE ON STELLAR CLASSIFICATION ADOPTED BY THE INTERNATIONAL ASTRONOMICAL UNION AT THE MEETING AT ROME, MAY 10, 1922[1]

The Committee[2] on Classification of Stellar Spectra finds its work greatly simplified by the fact that a definite plan of classification—the Harvard System—has already been adopted by international agreement, and has been used in extensive works, such as the *Henry Draper Catalogue*, which will be of value for a long time. Its duty, therefore, is not to make radical alterations in this system, such as changing the significance of the existing letters, or substituting numbers for them, but to suggest such modifications and extensions of the existing notation as may increase its usefulness to students of astrophysics.

a) With increasing knowledge of stellar spectra, it is desirable to have additional symbols which may be used to designate many of those characteristics which were formerly dismissed as "peculiarities." The new notation may in some cases appear complicated, but it should be remembered that its use is permissive, and not mandatory. The older notation remains complete in itself, and for many purposes is all that

[1] Reprinted, by permission, from *Transactions of the International Astronomical Union*, 1, 1922. London: Imperial College Bookstall, S.W. 7. Pp. 247+viii. Price, 15 shillings.

[2] The committee consisted of W. S. Adams, chairman, Miss Annie J. Cannon, Messrs. R. H. Curtiss, A. Fowler, A. de Gramont, M. Hamy, H. F. Newall, J. S. Plaskett, and H. N. Russell. Messrs. N. Bohr and M. N. Saha were added to the committee.

will be required, while the new makes it possible to compress into small compass information which would otherwise require voluminous notes.

b) The additional distinctions, like the original classification, are based on the line and band absorption and emission. The distribution of intensity on the continuous background is of great importance; but it is not considered, for three reasons: (1) It is already known that the correlation between the intensity distribution and the spectral class is so far from perfect that two practically independent symbols are required to express them. (2) All instruments record the line-spectrum in substantially the same fashion—barring the effects of different dispersion and resolving power. This is not the case for the intensity of the background, which may be influenced by such factors as absorption in the prisms. Great care is, therefore, necessary in interpreting the results obtained from spectrograms. (3) This distribution is intimately related to the color index and to the energy distribution in the spectrum. The problem involves spectroscopy, photometry, and the measurement of heat radiation.

c) It is evidently ultimately desirable that each spectral class should be precisely defined, both by verbal description and by means of standard stars. The present moment, however, does not appear opportune for doing more in this way than has already been done at Harvard. The exact relation of some of the more unusual classes of spectra remain to be investigated. Moreover, the recent application of the theory of ionization to stellar spectra promises a greater insight into the physical meaning of the spectral lines. There is good reason to believe that the relative intensities of lines emitted by the neutral atoms depend almost exclusively upon the temperature, while the intensities of enhanced lines, relative to those of the former groups, will vary also with the pressure or density of the star's atmosphere. It seems, therefore, desirable to postpone the *precise* definition of the characteristics of each spectral class until the relations of these two types of lines to the temperatures and absolute magnitudes of the star have been further considered.

I. GUIDING PRINCIPLES

a) The classification should describe the *spectra*, not the stars; that is, it should be based solely on what can be seen in the spectrum of a given star, when observed at a suitable time and with appropriate instruments.

b) The Draper Classification, or "Harvard System," which has already been adopted internationally, should be the basis on which any

further extensions should be built. Classification on other and different systems should be abandoned permanently.

c) Designations at present forming part of the Draper Classification should either be retained with the old meaning or abandoned entirely. No attempt should be made to retain the symbol but alter the meaning.

d) The capital letters, B, A, F, etc., *standing alone*, should be used to describe a spectrum only in those cases, when, on account of poor photographs or for other reasons, nothing more than the general character of the spectrum can be determined.

Similarly B–A, K–M may denote spectra which lie somewhere between the classes mentioned when more precise specifications cannot be made.

e) In cases of great uncertainty, Secchi's types may be employed. These may be characterized briefly as follows:

		Harvard Types
Type I.	Predominant hydrogen lines	Oe, B, A, F, F5
Type II.	Prominent metallic lines	F8, G, K, K5
Type III.	Titanium oxide bands	M
Type IV.	Carbon bands	N, R
Type V.	Bright Wolf-Rayet lines	Oa, Ob, Oc, Od

It is not recommended that this grouping be used in statistical work on account of the extreme heterogeneity of Type I. Stars of spectra Oe to B5 should, in any case, be separated from the other stars of Type I.

f) The decimal system of classification (G2, etc.) should be used in all cases in which it is established that a continuous spectral sequence exists. Such combinations will hereafter be called the "main symbol."

g) The present notation by means of small letters appended to the capital letter (as Oa) should be retained in cases in which it has not been clearly established that a continuous and unique sequence exists.

h) The terms "early" and "late" are very convenient. It is well, however, to emphasize that they denote positions early or late in the spectral sequence O—B—A—F—G—K—M, without any necessary connection whatever with an early or late stage of physical evolution. The terms "hotter" and "cooler," "whiter" or "redder" sometimes cover the same characteristics, but describe the star rather than the spectrum.

i) Composite spectra should be denoted by the sign + connecting the two superposed types (as K0+B9), or by two separate lines as is done in the *Henry Draper Catalogue*. The latter is convenient for purposes of tabulation.

j) The spectra of variable stars should normally be recorded as at maximum brightness. When the spectrum varies continuously, as in Cepheid variables, it may be recorded, for example, as F7 to G4; when discontinuously as in eclipsing variables, like a composite spectrum, e.g., Ao+K or $\left\{\begin{matrix}\text{Ao}\\ \text{K}\end{matrix}\right.$

k) Additional notation should be devised to describe as many as may be convenient of those spectral characteristics which are known to be common to any considerable number of stars. Such notations should be simple to print, and convey as much information as may be practicable in a small compass.

II. SPECIFIC EXAMPLES OF THESE RULES

I.*c*. Although the evidence of color index suggests strongly that what is now called K5 is really much nearer to M than to K, it is inadmissible to change the designation of this type to K8. The meaning of the symbols in the existing catalogues must be preserved.

Again, the notation Md has been found to include cases in which the "underlying spectrum" is not of Class M at all. This symbol may, therefore, advantageously be dropped.

I.*f*. and *g*. (i) Although the evidence is strong that Oe and Oe5 represent spectra immediately preceding Bo, the spectral sequence in Classes Oa to Od is not yet certainly worked out. Hence it seems desirable to retain for the present the existing notation for all these classes.

(ii) Again, Ma, Mb, and Mc clearly form a sequence, running on continuously from K5. It is suggested that they be called in future Mo, M3, and M8, the second interval on the decimal classification being taken wider than the other, because it appears to correspond to a greater difference in the spectra.

(iii) Similarly, Na and Nb may be called in future No and N3. The relations of the very red stars called Nc by Miss Cannon are not yet clear enough to justify giving their spectra a decimal notation.

Spectra intermediate between K and R (should such occur) will have to be called K5R.

(iv) The question of the notation for the spectra of gaseous nebulae should be deferred until further investigations have been made. Attention should be called, however, to the strong desirability of classifying the spectra of the nebula and the nucleus separately whenever possible.

III. NEW SPECTRAL CLASSES

a) Novae

It seems very desirable to have some less cumbersome description than at present of the successive stages through which the spectrum of a Nova ordinarily passes. The letter Q has been used for spectra of this type (*Harvard Annals,* 28) and should be retained. In view of the uncertainties of progression in sequence, the differences between various novae and the intermingling of spectral characteristics, the decimal notation cannot at present be used. The following quite provisional system of notation is suggested, not for immediate use, but as a basis of discussion, in the anticipation that it will be revised and improved in the future. The use of the letter "e" to indicate the presence of bright lines in other classes of spectra precludes its use in this connection. Bright bands due to hydrogen appear always to be present, except in Class Qz, and are not referred to specifically.

Qa Absorption spectrum of faint lines. Bright bands inconspicuous.

Qb Absorption spectrum of stronger lines, mainly enhanced metallic, many of which are double. Bright bands stronger.

Qc Absorption spectrum of enhanced lines, oxygen, nitrogen, helium and associated elements. Bright lines of all of these elements.

Qu Broad nebulous emission bands near 3480, 4515, and 4640A, accompanied at times by one at 4379A. The spectrum appears usually to occur in conjunction with other typical forms which it may modify through the extinction of some of their characteristic radiations, particularly 3445 and 4686A.

Qx Bright bands due to enhanced lines, oxygen, nitrogen, and helium. Absorption lines faint.

Qy Bright nebular bands in addition to preceding.

Qz Bright nebular bands. Weak Wolf-Rayet bands.

The stage in which Wolf-Rayet bands are strong in addition to the nebular bands may be indicated by Qz5O. (Capital letter O.)

Combinations of any of these spectra may be indicated by combinations of the letters. Thus Qbc would indicate that spectrum Qb was more prominent than Qc, and Qcb would show the reverse.

b) A New Class of Red Stars

Miss Cannon has found that a number of long-period variables and some other red stars, such as π_1 Gruis, R Cygni, and R Andromedae, have underlying spectra which are similar to one another, but do not

resemble Class M, and Dr. Merrill has shown from slit spectrograms that they do not resemble Classes R or N. Their spectrum in the region λ 4500 to λ 4700 is of a most complicated nature, and appears to consist of both absorption and emission lines, with absorption bands present at about λ 4650 and λ 6470. Most of the stars belonging to this type are long-period variables, and show bright hydrogen lines. The type may represent a third branch of the main spectral sequence, cognate with the K5-M and R-N branches. The letter S is suggested for this type.

IV. NOTATION FOR PECULIARITIES

a) Characteristics Connected with Absolute Magnitude

When observations of sufficient delicacy have been made, the spectroscopic absolute magnitude, upon the Mount Wilson system, provides a detailed description of these peculiarities. The more conspicuous differences, however, which can be detected upon inspection by an observer once familiar with them, suffice to divide the spectra into three groups. These may be denoted by small letters placed before the main symbol. They are defined as follows:

1. *Very Bright Stars*

All lines normally are narrow and sharp. In spectra later than Ao, the hydrogen lines are abnormally strong for the general spectral type. So also are the enhanced lines. λ 4227 *Ca* is abnormally weak compared with $H\gamma$ or λ 4215 *Sr*+.[1]

This set of characteristics, which is very conspicuous, is shown by Miss Maury's "c-stars," by the Cepheid variables, the stars called "pseudo-Cepheids" by Adams and Joy, and by practically all other stars of exceptionally great luminosity including some cases, like ζ^1 Scorpii and β Orionis, of type B.

It is suggested that these be denoted by the prefix *c* and be called "c-stars," leaving the term Cepheids for variables.

2. *Bright Stars*

In these spectra the enhanced lines are fairly strong. λ 4227 has a moderate intensity for the spectral type. The low-temperature lines, such as λλ 4435 *Ca*, and 4454 *Ca*, are relatively weak. The hydrogen lines are strong.

In class F, λλ 4077 *Sr*+, 4215 *Sr*+, 4290 *Ti*+ are strong.

[1] The sign "+" following a chemical symbol indicates that the line in question is an enhanced line, originating in ionized (positively charged) atoms.

In classes G, K, and M, λλ 4077 and 4215 are strong.

These are ordinary giant stars, and their spectra may be denoted by the prefix *g*.

3. *Faint Stars*

In these spectra λ 4227 is strong for the class, and λλ 4435, 4454 *Ca* and 4535 *Ti* are strong. The enhanced lines are weak.

In Class F, λλ 4077, 4215 and 4290 are weak.

In Class M, λ 4607 *Sr* is relatively strong, the hydrogen lines are weak.

These spectra may be denoted by the prefix *d* (dwarf stars).

For spectra earlier than B0 these differences disappear, so far as is known, all the stars being bright. The difference between ordinary giant and dwarf stars does not become prominent until spectra later than F0 are reached. The prefix *c*, therefore, will not at present be used with spectra earlier than B0, or *g* and *d* with spectra earlier than F0.

In the selection of standard or typical spectra, it is recommended that the fundamental types shall be giant stars, preferably of absolute magnitude about 0 or +1 (except in Class B, where they must necessarily be brighter). Stars showing either the c-star or dwarf characteristics should be selected as auxiliary standards.

There is additional reason for this, because in the classification of the spectra of dwarf stars developed at Mount Wilson, the decimal subdivisions are of much less unequal value than in the Harvard classification for giant stars. A dwarf K5 is much more nearly midway between K0 and M0 than a giant K5. It may be remarked incidentally that in plotting the physical data, it is well to plot K2 and K5 for giant stars as if they were K5 and K8, respectively. This should not, however, be done in the case of the dwarfs.

b) Width of Lines

Exceptionally narrow lines usually appear to be associated with the "c" peculiarity, and are already accounted for. Spectra showing all the lines unusually wide or diffuse on good plates may be denoted by "n," following Rowland's designation for diffuse (nebulous) lines in the solar spectrum. Similarly the letter "s" may be used to qualify spectra in which the lines are sharp, but in which the "c" characteristics (such as abnormally strong hydrogen and enhanced lines) are not present.

c) Double Lines

Spectra in which the lines are double rather than reversed belong to spectroscopic binaries with both components bright. In such cases the notation for composite spectra should be used.

The spectra of spectroscopic binaries in which only one component is visible present, as such, no peculiarity. Variable radial velocity cannot be detected by mere inspection of a single spectrum. They should not receive any special notation.

d) Stationary Lines

On the other hand so-called "stationary" (H) and (K) lines (so far found only in types O and B) are recognizable by their appearance, being very much sharper than the other lines in the spectrum. The symbol "k" suggesting the (K) line is proposed for this class of stars. The same symbol may be used to describe spectra in which the (D) lines and possibly others show the same characteristics. To illustrate, δ Orionis would be designated Bonk.

e) Bright Lines

It is suggested that spectra showing bright lines be denoted by the letter "e" (emission), except in classes where bright lines are normally present (as in O, P, and Q).[1]

In certain classes, most of the bright line stars have fairly definite characteristics, and may be considered as forming a recognized group. In these spectral classes, a spectrum which has emission lines differing considerably from those of the recognized group may be denoted by "ep." Cases in which the bright lines are conspicuously "reversed" (with a dark center) may be denoted by "er." These recognized groups are as follows:

1. *Classes A and B*

The hydrogen series may be thought of as composed of the normal Class B absorption lines, increasing in strength from Hα toward the violet, each having superposed upon it, in a nearly symmetrical position, one of a series of bright lines which decrease in strength from Hα toward the violet. Frequently in the hydrogen series (with a one-prism slit-spectrograph) one or more of the lines Hβ to Hϵ will show *both* emission and absorption components, the lines toward the violet showing no emission, and those toward the red showing no absorption; but in some cases Hα is the only distinct emission line. The emission lines often are double, and in some cases may appear as bright edges to a well-defined absorption line. Fainter emission lines (enhanced metallic) may or may not be present.

[1] The notation Pe, already in use, is still admissible, as it denotes a type of bright line spectrum.

Slight lack of symmetry of the combined bright and dark hydrogen lines need not require the suffix "p."

If desired, the letters α, β, etc., may be appended to indicate which is the last visible bright line of the hydrogen series.

In certain stars, such as P Cygni, the lines consist of a bright emission line with an absorption line bounding it on the violet side. These spectra may be denoted by "eq," the "q" recalling their similarity to certain stages of the spectra of Novae.

2. *Classes M, N, R, and S*

In all these the bright lines are usually of the type associated with long-period variability. The hydrogen lines are bright and narrow, with no absorption components visible. In Class M, $H\gamma$ is usually bright and conspicuous, while $H\delta$ is still stronger. $H\epsilon$ is absent or extremely weak, $H\beta$ and $H\alpha$ are weak or absent, except when the underlying spectrum is of an early division of Class M, or possibly a late one of Class K, in which case $H\beta$, $H\gamma$, and $H\delta$ may have approximately equal intensities. Weaker bright lines, especially at $\lambda\lambda$ 3905, 4138, 4178, and 4202, are not unusual. In stars of Class S, which show bright hydrogen lines, $H\beta$ is much stronger than $H\gamma$ or $H\delta$, though $H\gamma$ is usually conspicuous.

As these differences in the intensity of the hydrogen lines appear to be closely correlated with the underlying spectrum, no additional notation to describe them appears to be called for at present.

Consideration of a notation for the spectra of long-period variables near minimum should be deferred till they have been more fully investigated. It is already known that in *o* Ceti at least, the spectrum at minimum is very different from that at maximum, or from any other known spectrum.

3. *Classes F, G, and K*

Bright line spectra are here so rare that no characteristic group can be recognized. The mere addition of the suffix "e" may serve for the few known cases, which are far from similar to one another.

f) Variable Spectra

In certain cases the spectrum of a star varies. Although this is a peculiarity of the star and not of the spectrum, it is obviously desirable to refer to it in catalogues of spectra. This may be done either by giving the limiting types between which the spectrum varies, as for example, cF5–cG2, or simply by annexing the letter "v" to the spectral designa-

tion. The symbol "ev" will denote variability in emission lines such as has been observed in many B-type stars. In such cases, as in those in which the letter "p" is used, details should be given in notes.

g) OTHER PECULIARITIES

The letter "p" should be used to denote miscellaneous peculiarities, not sufficiently frequent or important to justify individual designations. It may be suggested that this should be understood to qualify the symbol immediately preceding. Thus B2pe would denote a star of Class B2, with peculiarities in the absorption spectrum, and emission lines of the normal type, while B2ep would denote a star with peculiar emission lines.

Similarly, A2pn would denote a peculiar A2 spectrum in which all the lines were wide; A2np one in which the lines were widened in some peculiar fashion.

The same principle may be extended to other symbols. Thus F8ne would denote an F8 spectrum with all lines wide, and with bright lines; F8en, one in which the dark lines were normal, but the bright lines abnormally wide.

V. NOTATION OF INDIVIDUAL LINES

a) The Fraunhofer letters for certain lines are so well established that it does not seem desirable to abandon them. Following a suggestion of the Committee on Notations, these lines should be denoted by letters in brackets or parentheses. Those symbols which it is proposed to preserve are (A), (a), (B), (α), (D), (b), (G), (H), and (K). It is worthy of note that each of these with the exception of the last two, denotes a group of lines having a common origin. Since the lines of the (E) group do not all belong to the same element, there is no reason for retaining this symbol.

For the hydrogen lines, the notation Hα, Hβ, etc., should be adopted.

d) In giving the origin of a line, the chemical symbol of the element should be printed in italics, as recommended by the Committee on Notations. When the line is known to originate in an ionized atom, or shows other strong evidence of being an enhanced line, the chemical symbol should be followed by the sign +, in accordance with the usage now prevailing among physicists. Thus λ 4571 *Mg*, λ 4481 *Mg*+, λ 4045 *Fe*, λ 4233 *Fe*+.

The symbols "pq" are used to indicate peculiarities of a character suggestive of the spectrum of Novae.

The exclamation symbol "!" may be used as a modifier to indicate very marked degree in a phenomenon. Thus "e !" means that the

emission lines are exceptionally strong; "p !" that the peculiarities are remarkable.

Examination of the notes to Miss Cannon's classification of spectra in *Harvard Annals*, 28, 56, and 93, shows that the proposed notation will cover almost all the peculiarities which occur at all frequently, with certain exceptions. The most notable of these are spectra of Class A showing unusual strength of the silicon lines λλ 4128, 4131 (as in α Doradus), or of the strontium line λ 4077 (as in δ Normae). An examination of the proper motions indicates that stars showing these peculiarities are distinctly brighter than the average; but further study will be required before it can be determined with certainty whether these characteristics are associated with the "c" or "g" characters, or are independent of these, and to what degree they are connected with one another.

Pending such study, they may still be called A0p, A2p, etc., the question of a notation for their peculiarities being postponed.

Some of the more difficult objects, such as η Carinae and SS Cygni have been classified on a provisional basis to show the capabilities of the method.

VI. EXAMPLES OF THE PROPOSED NOTATION

c-Stars		*Giants*		*Dwarfs*	
ε Canis Majoris	cB1	γ Velorum	Oap	α Canis Minoris	dF5
β Orionis	cB8	ζ Puppis	Od	β Virginis	dF8
η Leonis	cA0	29 Can. Maj	Oe	The Sun	dG0
α Carinae	cF0	ι Orionis	Oe5	μ Herculis	dG5
α Persei	cF5	ε Orionis	B0	70 Ophiuchi Br.	dK0
α Ursae Minoris	cF8	γ Orionis	B2	70 Ophiuchi Ft.	dK4
ζ Geminorum	cG0	α Gruis	B5	61_1 Cygni	dK8
5 Lacertae (Boss 5804)	cK2	α Lyrae	A0	Lal. 21185	dM3
α Scorpii	cM0	β Leonis	A5	Barnard's Star	dM0
		θ Scorpii	gF0		
		ε Ceti	gF5		
		τ Persei	gG1		
		η Piscium	gG5		
		α Boötis	gK0		
		α Tauri	gK5		
		δ Virginis	gM0		
		β Pegasi	gM3		
		45 Arietis	gM8		
		B.D.+42°2811	R0		
		−3°1685	R5		
		19 Piscium	N0		
		+67°350	N2		
		VX Andromedae	Nc		

Nova Type

Qa	Nova Aquilae	1918 June 8–9
Qb	Nova Aquilae	June 10–13
Qc	Nova Aquilae	June 14–20
Qx	Nova Aquilae	June 21–July 1
Qy	Nova Aquilae	July 1, 1918–Oct., 1919
Qz	Nova Aquilae	1920–21
QzO	Nova Geminorum	February, 1914

Red Stars

π_1 Gruis	S
R Cygni	Se
R Geminorum	Se
R Andromedae	Se

Wide and Double Lines

δ Orionis	Bonk	
α Eridani	B5n	
α Leonis	B8n	
α Aquilae	A5n	
δ Geminorum	A7n	
S Antliae	A6n / A6n	or A6n+A6n
W Urs. Maj.	dF8n / dF8n	or dF8n+dF8n
β Scorpii	B1k / B1	or B1k+B1
μ Scorpii	B3 / B3	or B3+B3
β Aurigae	Ao / Ao	or Ao+Ao
α Aurigae	gGo / F5	or gGo+F5
V Puppis	B1 / B3	or B1+B3
o Leonis	cF5 / A	or cF5+A
U Sagittae	B8 / gG2	or B8+gG2

Bright Lines

γ	Cassiopeiae	Boe
η	Tauri	B5eα
β	Lyrae	cB8+B2nep
P	Cygni	B4eq
α	Cygni	cA2eα
RT	Serpentis	cFoe
RZ	Ophiuchi	cF8e
W	Virginis	cF7e
σ	Geminorum	gK1e
T	Tauri	gG5e
	Comp. α Gemin	dMo+dMoe
T	Aquarii	M1e
o	Ceti (max.)	M6e
o	Ceti (min.)	Mvep !
R	Leonis	M8e
T	Coronae Bor.	M5epn
R	Aquarii	M6e+P
28	Tauri (Pleione)	B8ev
φ	Persei	Bpevr
	+11°4673	Beqpv
η	Carinae	Qp !
SS	Cygni maximum	B8n !q
SS	Cygni minimum	Opq
R	Cor. Bor.	cF5ep
H.D.	42474	gMoep !

DISTRIBUTION OF THE VELOCITIES OF STARS OF SPECTRAL TYPE A[1]

By GUSTAF STRÖMBERG

ABSTRACT

Velocity-distribution of A-type stars.—The three velocity-components for 332 stars of spectral types B7 to F2 have been computed from their proper motions, radial velocities, and spectroscopic parallaxes. The method used is that previously used for types F and M. The study of the distribution of the velocities reveals the existence of three well-defined groups, the *Central group*, the *Ursa Major group*, and the *Taurus group*, whose relative proportions among the stars studied are 69, 23, and 8 per cent. The Central group, which can be identified with the stream O of Halm and the antapex stream of Eddington, is shown to have an ellipsoidal distribution. Among the A stars, Kapteyn's First and Second streams can be identified with the Taurus and the Ursa Major groups, respectively. The elements of ellipsoidal distribution-functions for all three groups are given in Table III.

The Sun's velocity referred to the A stars is somewhat smaller than that found for other spectral types, on account of the presence of a large proportion of stars whose motion is nearly the same as that of the Ursa Major group.

A study of the velocities of stars of spectral types F to M was recently published in this journal.[2] A similar investigation of the stars of type A has been made possible by a recent application of the spectroscopic method by Adams and Joy[3] to the determination of the absolute magnitudes of stars of this type. In the latter paper the spectroscopic parallaxes of about 500 stars of spectral types B7 to F2 are given, all of known proper motion.[4] For 332 of these, radial velocities have been determined at the Lick, Mount Wilson, and the Dominion Astrophysical observatories, and hence it was possible to calculate the three velocity components.

These velocity components, relative to the sun and in the equatorial system of co-ordinates, were computed by the formulae:

$$\left.\begin{aligned}
\xi &= V\cos\alpha\cos\delta - \frac{k}{\pi}\,(\mu_\alpha\sin\alpha\cos\delta + \mu_\delta\cos\alpha\sin\delta)\\
\eta &= V\sin\alpha\cos\delta + \frac{k}{\pi}\,(\mu_\alpha\cos\alpha\cos\delta - \mu_\delta\sin\alpha\sin\delta)\\
\delta &= V\sin\delta \qquad\qquad + \frac{k}{\pi}\,\mu_\delta\cos\delta
\end{aligned}\right\} \qquad (1)$$

[1] *Contributions from the Mount Wilson Observatory*, No. 257.

[2] *Mt. Wilson Contr.*, No. 245; *Astrophysical Journal*, **56**, 265, 1922.

[3] *Mt. Wilson Contr.*, No. 244; *Astrophysical Journal*, **56**, 242, 1922.

[4] In the following the stars belonging to this range of spectral class are all called "A stars."

where V is the radial velocity relative to the sun, μ_α and μ_δ, the proper motions in right ascension and declination expressed in seconds of arc per year, π, the parallax, and where $k = 4.737$ km/sec.

For the study of the distribution of the velocities, these components were referred to the galactic system of co-ordinates and to the origin used in the study of the velocity distribution for stars of later types. The reduction to this origin assumes a solar motion of 20 km toward the apex $A_0 = 270°$, $D_0 = +30°$. The equations for finding the galactic co-ordinates, referred to the adopted origin, from the equatorial co-ordinates are:

$$\left.\begin{aligned} x &= +0.185\,\xi - 0.983\,\eta + 17.0 \\ y &= +0.449\,\xi + 0.084\,\eta + 0.889\,\zeta + 7.4 \\ z &= -0.874\,\xi - 0.164\,\eta + 0.457\,\zeta + 7.4 \end{aligned}\right\} \quad (2)$$

The stars were divided into two groups, according to their absolute magnitudes. Various data for these groups are given in Table I.

TABLE I

	Group I	Group II
Absolute Magnitude	≤ 1.1	≥ 1.2
Spectrum	B7 to A3	A1 to F2
c_1	40 km	40 km
c_2	30 km	30 km
c_3	30 km	30 km
Number of stars	159	173
Number within limits	152	167
Number of stars belonging to the Taurus group	0	17
Number of stars belonging to the Ursa Major group	10	7

The quantities c are the limits of x, y, and z, respectively, within which the distribution of the velocities has been represented by a three-dimensional cosine-series according to the method devised in *Contribution* No. 245.[1] The numbers of recognized members of the Taurus and the Ursa Major groups are given at the end of the table.

THE SOLAR MOTION

If we take the algebraic means of the equatorial velocity components, referred to the sun as origin, we obtain the velocity of the

[1] *Astrophysical Journal*, **56**, 265, 1922.

center of mass of the group studied relative to the sun. The opposite vector is the velocity of the sun relative to this "centroid." The velocity of the sun defined in this way is given in Table II for the two groups in question, both in equatorial and galactic coordinates.

TABLE II

Groups	Number	A_0	D_0	V_0	L_0	B_0
				km		
I	159	258°4	+38°0	12.0	29°2	+33°1
II	173	262.0	+19.2	14.0	9.7	+24.6
Omitting Taurus stars	156	256.5	+29.4	11.5	18.8	+32.7
All	332	260.7	+27.4	12.8	17.9	+28.7
Omitting Taurus stars	315	257.4	+33.8	11.7	24.1	+33.0

A remarkable fact is the small velocity obtained for the sun relative to these stars. As will be shown later, this is due to the circumstance that there exists a large number of stars whose motion is nearly the same as that of the Ursa Major group.

DISTRIBUTION OF VELOCITIES

An unbiased idea of the general character of the distribution of the velocities can be secured from a representation of the velocity-vectors with the aid of a trigonometric series in three dimensions. The formulae for this computation are all given in *Contribution* No. 245. No stars were omitted in this study except those outside the limits given in Table I. In figures 1 and 2 are shown the results of the synthesis of the trigonometric series in the form of closed curves. These are the intersections of the equi-frequential surfaces with the galactic plane. The numbers attached to the curves are the corresponding "densities," i.e., the number of velocity-vectors which terminate in a cube of unit size whose center is situated on the curve. These numbers are comparable with those given in the previous paper, and refer to a total of 1000 stars and a unit volume of 1000 cu. km. In order to emphasize that we are dealing with the frequency-function itself, and not with its integrated values, we might rather say that the density numbers are reduced to a total of 1,000,000 stars and a unit volume of 1 cu. km. The dots in the

diagrams represent velocities of individual stars, so that the vector from the origin to a dot is the projection on the xy- or the xz-plane of the velocity-vector referred to the same origin. In the xy-plane only those velocities are indicated whose z-components are numerically smaller than 10 km, and similarly for the xz-plane. The recognized members of the Taurus group are indicated by crosses,

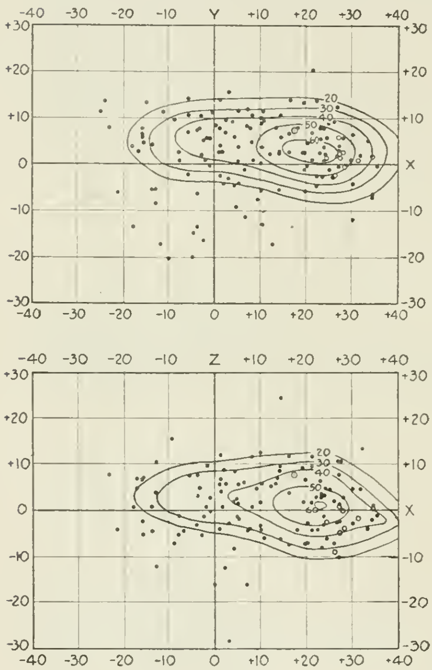


Fig. 1. $M \leq 1.1$

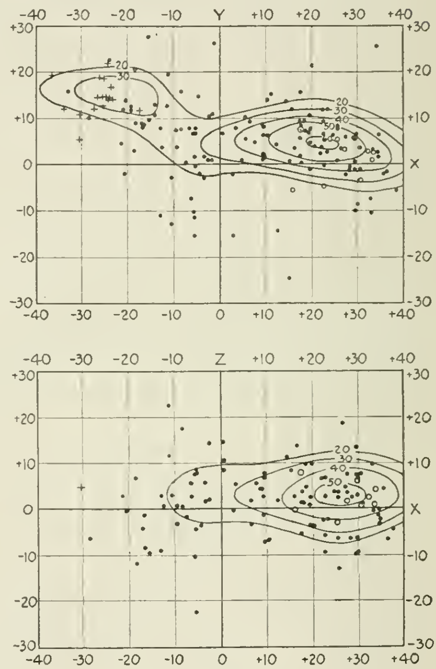


Fig. 2. $M \geq 1.2$

those of the Ursa Major group by open circles. The velocity of the sun is indicated by the symbol ⊙.

An inspection of these diagrams gives us immediately some valuable information. In the case of the brighter A stars (Fig. 1), the point of highest density, the condensation point, does not fall at the origin, and not even near the center of the outer equi-density curves, but at a point which corresponds nearly with the group-motion of the Ursa Major stream. The most frequent velocity among these A stars is thus nearly the same as that of the Ursa Major group. The same holds for the group of fainter A stars

(Fig. 2); but in addition we have a group of stars forming a second condensation point in the second quadrant of the xy-plane. Most of the stars belonging to this last group are recognized members of the Taurus group. Actually we can distinguish three groups of stars, the Central, which is the most numerous, the Ursa Major group, and the Taurus group. As is well known, there are other moving groups, but their members are few in number, and the real existence of the groups can be proved only by some distinguishing characteristic other than apparent parallelism of motion, as, for instance, an actual grouping of the stars in the sky.

Having found these qualitative data, an attempt was made to determine the three frequency-functions the sum of which constitutes the actual distribution function. All the A stars except those belonging to the Taurus group were studied together. It was thus sufficient to determine the constants of two frequency-functions, corresponding to the Central group and to the Ursa Major group. Both of these frequency-functions were supposed to be ellipsoidal and the method used in *Contribution* No. 245 could accordingly be applied. The equations of condition thus take the form:

$$\left.\begin{aligned} dn=\mu dN_1+(1-\mu)dN_2+(N_1-N_2)d\mu+n_c\kappa=n_0-n_c \\ dN_1=\frac{dN_1}{dh_1}\Delta h_1+\frac{dN_1}{dh_2}\Delta h_2+\frac{dN_1}{dh_3}\Delta h_3 \\ +\frac{dN_1}{dx_0}\Delta x_0+\frac{dN_1}{dy_0}\Delta y_0+\frac{dN_1}{dz_0}\Delta z_0 \\ +\frac{dN_1}{dL_1}L_1+\frac{dN_1}{dB_1}B_1+\frac{dN_1}{dB_2}B_2 \end{aligned}\right\} \quad (3)$$

with a similar expression for dN_2. The expressions for

$$N_1, \quad \frac{dN_1}{dh_1}, \quad \frac{dN_1}{dh_2}, \quad \text{etc.,}$$

are given in the paper cited. Further,

$$n_c=\mu N_1+(1-\mu)N_2$$

where $\mu/(1-\mu)$ is the proportion of stars belonging to the Central group relative to that of the Ursa Major group. There are in all

twenty unknowns, but by choosing the volumes in which the number of velocity points are counted in such a way that they are symmetrically placed relatively to the assumed center of one of the groups, we can make several of the product-sums equal to zero, which facilitates the solution considerably. The results of this computation are given in Table III, in galactic co-ordinates relative to the adopted origin, and in equatorial co-ordinates relative to the sun.

The position of the center of the Central group agrees well with that found for the center of the velocity-ellipsoids for later-type stars in the study of those objects. The large uncertainty found for the major axis of the ellipsoid belonging to this group, and for the number μ which determines the relative proportion of stars belonging to the Central and to the Ursa Major groups, is due to the fact that it is possible to represent the distribution (excluding the stars belonging to the Taurus group) by a single frequency-function of the ellipsoidal type. In this case we have $\mu=1$. But there is no doubt that a much better representation could be secured by using the sum of two frequency-functions, one representing the distribution in the Central group, the other that of the Ursa Major group.

The position of the center of the Ursa Major group as given in Table III does not agree exactly with the values ordinarily given for the convergence-point and for the velocity of this group. The values recently published by Rasmuson[1] are:

$$\alpha=307\overset{\circ}{.}6\,, \qquad \delta=-39\overset{\circ}{.}9\,, \qquad v=18.6 \text{ km}$$

while our values are

$$\alpha=306\overset{\circ}{.}5\,, \qquad \delta=-25\overset{\circ}{.}9\,, \qquad v=13.0 \text{ km}$$

As can be seen from the diagrams, the recognized members of the Ursa Major group, which are marked by open circles, lie somewhat to the right of the point of highest condensation. We cannot distinguish the Ursa Major group, as defined by its recognized members, from that revealed by the general excess of stars moving in this direction, and we are thus justified in using the name of the Ursa Major group to designate this whole group of stars.

[1] *Meddelanden från Lunds Observatorium,* Serie II, No. 26.

The orientation of the velocity-ellipsoid for the Central group agrees well with that for the giant stars of later types, and the dispersion along the axis is similar to that for the F stars of high luminosity. The velocity-ellipsoid of the Ursa Major group presents the remarkable feature of a prolate ellipsoid with the longest axis pointing toward the pole of the galaxy. The spread, both for the Central group and for the Ursa Major group, is to some extent increased by the effect of errors in the proper motions, radial velocities, and distances; but the general features of the frequency-functions cannot be affected much by this circumstance.

The algebraic means of the velocity components of twelve recognized members of the Taurus group are given in the fourth column of Table III, together with the equatorial co-ordinates relative to the sun of the velocity-vector of the group. These may be compared with Rasmuson's elements,[1] which are nearly identical with Boss's original values, viz.,

$$\alpha = 93\overset{\circ}{.}2\,, \qquad \delta = +7\overset{\circ}{.}0\,, \qquad v = 40.5 \text{ km}$$

In the last column are given the elements for the Taurus group, including eight additional stars, which, as judged from their motions, are members of the group, although their position in the sky differs from that of the recognized members.

The constants of the ellipsoidal distribution of the Taurus group are given in the last column of Table III and are computed directly from the moments. The spread is small and can well be accounted for by accidental errors in the quantities involved. It is significant, however, that the Taurus group among the A stars has a small spread and is at the same time largely limited to the constellation of Taurus, whereas the Taurus group as defined by the F stars has a larger spread and its members are scattered all over the sky. (See remark on the Taurus group in *Contribution* No. 245.) This indicates that the central part of the Taurus group is actually situated in the constellation of Taurus, but there are members, presumably of smaller mass, which, on account of larger "peculiar" motions, have been scattered over a larger space.

[1] *Loc. cit.*

The Central group found among the A stars can be identified with the antapex-stream of Eddington[1] and with the O stream of Halm.[2] As mentioned before, it can also be identified with the ellipsoidal group to which all giant stars of later types belong,

TABLE III

CONSTANTS OF THE DISTRIBUTION-FUNCTIONS

		Central Group	Ursa Major Group	Taurus Group	
		km/sec	km/sec	km/sec	km/sec
Position of center	X_0...	+5.0±6.1	+27.5± 1.3	− 26.4	− 25.1
	Y_0...	+1.8±0.8	+ 4.7± 0.9	+ 14.3	+ 13.5
	Z_0...	+2.4±0.5	+ 0.3± 1.3	+ 5.4	+ 5.6
	α_0...	91°.7± 6°.0	306°.5± 6°.1	105°.1	114°.9
	δ_0...	−31°.0±13°.1	− 25°.9± 5°.6	+ 6.8	+ 6.1
	V...	14.2± 5.2 km	13.0± 1.3 km	44.0km	42.6km
Orientation of ellipsoids and dispersion along principal axis	a....	$21.4\begin{cases}+36.1\\-\ 8.2\end{cases}$ km	$9.69\begin{cases}+3.91\\-2.17\end{cases}$ km		5.2km
	L_1...	158° ±22°	146°.0		324°
	B_1...	− 6.8± 4.2	+ 83.2±16.8		+ 21
	α_1...	84.0	184.1		244
	δ_1...	+ 16.1	+ 30.8		− 18
	b....	8.54± 0.66km	5.52±0.34km		4.3km
	L_2...	65° ±20°			48°
	B_2...	− 22 ± 9			− 16
	α_2...	342			323
	δ_2...	+ 34.9			+ 31
	c....	6.73± 0.59km	5.48±0.94 km		2.1km
	L_3...	84°. ±12°			104°
	B_3...	+67 ± 8			+ 63
	α_3...	194			180
	δ_3...	+50			+ 53
Relative proportion..		75.0±23.4 per cent	25.0±23.4 per cent		
Number of stars used in determining con stants............		307		17	25

except 20 per cent of the fainter F stars, which belong to the Taurus group. This Central group, to which the vast majority of the apparently bright stars seem to belong, has a fixed group-motion and orientation of the axis of its velocity-ellipsoid, but the dispersion increases regularly from the A stars to the M stars, while the prolateness of the velocity-ellipsoid decreases.

[1] *Monthly Notices*, 71, 40, 1910. [2] *Ibid.*, p. 610, 1911.

So far as the A stars are concerned, the Taurus and Ursa Major groups can also be identified with Kapteyn's First and Second streams. Since this class of stars is very numerous, the division into two streams is very marked among the stars in general. The ellipsoidal nature of the frequency-distribution for the Central group of types A to G, when studied from the standpoint of proper motion or radial velocity, could not be distinguished from two separate streams. When Kapteyn made his original discovery and determination of stream-motion, the frequency-distribution of proper motions for different regions in a zone at the same angular distance from the sun's apex was determined. The mean of these distributions was formed and the deviations from this mean distribution were determined for each region of the zone. Since the mean distribution corresponds closely with that of the Central group, this group, as has been pointed out to the author by Mr. Seares, was almost entirely eliminated from Kapteyn's discussion when the differences were studied. The distribution described by Kapteyn was that remaining after a central, random (spherical) distribution had been subtracted. This Central group of stars, appearing later in the investigations of Halm and Eddington, is now found to be the most numerous and to have an ellipsoidal distribution in agreement with the theory of Schwarzschild and Charlier.

Mount Wilson Observatory
October 1922

VACUUM ARC FOR OBTAINING SPECTRA EXTENDING FROM VISIBLE LIGHT TO SOFT X-RAYS

By H. NAGAOKA AND Y. SUGIURA

ABSTRACT

New vacuum arc with salted carbon cathode.—A simple type of arc is described which, with 2–5.5 amperes at from 80–150 volts, is an intense source of very sharp lines. The cathode is a carbon rod covered with a layer of the oxides of Ba and Sr and sheathed up to the end in a fused silica tube. The anode consists of the metal or salt to be tested placed in another silica tube. These tubes are fastened in necks of the vacuum flask by means of rubber stoppers made tight with Khotinsky cement and water-cooled. Light passes to the spectrograph through a quartz window or right-angled prism. After exhausting the flask to a low pressure, a glow discharge is started by means of an induction coil and then the electrodes are rapidly switched to the poles of the 500 volt D.C. generator. Low melting-point metals tend to deposit on the window, but spectra of all metals including tungsten may be obtained. The oxide layer on the cathode must be renewed occasionally. The cyanogen bands appear with a weak current, but vanish with a strong current. The *sharpness of the lines* was tested with a Fabry-Perot etalon interferometer and more distinct interference fringes were obtained than in the case of lines from a Pfund arc (Plates III*a* and III*b*). As a *standard source* the vacuum arc has other obvious advantages, since it eliminates various uncertain effects of pressure and electric field.

Vacuum arc as source of soft X-rays.—When the potential was raised to 1500 volts, using the rectified current from a transformer, rays were emitted which affected a photographic plate covered with one to three aluminum foils, placed within the flask. The minimum wave-length is computed, from the quantum relation, to be 10 A. No effect was obtained at 80 volts.

For purposes of spectroscopy, it is always desirable to obtain a light-source of sufficient purity and intensity, so that the lines can be examined by means of interferometers. Sparks and arcs in air are unfit for the purpose; the light excited by cathode bombardment fulfils this condition, but the arrangement is generally complex, and the duration of light-emission not long enough to obtain a good impression on a photographic plate. To overcome some of the inconveniences attending the handling of a limed cathode and to increase the effective duration of light, the lamp here to be described was designed.

One[1] of us has already solved a portion of the problem by replacing an oxide-covered platinum strip in the Wehnelt cathode by a thin carbon plate. With this change, the Wehnelt cathode was

[1] *Astrophysical Journal*, 53, 323, 1921.

somewhat simplified, but the tedious process of excitation remained unimproved. In the present form the same idea was followed, but great simplification in removing the material and in making the oxide-covered cathode is introduced.

The essential part of the apparatus consists of a three-necked flask of about a liter capacity, as shown in the figure. An India-rubber stopper *a* has a carbon rod *c* inserted into it, and the rod is sheathed in a closely fitting silica glass tube: the end of the rod is first dipped in a solution of barium and strontium nitrate, and heated in a Bunsen burner, so that the surface of the carbon appears white, being covered with the oxides of barium and strontium. A thick copper wire *b* is screwed into the rod to have good contact, and passes through an India-rubber stopper which is covered with Khotinsky cement to make it air-tight. The substance to be tested is put into another fused silica tube *d*, which fits into an India-rubber stopper *e*, the substance and the electrode being electrically connected within the tube by means of a carbon rod. On inserting the stopper, the neck is sealed by Khotinsky cement. The end of the neck *f* is ground plane, and a right-angled prism *p* of quartz cemented on it. The air within the flask is evacuated by means of a Gaede mercury pump, having a condensation pump in series to obtain a good vacuum. On running the pumps for a few minutes, the manometer indicates that the flask is pretty well exhausted; the carbon rod is then connected to the cathode and the substance to be tested to the anode of a small induction coil, by which the carbon pole is covered with "Glimmlicht." The electrodes are then rapidly switched to the poles of a 500-volt D.C. generator, and the current kept at proper strength, which

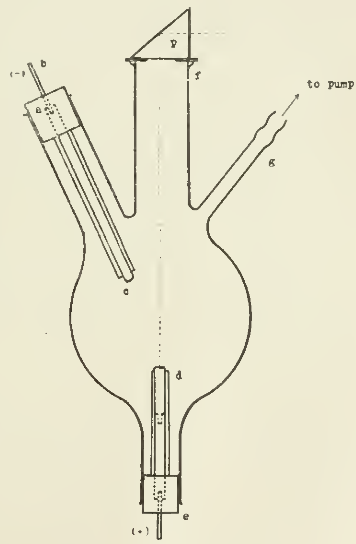


FIG. 1

varies with the nature of the substance to be tested. With most metals, 2 or 3 amperes with terminal voltage of 80 to 90 volts are sufficient to give intense light, the fluctuation of the current being scarcely noticeable. With the spectrograph, described below, a few seconds suffice to obtain spectrograms of the violet region. To obtain a very intense source of light, the current is pushed to 5.5 amperes; the evaporation of the anode material is very rapid, the terminal voltage being kept at 150 volts. Care is taken not to make the tip of the carbon interfere with the passage of light into the total reflecting prism *p*. The cathode carbon is heated red hot, but its light is faint compared with that from the anode, and does not mix in the spectrum to be examined. With mercury, the stopper is dispensed with; the neck is made smaller, and mercury poured into it, the electrode of iron wire being cemented at the end of the neck. It was at first thought that the light would play round the surface of the mercury; this takes place only when the current is strong, otherwise a steady bright bundle of light, resembling a tuft, is seen protruding from the surface. To prevent heating and the softening of the cement, two-thirds of the flask is immersed in a trough of running water. Salts may also be introduced into the silica tube *d* and their spectra investigated. So long as the current is not broken, the light continues to be emitted till the greater part of the anode material is evaporated by the bombardment. When the oxide is evaporated, it is difficult to relight the lamp. With certain substances as zinc, the wall is covered with fine particles of evaporated material, and the fogging is so great that a bright source of light is hardly visible through the wall. There is, however, no difficulty in obtaining spectra of highly refractory metals, such as tungsten. With easily evaporating substances as mercury, the face of the prism is slightly dimmed by the vapor rising from the anode; to prevent this it is advisable to introduce diaphragms in the neck below the prism.

By making the capacity of the flask bigger, we may perhaps dispense with the water-cooling. The necks *a* and *e* may be made somewhat conical and ground so as to fit tightly into glass stoppers, which have the electrodes fused into them. A wire of platinum

substitute, used in electric lamps, will form suitable external electrodes: they will secure good contact with glass, as the thermal expansion of both substances may be made equal.

As will be easily seen from the construction of the flask exciting the light, the arrangement of the anode and cathode is exactly analogous to a Coolidge X-ray tube; if the difference of potential between the electrodes be made sufficiently high, X-rays must be emitted. The current of a small step-up transformer was rectified by means of a kenotron and passed into the flask, the potential difference between the electrodes being about 1500 volts. Three strips of thin aluminium foil were placed one over the other, so that they formed layers of three different thicknesses, and laid on a photographic plate. It was wrapped with black paper and placed in the neck of the flask under p, and exposed to the action of rays for 15 minutes. Zinc, silver, and mercury were used as anodes; on developing, the plate was blackened each time, showing the gradations according to the thickness of the aluminium foil. By placing a wing of a butterfly on the plate, and photographing in the same manner, the veins distinctly screened the photographic action. Calculated from quantum relation, the wave-length in these experiments is a little shorter than 10 A. The same experiment was repeated with a terminal voltage of 80 volts, but the plate showed no sign of blackening, although intense visible light was excited under it. By applying sufficiently high voltage, it would be possible to obtain hard X-rays, if good vacuum be maintained.

Since there was no vacuum spectrograph at hand, the Schumann region was not examined, but the emission of soft X-rays sufficiently proves that the emitted light extends to the extreme ultra-violet. In the present apparatus, it is evident that by applying proper terminal voltage, we can excite light waves ranging from the visible to the ultra-violet, and extending even to soft X-rays.

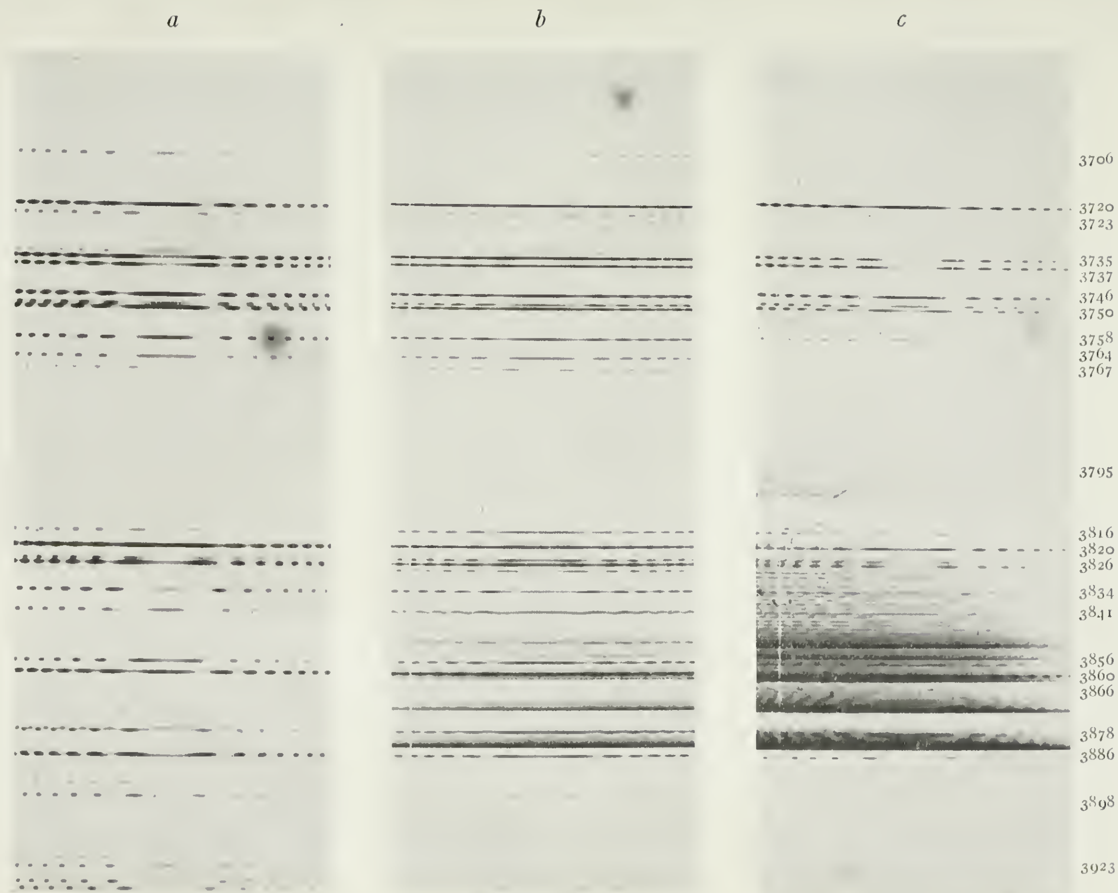
The purity of the spectrum was tested in the following manner. An arc of electrolytic pure iron was excited and compared with the Pfund arc of ordinary iron, by passing the light through a Fabry-Perot interferometer of etalon type, the silvered plates being separated by a fused silica cylinder 1.01625 cm long. The interference

rings were projected on the slit of a Hilger quartz spectrograph on a Littrow mounting; it was provided with a quartz lens (aperture 7 cm, focal length 170 cm) and a quartz prism (refracting angle 30°, 9.8×5.7 cm face) silvered on the back, and so arranged that the spectrum could be photographed nearly at minimum deviation. The Pfund arc was 6 cm long at 500 volts. The middle portion of the arc was used. The fringes from the vacuum arc were sharply defined as shown in Plate III*a*, which represents a small portion of the violet region. With the Pfund arc the fringes are not sharp (Plate III*b*); they are bridged and the mean points difficult to measure. Usually the length of the Pfund arc is 6 mm or so at 200 volts, but the length of the arc can be extended to 6 cm working at 500 volts, making the fringes more distinct, although they are lacking in definition compared with those of the vacuum arc, as the inspection of the figures will show. By increasing the thickness of the air plate in the Fabry-Perot interferometer, they can be made more distinct apparently, but the same relative difference in fringes of these two sources of light will persist.

For accurate comparison of wave-lengths, the interference method is generally applied; for this purpose it is necessary to obtain sharp fringes, whose position can be measured with great accuracy. The preference for the vacuum arc instead of that in air requires no further comment.

The curious characteristic of the vacuum arc here described is the appearance of cyanogen bands for weak electronic current. With iron as the anode, most of the iron lines appear, but those lying in the region of the cyanogen bands are mostly obliterated so that it is difficult to trace their faint presence. The bands are, however, very strong and show their characteristics to a high degree. They gradually disappear as the current is increased; when a certain strength is exceeded, they vanish and only the iron lines remain. Plate III*c* shows the latter stage. The bands are all capable of high interference, and can be examined by a Fabry-Perot interferometer or a Lummer-Gehrcke plate. The Pfund arc shows the same phenomenon, especially at high voltage. In the vacuum arc, the bands are probably excited by electronic bombardment of nitrogen, to

PLATE III



Vacuum arc 5 amps. Pfund arc, 3 amps. Vacuum arc, 3 amps.

which they are due, the presence of cyanogen not being necessary according to Runge. Owing to dissemination of carbon particles within the flask, there is, however, great chance for the production of cyanogen within the apparatus.

The study of the structure of the iron lines and of the cyanogen bands obtained by the present method will be reserved for a future communication.

INSTITUTE OF PHYSICAL AND CHEMICAL RESEARCH
TOKYO, JAPAN
August 2, 1922

A COMBINATION OF A CONCAVE GRATING WITH A LUMMER-GEHRCKE PLATE OR AN ECHELON GRATING FOR EXAMINING FINE STRUCTURE OF SPECTRAL LINES

By H. NAGAOKA AND T. MISHIMA

ABSTRACT

New method of studying the fine structure of spectrum lines; concave grating in series with a Lummer-Gehrcke plate or an echelon grating.—Because of the astigmatic property of the concave grating, fringes projected on to the slit parallel to the rulings are drawn out, and the components, if not too faint or too close together, are made more distinct. The slit should be wide enough to include two or three orders. In the case of the L.G. plate, photographs with two plates of different thickness will give the order numbers. This method should be useful in the study of the Stark and Zeeman effects and for the rough analysis of diffuse lines such as those from arcs and sparks in air. For a minute study of the structure, the method of crossed spectra is superior though the loss of intensity is considerable. Results obtained are illustrated by four spectrograms showing: Hg, λ 2536; the Hg triple, $\lambda\lambda$ 3663, 3655, 3650; Hg, λ 5461; and some iron lines from a Pfund arc.

Of the various methods of examining the structure of spectral lines, most accurate results are obtained by forming crossed spectra of lines with interferometers. The lines are, however, generally diffuse, and not sharp enough to apply the method, when the arc or spark is excited in air. Only by using vacuum arcs can we effectively examine the fine structure of spectral lines. Some means of overcoming this difficulty and of making the study of the structure easier is necessary, in order that we may be able to get some insight into the perturbations of electronic movement in atoms. The method which we shall now develop will be of some significance in investigations of similar nature, such as the study of the Zeeman or Stark effect.

Of the different interferometers which are used for examining the structure of lines, we may mention Michelson's echelon grating, the Lummer-Gehrcke plate, and the Fabry-Perot interferometer. All these instruments have the common defect that for using them the source of light must be pure; diffuse lines, although some difference in intensity may exist among the components, cannot be analyzed by such means. The fringes of interference are merely blurred images of lines without definite boundary. Such, for

example, is the case with the sodium lines from a Bunsen flame, or the cadmium lines excited by an electric spark in air. It is, however, well known that the concave grating, by virtue of its astigmatic property, can effectually analyze most of the lines, which are not resolvable by the interferometers above mentioned; this is especially the case in the study of the Zeeman effect of most of the light metals. This important characteristic can be taken advantage of in the investigation of the fine structure of lines, but owing to the low resolving power of most of the gratings, there is still some advantage in using the interferometers. The method which we shall describe consists in the combination of these two instruments, so that we may be able to utilize, on the one hand, the high resolving power of the interferometers, and on the other, the astigmatism of the concave grating.

The usual practice is to analyze the light roughly into its spectrum and examine one of its lines by means of an interferometer. In the present method the order is inverted. The light from a source is directly received on the slit of an interferometer, and the interference spectra thus obtained are projected on the slit of a concave grating. The slit is kept tolerably wide, so that two or three successive orders of interference spectra can be received on it. Great care is needed to bring the projected lines parallel to the rulings of the grating. The parallelism can be easily obtained by a direct observation on the visible part of the spectrum, for the lines appear very distinct when this condition is fulfilled.

The absence of the dust lines of the slit used in observations with the concave grating is a point which we can properly turn into an important application to the problem at hand. Owing to the astigmatic property of the spherical surface, a point is drawn out into a line, whose length is approximately given by $l \sin \theta \cos \theta$, where l is the length of the rulings, θ the angle of diffraction in the case of Rowland mounting. Thus the length of the slit is very much elongated from its natural length; consequently the distribution of complex lines projected on the slit is more distinct, and gives better definition to the appearance of the lines than when observed with interferometers only. The spectrum as given by a Lummer-Gehrcke plate consists of a number of straight lines; if we project

these lines on to the slit of a concave grating, they will, after diffraction, appear as distinct lines, the blurred edge of the component lines being wiped away just as the dust lines of the slit are eliminated. The complex lines, as observed in this manner, are not exactly similar to lines analyzed by the plate. Owing to the slight difference of wave-lengths among the components, the mutual position will be affected to a small extent by diffraction, and thus indicate minute differences in the interval between them. The appearance of the lines is made very distinct by this method of observation, so that we have no difficulty in delineating the separate components, which otherwise would have presented a diffuse appearance, especially when we use an arc at ordinary atmospheric pressure. Even with a vacuum arc, we may sometimes find, by the usual method of observation, no great difference from the condition in air, especially with light metals and metalloids.

Denote the angle between the line joining the slit with the center of the grating and the line passing through the center of the sphere by θ, and the angle of diffraction by θ', then we have the approximate relation

$$\epsilon(\sin\theta+\sin\theta')=\pm h\lambda$$

where ϵ is the interval between successive rulings, and h the order of the spectrum. In the present method, we have to use a tolerably large slit, in order that more than one of the spectra from the plate may appear projected on the slit. Denoting the small change of angle by $d\theta$ and $d\theta'$, we have the relation

$$\epsilon(\cos\theta d\theta+\cos\theta' d\theta')=\pm h d\lambda.$$

For constant wave-length $d\lambda=0$, and

$$\cos\theta d\theta=-\cos\theta' d\theta',$$

giving the relation between the consecutive spectra. For Rowland mounting

$$d\theta'=-\cos\theta d\theta,$$

and for Littrow mounting

$$d\theta'=-d\theta.$$

Since λ is nearly constant, we have the images of the components of luminous lines in inverse order at the point of observation from

those projected on the slit. From the constant of the grating, we can easily calculate the displacement of the line due to the small difference in the wave-lengths of the satellites from the principal; it is necessary to introduce this correction in exact measurements of the satellites. Another way is to take two photographs, such that the wave-length in one increases in the opposite direction from that on the other. The mean of these two readings will give the true value. This process can be easily effected by deflecting the plate in opposite directions and adjusting the interval so that it is the same in both cases between successive spectra. The correction due to this cause is generally small. Further, we notice the advantage of the Littrow mounting over that of Rowland, as is evident from the formula.

The great disadvantage of using the Lummer-Gehrcke plate lies in the difficulty of discriminating the order of the spectra to which the component belongs. There is no easy means of finding out the proper order except by crossing the spectra at right angles to each other; this entails a great deal of labor on the part of the experimenter, and what is worse there is great loss of intensity by this process. The method here given has some advantage over that of crossed spectra, as the question of the order of the spectra can be easily settled if we have two plates of different thickness. The advantage thus gained by the combination of the grating with the interferometers is not easy to estimate, since the study of some faint components is made possible by this means, and the discrimination as to the doubtful position of many lines with respect to the principal rendered possible without crossing. When there are faint components grouped together, the present method is still inferior to that of crossing, as will be shown farther on.

A quartz plate of thickness 4.529 mm made by Hilger was mounted vertically on a stand, and light of a mercury vacuum arc was made to fall on the plate after passing through a quartz lens and a Wollaston prism, so as to receive only the ordinary or extraordinary ray in the plate. The light which passed through the plate was projected on the slit of the grating by a quartz-fluorite lens; the slit was 2 to 3 mm wide; this was necessary as the spectra to be projected on the slit must be at least two or three in number, in order that the evaluation of the position of the satellites may

attain sufficient accuracy. Plate IV*a* shows the doublet structure of the mercury line 2536. The interval between successive spectra amounts to 0.0474 A, and that between the doublets is 0.0142 A, which coincides with the measurement of Miss L. Wilson[1] by a different process.

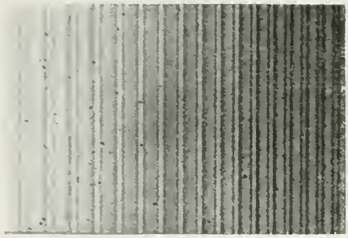
Experiments with an echelon spectroscope taught us that the structure of iron lines cannot be easily examined, and our knowledge of the fine structure of the numerous lines of that element is still vague. Recently we have found out that iron lines are resolvable by the L.G. plate, when analyzed by combining it with a concave grating. With the plate alone, the lines appear as strips with no definite boundary, but examined by interposing a concave grating, the lines become distinct; the slight blurring of the lines is probably due to the pole effect, as the focusing lens generally projects all parts of the arc on the plate. By taking the precaution to make only the central portion of the Pfund arc enter the plate, the lines become tolerably fine, so that, with a grating of 1.85-meters radius and 4-cm ruled spacing, and in the second-order spectrum of a Littrow mounting, the interval between the interference fringes belonging to the same line may be made as wide as 1 mm, and the interval can be measured with great accuracy. For $\lambda = 0.44\ \mu$, the difference of wave-length between successive spectra amounts to 0.17 A, equivalent to 1 mm on the photograph, and thus the measurement can be easily pushed to milliangstroms without incurring much error in the last figure. This point is of importance in the measurement of changes of wave-length of different elements in electric and magnetic fields.

Plate IV*b* shows three orders of interference spectra of iron lines. They are all simple. Of course different exposures are necessary for weak and strong lines; in the figure, strong lines are obliterated, but they come out distinctly by shortening the exposure.

With the echelon grating, the observation is made in the same manner as with the L.G. plate. On account of the use of glass plates in the construction of the grating, it is difficult to extend the investigation into the ultra-violet, but we were fortunate to photograph the triplet lines 3663, 3655, 3650, in the third-order spectrum, revealing the complex structure as shown in Plate IV*c*. The echelon

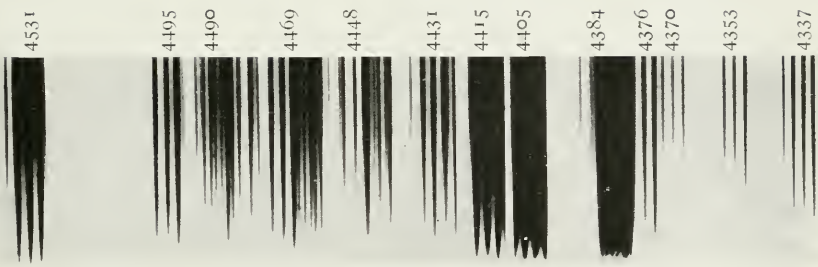
[1] *Astrophysical Journal*, 46, 340, 1917.

PLATE IV



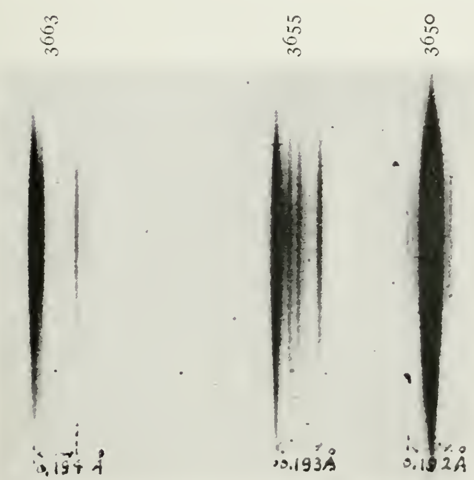
a

Hg 2536. L.-G. plate and concave grating.



b

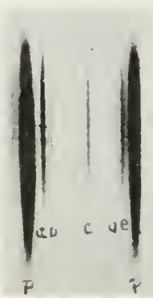
Fe lines. L.-G. plate and concave grating



c

Hg lines. Echelon and concave grating

d



$a: +0.083\text{A}$
$b: +0.128\text{A}$
$c: -0.237\text{A}$
$d: -0.103\text{A}$
$e: -0.069\text{A}$

Hg. 5461
Echelon and concave grating

grating had thirty-five plates, of 9.36 mm thickness, with steps of 1 mm; the concave grating was the one already cited. Dr. T. Takamine has already photographed these lines with the echelon grating, by interposing a constant deviation prism; on account of the small dispersion of the prism the lines are closely packed, but with the concave grating they are well separated and the components decidedly better defined. The approximate position of the components is given in the diagram; the exact position obtained from crossed spectra of two quartz plates will be given in a future paper, which is now in course of preparation. Plate IV*d* shows the well-known green line of mercury obtained in the same manner; only five out of twelve satellites are found, the rest lying mostly in the neighborhood of the principal.

On examining the spectra thus obtained with concave grating combined with interferometers, we notice the advantage of the method when the component lines are not very near the principal. For obtaining exact position, the crossed spectra of two interferometers are still preferable. This is well exemplified in the mercury line 2536; Plate IV*a* shows that the principal components are a doublet, but there are vague traces of faint components whose position is difficult to make out. We lately found that there are, besides the doublet, at least four minor components, whose position can be exactly located by crossing two quartz plates. It is mostly in work on Zeeman and Stark effects that the present method of observation can be effectively used, provided the intensity of the components does not fall off or when they do not lie closely together. When the lines are diffuse, and have apparently no definite boundary, the combination here described will give better definition and facilitate the location of the mean position of the lines for measurements. Thus the advantage of using the present method will mostly lie in dealing with the arcs or sparks in air, while the crossed spectra can be used in accurate measurements with the light from vacuum arcs.

Finally, it may be pointed out that the combination of Fabry-Perot interferometer with a concave grating is impossible, as the fringes come out in circles instead of in straight lines.

Institute of Physical and Chemical Research
Tokyo, Japan
July 28, 1922

THE EXPLOSION SPECTRA OF THE ALKALINE EARTH METALS

By R. A. SAWYER and A. L. BECKER

ABSTRACT

Modification of Anderson's exploded wire source of light.—In 1920 Anderson obtained extremely high temperature spectra of certain metals by discharging a large condenser through fine metal wires. The authors have found that the metal wire may be replaced by an *asbestos fiber saturated with an aqueous solution of a salt* of the chosen metal. Thus the explosion spectrum of any soluble salt may be obtained. The fiber is uninjured by the explosion and may be used repeatedly. With a 6-foot grating, from six to twenty explosions are necessary for a first-order spectrogram. The condensers (0.3 M.F.) were charged to 40,000 volts through a high resistance with the rectified current from a 1-kilowatt transformer, and were discharged by means of a special switch. The apparatus and connections are described. The temperature of the explosion is probably about 15,000° C. and the pressure 10 to 20 atmospheres.

Explosion spectra of chlorides of Ba, Ca, Mg, and Sr, λ 2280–λ 4550.—When lines due to impurities, Cu, Zn, Al, Pb, C, and N, are eliminated, the spectra are found to be almost pure spark spectra, consisting chiefly of the doublets of the first and second subordinate series, $2p-md$ and $2p-ms$. Of the arc lines, only the fundamental singlet line, $1S-2P$, appears and its intensity with reference to the spark lines is only one-tenth as much as in the vacuum arc and about the same as in the spectra of the solar chromosphere and of class B stars. The first member of the Bergman doublet series of Mg, $3d-4f$, λ 4481, appears and also a line of Ba, λ 4350, whose series relation is unknown. No lines of Cl, H, or O were detected. The prominence of impurity lines shows how small an amount of material is needed to give explosion spectra.

Relation of explosion spectra to the theory of spectra.—According to Saha's theory, the metals used would be ionized at the expense of the less readily ionized Cl, H, and O; in fact at 15,000° C. the alkali earths should be almost completely ionized. Hence, according to Sommerfeld, the explosion spectra should be almost pure spark spectra. These conclusions agree with the facts. The faint arc lines may be emitted during the initial and final stages of the explosion. The Bergman line indicates an approach toward double ionization.

I. INTRODUCTION

In 1920 Dr. J. A. Anderson[1] announced a new method of producing spectra. This method consists of exploding a short fine wire of the metal chosen by discharging through the wire a large condenser charged to several thousand volts. If the explosion takes place in a confined space an absorption spectrum is produced; while if the explosion occurs in the open air or in a partial vacuum the spectrum partakes more of the nature of an emission spectrum. The exact type of spectrum produced depends, however, upon other factors besides the pressure.

[1] *Astrophysical Journal*, 51, 37, 1920.

One of the interesting features of this source of light is that it enables a very powerful stimulus to be applied very abruptly to the molecules of an element. It is thus possible to study spectra under conditions widely different from those obtaining in most laboratory sources.

In the course of some investigations with these explosions, which have been briefly reported elsewhere,[1] a modification of Anderson's source was announced. The fine wire used by Anderson is replaced by a fine asbestos fiber which is saturated with a solution of some salt of the desired metal. When the high tension condensers are discharged through this saturated fiber, with the fiber in the open air, an emission spectrum is obtained characteristic of the metallic ions in the solution.

It is the purpose of this paper to discuss the spectra produced by exploding, in this manner, solutions of the alkali earth metals.

II. THE APPARATUS

The apparatus, as used, differed in several minor respects from that described by Anderson. Changes were made with a view to greater convenience and efficiency and are perhaps worth describing briefly.

The condensers were built up of sheets of double strength window glass, $10 \times 12.5''$, alternating with sheets of roofing tin, $8 \times 10''$. The tin sheets were pressed and their edges carefully smoothed. Tabs on the corners of the tin sheets projected alternately on either side of the pile and were bolted together. The condensers were built up in piles of sixteen tin plates and firmly tied with cord. Sixteen such piles were constructed. The condensers thus obtained were placed in pairs in large glass battery jars and covered with transil oil. The total capacity thus obtained was of the order of 0.3 M.F. The advantage of such a condenser is that its units are readily portable and easily repaired in case of rupture—no small advantage in the present work since the voltages used were near the rupture voltage of the dielectric and transient surges sometimes resulted disastrously to the glass plates.

The charging voltage was obtained from a 50,000-volt 1-kilowatt transformer. The primary of this transformer was fed with a 110-

[1] *Science*, 54, 305, 1921.

volt alternating current obtained from the 200-volt alternating current power line of the university by use of a second transformer. The second transformer was employed to protect the university power circuits from the possibility of high frequency transient surges. Rectification was obtained by means of a 100,000-volt 100-milliampere kenotron.

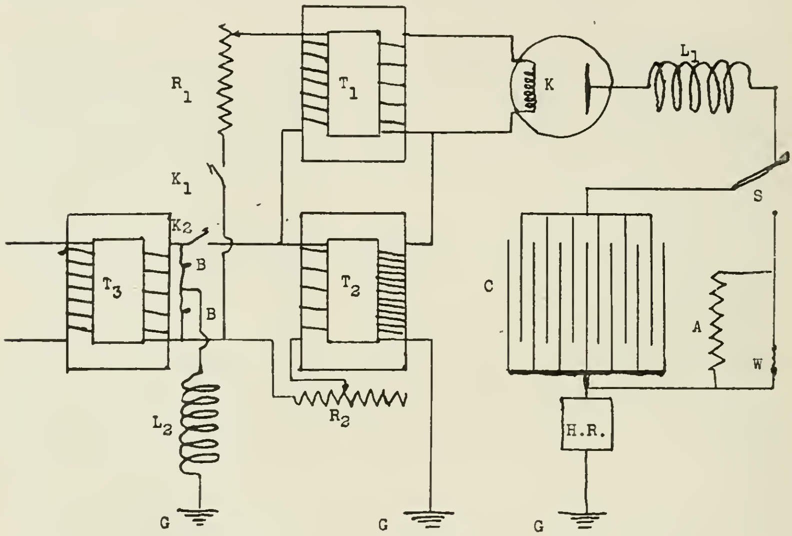


FIG. 1

The discharge circuit was closed by means of a heavy single pole, single throw-knife switch immersed in transil oil. A heavy spring served to close this switch rapidly. It was held open by a trigger device while the condensers were charging. A lever was arranged so that a single pull released the trigger holding the switch open, opened the primary circuit of the high voltage transformer, and opened the high-voltage charging circuit.

The electrical circuit is shown diagrammatically in Figure 1. The features described above will be noted. There is shown, also, a water rheostat which was used to keep down the charging current and protect the kenotron. The water rheostat consisted of a glass tube, about 5 mm bore and 120 cm long. Its resistance was so high that several seconds were required to charge the condensers.

The leak resistance, *A*, was made from a piece of No. 30 Nichrome wire about 3 m long. It allowed any absorbed charge on the condensers or any charge that did not escape in the first surge across the asbestos fiber to leak off gradually. Its resistance, 65 ohms, was sufficiently high so that only an inappreciable part of the first surge escaped through it.

The inductances, L_1 and L_2, consisted merely of twenty-five turns of wire. They were intended to act as choke coils to stop any high frequency oscillations. To reduce further the possibility of damage by high frequency oscillations a 220-volt carbon lamp, *B*, was shunted across each half of the secondary of the transformer, T_3, between the ground and the outside terminals.

The fibers were supported on an insulating stand which could be adjusted both vertically and laterally by rack-and-pinion movements. The upper end of the fiber was held in a brass clamp on the stand while the lower end rested against a brass finger.

The spectra were photographed with a 6-foot concave grating, Rowland mounting. The first-order spectrum was used, with a dispersion of 9.4 A per mm. A region of about 700 A could be photographed on the plates employed.

The explosions were focused on the slit with a quartz or glass lens, as the spectral region demanded. The adjustment was made as described by G. A. Hemsalech.[1] A light was placed behind the fiber to illuminate it and the image of the illuminated fiber was focused on the slit by the aid of the vertical and lateral adjustments of the fiber mounting. It is difficult to make this adjustment with great accuracy and so a rather wide slit had to be used. A wide slit was useful also in reducing the number of explosions necessary.

III. PROCEDURE

To take a spectrogram the fiber was saturated with an aqueous solution of some salt of the desired metal; the fiber was focused on the slit as described, the switch *S* opened, and the high-tension lead attached to it by a spring clip; and the switches K_1 and K_2 closed. A few seconds were allowed for the condensers to charge. The lever was then pulled which simultaneously opened the switch

[1] *Philosophical Magazine*, 40, 37, 1920.

K_2 and released the trigger on S. As the blade of S descended, the spring clip of the high-tension lead was pulled off. When the switch closed, the charge of the high-tension condenser was discharged through the fiber, with a sharp crack and a brilliant flash. The fiber was uninjured by these discharges and could be moistened and the procedure repeated. From six to twenty such explosions were required, the number varying with the spectral region.

The copper arc was used as a comparison to aid in the identification of the spectra. The copper arc was particularly useful in this work as copper, from the brass clamps that hold the fiber, is the principal impurity in the spectra. The plates were measured on a small comparator and the wave-lengths computed to tenths of angstroms. Greater accuracy was rendered difficult by the character of the lines; because of the width of the slit and because of the high pressure and gaseous velocities in the explosion, the lines were rather broad. The problem, however, was principally one of identification and the accuracy attained was sufficient for that.

IV. DATA

By the use of aqueous solutions of their chlorides, photographs were taken of the spectra of calcium, magnesium, barium, and strontium. The region λ 2280–λ 4550 was completely covered for calcium, while of this range the region λ 3381–λ 3890 for magnesium and barium, and λ 3381–λ 4550 for strontium was not covered.

The chief impurities in the spectrograms are copper and zinc from the brass clamps, as mentioned. There also appeared the strong aluminum pair, λ 3944 and λ 3961, the lead line at λ 4058, carbon at λ 2479, and nitrogen at λ 3995 and λ 4447. A striking fact is that in the spectrum obtained from a salt of any one of the four metals used there always appear many of the strong lines of the other three and of cadmium which is also a member of this group of metals. Although no great effort was made to secure purity of the salts used, the other metals of the group could have been present in the solutions only in very minute quantities. This source seems to be effective in producing spectra of substances which are present in the solution only in extremely low concentrations. It is of interest to note that no lines due to the acid radical employed, chlorine, nor

any lines of hydrogen or oxygen have been identified. If any radiations of these elements were produced, their intensities were too low to register with the exposures used.

The wave-lengths of calcium, magnesium, barium, and strontium, remaining after the elimination of the known impurities,

TABLE I

WAVE-LENGTHS IN EXPLOSION SPECTRA OF THE ALKALI EARTHS

$CaCl_2$		$MgCl_2$		$SrCl_2$		$BaCl_2$		Iden.	True Wave-Length
Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity		
2585.8	0	....	......	....	......	....	......	?	
2592.6	0	....	......	....	......	....	......	?	....
2606.5	0	....	......	....	......	....	......	?	....
2612.7	0	....	......	....	......	....	......	?	....
2630.7	1	....	......	....	......	....	......	?	....
2739.3	1	....	......	....	......	....	......	?	....
2756.0	1	2755.7	1	2755.8	1	....	......	?	....
2790.8	3	2790.8	5	2790.8	5	2790.8	4	Mg	2790.80
2796.5	10	2796.5	12	2796.8	12	2796.7	10	Mg	2795.53
2802.8	8	2802.7	8	2802.6	7	2802.8	6	Mg	2802.69
2852.6	1	2852.2	1	2852.2	1	....	......	Mg	2852.13
2928.6	2	2928.6	2	2928.6	1	....	......	Mg	2928.64
2936.9	3	2936.8	2	2936.8	1	....	......	Mg	2936.76
3107.5	1	....	......	....	......	....	......	?	....
3126.5	1	....	......	....	......	....	......	?	....
3159.0	13	3159.0	6	3159.0	5	....	......	Ca	3158.88
3180.7	17	3180.8	8	3180.7	7	....	......	Ca	3181.27
3380.7	8	3381.0	1	3381.0	8	....	......	Sr	3380.8
3706.1	10	....	......	....	......	....	......	Ca	3706.03
3737.0	12	....	......	....	......	....	......	Ca	3736.91
....	......	....	......	....	......	3891.9	15	Ba	3891.79
3933.7	25	....	......	....	......	....	......	Ca	3933.67
3968.5	20	....	......	....	......	....	......	Ca	3968.48
....	......	....	......	....	......	4078.0	10	Sr	4077.75
....	......	....	......	....	......	4130.7	20	Ba	4130.68
4212.0	2	....	......	....	......	4166.0	7	Ba	4166.02
4215.9	1	....	......	....	......	....	......	?	....
....	......	....	......	....	......	4215.6	7	Sr	4215.52
4226.7	3	4227.0	1	....	......	4226.7	4	Ca	4226.72
....	......	....	......	....	......	4350.4	3	Ba	4350.38
4481.0	7	4481.1	15	....	......	....	......	Mg	4481.17
....	......	....	......	....	......	4525.1	7	Ba	4524.95
....	......	....	......	....	......	4554.0	30	Ba	4554.04

are tabulated in Table I. Here are listed for each salt used the wave-lengths obtained, and the intensity, source, and exact wave-length in International Units of each line. It will be noted that there are several unidentified lines, chiefly in the shorter wave-

lengths. It was impossible to identify these lines with the alkali earths or any of the impurities detected. As they may be due to any of a half-dozen or more elements, nothing can be said at this time as to their source.

V. DISCUSSION OF DATA

It will be noted that after the impurity and unidentified lines have been eliminated, strikingly few lines remain. This fact

TABLE II

DOUBLET SERIES OF THE EARTH ALKALIES*

First Subordinate Series $2p_1-md_2$
$2p_1-md_1$
$2p_2-md_2$

Element Ordinal Number	Mg		Ca		Sr		Ba	
	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity
3......	....	....	....	....	....	....	7485.4	*x*
	....	....	8498.3	*x*	10038.3	*x*	8564.0	*x*
	2798.0	12	8542.5	*x*	10328.3	*x*	or	
	2790.8	5	8662.5	*x*	10915.0	*x*	10635.6	*x*
	....	....	....	....	....	....	10052.4	*x*
	....	....	....	....	....	....	12084.8	*x*
4......	....	....	3181.4	17	3475.0	*x*	4166.2	7
	1737.5	*x*	3179.5	17	3464.6	*x*	4130.9	20
	1734.7	*x*	3159.0	13	3380.9	8	3892.0	15
5......	....	....	....	....	2324.6	*a*	2641.5	*a*
	....	....	2113.0	*x*	2322.5	*a*	2634.9	*a*
	....	....	2103.5	*x*	2282.3	*a*	2528.5	*a*

Second Subordinate Series $2p_1-ms$
$2p_2-ms$

Element Ordinal Number	Mg		Ca		Sr		Ba	
	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity	Wave-Length	Intensity
1......	2795.5	12	3933.8	25	4077.9	10[c]	4554.2	30
	2802.7	8	3968.6	20	4215.7	7[c]	4934.2	*x*
2......	2928.6	2	3706.2	10	4161.9	*x*	4525.2	7
	2936.5	2	3737.1	12	4300.6	*x*	4900.1	*x*
3......	1753.3	*x*	2208.9	*x*	2471.7	*a*	2771.6	*a*
	1750.6	*x*	2198.0	*x*	2423.7	*a*	2647.4	*a*

[c] = On barium plate.

* These values are taken from Fues, *Annalen der Physik*, **63**, 1, 1920.

becomes even more striking when the lines of the alkali earth metals which do appear are classified according to their series relationships, as is done in Table II. In this table a number in the intensity column opposite a wave-length shows the relative intensity with which the line appeared; an "*a*" indicates that it did not appear on the plate, while an "*x*" indicates that the line lies in a region not photographed in this work. Comparison of the lines in Table II with those in Table I shows that, with one exception in the case of barium, one in calcium, and two in magnesium, all of the lines are members of the first and second subordinate series of the respective metals.

It will be seen that the first two members of each series for each metal appear, if they lie in the region photographed. In the few cases where higher members of the series lie in the region covered, they did not appear on the photograph—probably because of the usual decrease in intensity with increasing ordinal number in series.

Such a spectrum for the alkali earths is of course wholly different from that produced by ordinary laboratory sources. In the usual spark spectrum, members of the triplet and singlet series appear with intensities of the same order as those of the lines of the doublet series. (See for example the tables V and VI of this paper.) Nelthorpe,[1] however, using metallic salts in a discharge tube, obtained results for calcium, barium, and strontium quite similar to these in the region λ 6500–λ 3400.

VI. CORRELATION OF DATA AND THEORY

The explanation of such a spectrum as has been described above is readily found in the modern theories of atomic structure and spectral emission. Bohr has pointed out that, due to quantum changes of one of its outer electrons, a neutral atom will emit a spectrum which is called the "arc" spectrum. An ionized atom, however, has an effective nuclear charge $+2e$ now acting on the outer electron, instead of $+e$ as in the former case, and will consequently emit a totally different spectrum known as the "spark" spectrum.

Sommerfeld[2] has stated in his "Verschiebungssatz" that the "spark" spectrum of any element is similar in series structure to the

[1] *Astrophysical Journal*, 41, 16, 1915.

[2] Sommerfeld, *Atombau und Spektrallinien*, 3d ed., chap. vi.

"arc" spectrum of elements in the preceding column of the periodic table. He has further pointed out that for the alkali earths the "spark" spectrum consists of the lines of the doublet series, while the triplet and single line series comprise the "arc" spectrum. The "arc" spectrum of the alkalies, then, consists of doublets, while the "spark" spectrum of the alkalies is a complicated spectrum without known series, similar to the spectra of the noble gases. Laboratory evidence in support of the "Verschiebungssatz" has been rather meager. Goldstein,[1] by means of a powerful excitation, succeeded in producing spectra of the alkalies, in which the usual doublet spectra were entirely missing, and which by their complexity, resembled the spectra of the noble gases. It seemed, therefore, that he had not only succeeded in producing the "spark" spectrum of the alkalies, but in doing so had so completely ionized the emitting atoms that the "arc" spectrum was wholly absent. For the alkalies, it was then possible to produce at will either the "arc" or the "spark" spectrum.

The spectra produced in these experiments, it will be seen, then, are almost pure "spark" spectra of the alkali earths. They do for the alkali earths what Goldstein did for the alkalies. They show that it is possible to produce for these metals the "spark spectrum" with practically no trace of the "arc" lines.

It is possible to carry farther the application to the spectra at hand of the quantum theory of emission. In the source under discussion a considerable energy, 50 calories or so, was applied to 1 or 2 milligrams of material. The effect was to heat the material very quickly to an extreme temperature, estimated by Anderson at 20,000° in his case. This heating is so abrupt and its dissipation so rapid that it seems not unreasonable to say that the emission takes place principally, if not entirely, at this high temperature. The reason for thinking this to be the case will be apparent later.

Dr. M. N. Saha in several recent papers has discussed the temperature radiations of gases.[2] Saha's work has been amended

[1] *Verh. d. D. Phys. Ges.*, 10, 321, 1907.

[2] *Philosophical Magazine*, 41, 267, 1921, and *Proceedings of the Royal Society* Series A, **99**, 135, 1921.

and extended by Russell[1] and Milne.[2] The conclusions of these writers would seem to be applicable to the explosion spectra. In brief they are as follows (to a large extent Saha's wording is followed).

An atom, cool and unstimulated, has its vibrating electron in the $1s$-orbit. Such an atom or mass of atoms does not emit and can absorb only lines of the principal series, $v=1s-mp$. If the atoms are heated, the vibrating electron proceeds toward ionization through the various quasi-stationary states and radiation occurs. First we have emission of $1s-2p$ lines and then of $1s-mp$ or principal series. When the gas emits the line $1s-2p$ strongly it can absorb the lines of the $2p-md$ and $2p-ms$, diffuse and sharp series. At a higher temperature, the $2p-md$ and $2p-ms$ lines begin to be emitted and the $3d-4f$ or fundamental series can be absorbed. Thus, as the heating of the atoms continues, the electron proceeds toward ionization, being able progressively to absorb and to emit series corresponding to higher and higher levels of energy.

The theory is not sufficiently developed to permit the calculation of the proportion of atoms in the various quasi-stationary states but the qualitative situation is clear. The temperature at which the fundamental line $1s-2p$ is emitted and the $2p$-orbits commence to form will be higher, the higher the ionization potential of the element. Other orbits appear at temperatures intermediate between this temperature and the temperature at which ionization is complete. At any state, the proportion of atoms at any energy level will be smaller, the higher the quantum number corresponding to this stage. Thus the $1s-2p$ and $1s-mp$ series will be the first to appear, the last to disappear, and the most intense always. The $2p-md$, $2p-ms$, and other series will appear later, disappear sooner, and be weaker. The disappearance of $1s-2p$ marks complete ionization.

After the first step ionization is completed, if heating of the mass of atoms continues, the singly ionized atoms then proceed toward double-ionization through the same cycle of processes described for the un-ionized atoms. The series in question will of course now

[1] *Astrophysical Journal*, 55, 119, 1922.

[2] *Observatory*, 46, 261, 1921.

be the series of the "spark" spectrum. As before, the series emitted will afford an index of the energy conditions of the mass of atoms.

Although it is impossible to compute the proportion of atoms in various stages, Saha has pointed out that the proportions of ionized and un-ionized atoms may be computed on the basis of modern thermodynamics by the aid of the Nernst theorem of the reaction-isochore. The equation for computing the percentage of ionization is

$$\log \frac{x^2}{1-x^2} P = -\frac{U}{4.571\, T} + 2.5 \log T - 6.5\,,$$

where x is the fraction of atoms ionized, P, the gaseous pressure, T, the temperature, U, heat of dissociation, or calories to ionize one gram-atom.

Also

$$U = \frac{eVN}{J\,300} = 2.302 \cdot 10^4\ V \text{ calories}\,,$$

where e is charge on the electron, V, ionization potential in volts, J, mechanical equivalent of heat, N, Avogadro's number.

The percentage of ionization is determined by the pressure, temperature, and ionization potential and may readily be computed.

Table III gives for the alkali earths the values of the ionization potential and of the heat of dissociation.

TABLE III

Element	Ionization Potential	U in Calories
Magnesium	7.65	1.761×10^5
Calcium	6.12	1.409×10^5
Strontium	5.7	1.313×10^5
Barium	5.12	1.178×10^5

Table IV gives the values computed by Saha for the percentage of ionization of the alkali earths at a pressure of one atmosphere at various temperatures.

Saha has extended his theory to second-stage ionization also. Corrections to his formulae have been pointed out by Russell and

by Milne (*loc. cit.*). The extent of second-stage ionization, however, need not be considered here since the spectroscopic criteria of such ionization are as yet unknown.

Russell (*loc. cit.*) has shown that if atoms of several kinds are heated together it is possible to compute the fraction of each kind ionized at any temperature and pressure. In general it may be said that if an element is more easily ionized than the average, its percentage of ionization will be greater than if it alone were present. For elements whose ionization potential is greater than the average, the reverse will be the case.

TABLE IV

Temperature of Element	Mg	Ca	Sr	Ba
5000	$5 \cdot 10^{-2}$	2	3.2	5.5
6000	2	8	13	19
7000	6	23	32	43
8000	17	46	58	70
9000	34	70	79	85
10,000	56	85	90	93
11,000	75	93	95	....
12,000	86	96.5	97.5	....
13,000	93	98.5	98.5	....
14,000	96	Complete Ionization		
15,000	98			
16,000	99			

It is now possible, at least qualitatively, to apply the theory discussed above to the spectra under consideration. The solutions used were aqueous solutions of chlorides of the alkali earths. The effect of the explosions was to dissociate these chlorides and the water and to heat the resultant atoms to a high temperature. Since hydrogen, oxygen, and chlorine each have higher ionization potentials than any of the alkali earth metals, the alkali earths were ionized at the expense of these other constituents and to a greater extent than they would have been if the material exploded had consisted of an equal amount of the pure metals. The effect of the explosion is not wholly one of temperature—the electric effects, however, probably act to further the work of the temperature.

The exact temperature and pressure obtained are, of course, unknown. Anderson estimated from the intrinsic brilliancy that the temperature attained was of the order of 20,000° C. From two

different considerations he inferred that the pressure attained from an explosion in a slot in a block of wood was of the order of 20 atmospheres. Since the explosions here discussed took place in the open air, both pressure and temperature were probably somewhat less, although the voltage used, 40,000 volts, was higher than that used by Anderson. It must be remembered, moreover, that in the case of the exploded solutions the pressure to be used in computing the degree of ionization of an element is the partial pressure of that element which is much less, of course, than the total pressure of the explosion. It is probably safe to assume that throughout these experiments, the temperature attained and the partial pressure of the major constituents were in each case of the same order of magnitude. That is, we deal here with spectra of the alkali earths produced all under approximately the same conditions.

Examination of Table IV will show that at a temperature of 15,000° and a pressure of one atmosphere, ionization of each of the alkali earths is practically complete. At higher temperatures or lower pressures the certainty of complete ionization is, of course, greater. These figures are chosen in the belief that they are at least of the right order of magnitude.

Spectroscopic evidence that complete ionization had been attained in these explosions would be the complete suppression of the "arc" spectrum and the appearance of only the "spark" spectrum. As pointed out above, the last arc line to disappear is $1S-2P$; its disappearance indicates complete ionization. For the alkali earths the fundamental line, $1S-2P$, of the single line spectra has the following value for the various elements:

Magnesium	2852.13
Calcium	4226.72
Strontium	4607.34
Barium	5535.69

The line λ 4226.72 is the only calcium line in Table I which is not a member of either the first or second subordinate doublet series. Its intensity, 3, is only one-eighth that of the fundamental doublet pair, λ 3968 and λ 3934. The weakness of λ 4227 and the absence of all other "arc" lines, shows how nearly all radiation takes place from ionized atoms.

In the magnesium spectrum are only two lines which do not appear among the doublets of Table I. One of these lines is the fundamental singlet line, λ 2852, which appears with one-tenth the intensity of the fundamental doublet pair. The other line, λ 4481, is the unresolved first member of the Bergman doublet series, $3d-4f$. Its emission, as pointed out above, denotes the presence of radiating atoms in an even higher energy state than the atoms which emit the sharp and diffuse series.

The lines $1S-mP$ of strontium and barium unfortunately do not lie in the region photographed in this work. However, these lines, if they appeared, would be expected to be relatively fainter even than the corresponding lines of calcium and magnesium (since the heats of dissociation of strontium and of barium are less than those of calcium and magnesium). The appearance of the line, λ 4350, the only barium line, not in the sharp or diffuse doublet series, is puzzling. This has been classed as an "arc" line of barium; that is, its intensity is greater in the ordinary arc than in the spark. It does not seem to have been fitted into any of the known barium series and, therefore, no criterion exists for explaining its appearance.

With the exception of the barium line, λ 4350, the explosion spectra of the alkali earths consist of the fundamental singlet line, $1S-2P$, and of the doublet or "spark" lines. There would seem to be two possible explanations of the persistence of the fundamental "arc" line. First, the pressure and temperature conditions may not be sufficiently extreme to produce complete ionization. In this case it is evident that conditions in the explosion source as here used must be similar to those pointed out by Saha[1] as existing in class A2, A, B9, B8 stars; that is, calcium and magnesium are almost wholly ionized although λ 4227 of calcium still persists. The second possible explanation is that, since we start with our material to be exploded at a normal temperature and the final temperature and pressure cannot be built up instantly, some of the radiation of the neutral atom may appear while the atoms are being brought to their final ionized condition. If so, then, as pointed out above, the lines $1S-2P$ will predominate in the arc spectrum.

[1] *Proc. Roy. Soc.*, A, **99**, 135, 1921.

The effect of electrolytic dissociation, which of course exists in the solution used, somewhat complicates the situation. It is hard to say what effect the electronic rearrangements of dissociation may have on the radiation produced. Some evidence on this point is afforded by some spectrograms taken of the explosions of fine magnesium and calcium wires. The results so obtained are wholly similar to the results with exploded solutions of magnesium and calcium. In general the $1S-2P$ line is not so much weakened

TABLE V

IONIZATION OF MAGNESIUM

Source	Intensity of λ 2795–λ 2803	Intensity of λ 2852	Ratio of Intensities
King's electric furnace	....	1200	....
Crew and McCauley arc	8	10	0.8
Eder and Valenta spark	10	8	1.2
Fowler vacuum arc	50	30	1.7
Exploded wire	10	2	5.0
Exploded solution	10	1	10.0

TABLE VI

IONIZATION OF CALCIUM

Source	Intensity of λ 3933–λ 3968	Intensity of λ 4226	Ratio of Intensities
King's electric furnace	55	1000	.05
Crew and McCauley arc	400	500	.8
Lockyer spark	500	400	1.2
Loving vacuum arc	20	8	2.5
Exploded wire	10	2	5.
Exploded solution	25	3	8.
Chromosphere of sun	72	8	9.
Stars of type B	7	1	7.

in the exploded wire spectra as in the exploded solution. This fact may be due, however, to the lower partial pressure of the metal and the smaller amount of material exploded in the solution; both of these factors would tend toward more complete ionization. The effect of dissociation would then appear to be negligible.

For the purposes of comparison there are given in tables V and VI data on the ratio of the intensities for different sources of the fundamental spark doublet, $1s-2p$, to the fundamental arc singlet $1S-2P$, for calcium and for magnesium. This ratio is, as pointed

out above, an index of the degree of ionization produced in any emitting source. It will be noted that ionization is carried farther in the exploded solution than in the other sources, and that the ionization produced in the exploded solution compares with that in the sun and stars of class B.

In conclusion it may be pointed out that the exploded solution as source, discussed above, must contain a relatively small percentage of un-ionized atoms. Only the fundamental single lines $1S-2P$ and the barium line, λ 4350, appear of the "arc" spectrum and these lines are faint. It, of course, follows from Saha's theory that there must also be slight emission of other arc lines. This emission was, however, too weak to be detected by the means at hand.

Further studies of the spectra produced by this type of discharge are now being carried on.

VII. SUMMARY

1. A modification of Anderson's exploded wire as source of light is developed, in which there is exploded, in place of the wire, an aqueous solution of any salt of the desired metal. The method is thus made applicable to any metal, rather than only to those which can be obtained in the form of fine wires.

2. The application of this source to the alkali earth metals produced for these metals an almost pure "spark" spectrum.

3. The correlation of this data with Sommerfeld's "Verschiebungssatz" and with Saha's theories of the temperature radiation of gases is discussed. The data afford corroboratory evidence for both these theories.

Physical Laboratory
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THE EFFECT OF A PROBABLE ELECTRIC FIELD ON THE BANDS OF NITROGEN

By SNEHAMOY DATTA

ABSTRACT

Broadening of nitrogen positive bands in presence of bromine vapor.—The partial pressure of bromine vapor in a discharge tube containing some air was varied by immersing a side tube in different freezing mixtures. With the lowest partial pressure ($-78°$ C.), the bands were sharp, but as the pressure was raised the bands showed evidence of blurring, especially the more refrangible ones, the effect increasing with the pressure until for $-5°$ C. even the heads had disappeared, leaving only diffuse patches where the bands had been. Both the second and third bands showed the effect, and the bands below λ 3900 were absent entirely, though a quartz spectrograph was used. It is suggested that the blurring may be a Stark effect produced by the electric fields of the ionized bromine atoms. Though this effect has not been directly observed in the laboratory, it was predicted by Hettner from theoretical considerations. The similar blurring due to increased pressure in pure nitrogen may also be a Stark effect. However, the *cyanogen band* was unaffected by the bromine.

Suggested explanation of displacement of bands by pressure of foreign gas.—The displacements observed by Dufour and Clinkscales in the case of the bromine and sodium bands may also be a Stark effect due to the electric field of the ionized atoms of the foreign gas, even though the effect has not yet been observed for any bands in the laboratory.

I. INTRODUCTION

It is well known that, beside the characteristic line and band spectra, the majority of elements emit under proper conditions a continuous spectrum, which has not been resolved into fine lines by the highest dispersion used. It has also been definitely proved that some of these continuous spectra—especially those of the halogens—do not belong to the class of radiation emitted by glowing hot bodies. No satisfactory explanation, however, has yet been given. Lately, a possible suggestion has been made by Professor J. Franck,[1] who considers the continuous spectra to be the type of radiation one would expect during the formation of negative ions (atoms+electrons). From the frequency ν_0 of the head of the continuous spectrum, which must be on the less refrangible side, according to his theory, the energy necessary to remove the additional electron from the normal atom (i.e., the electron affinity of the atom) can be calculated. From the data furnished by W. Steubing,[2] Franck has calculated the electron affinity of iodine,

[1] *Zeitschrift für Physik*, 5, 428, 1921. [2] *Annalen der Physik*, 64, 673, 1921.

which appears to agree well with the values theoretically deduced by M. Born.[1] Bromine was known to possess a continuous spectrum, and it was thought worth while to investigate whether this could be regarded as its electron affinity spectrum. Work on this subject was at first undertaken in the Physical Institute of Göttingen University under the direction of Professor Franck.

2. EXPERIMENTS

A preliminary investigation was undertaken to determine the exact condition when the continuous spectrum could be obtained at its brightest. The type of discharge tube finally chosen for the purpose was an ordinary Jena glass tube of internal diameter about 1.5 cm, having as the electrodes a platinum point and a plate at a distance of about 2 cm. The discharge chamber was connected by glass tubes with a side reservoir for bromine. By warming the reservoir sufficient vapor was allowed to escape to displace the air in the tube. The tube was then sealed. The pressure in the discharge chamber was conveniently regulated by immersing the side tube in a Dewar flask filled with a pasty mass prepared by mixing solid carbon dioxide with acetone. At about -5° C., the color of the discharge changed from yellow to pink, and the discharge itself became very steady but feeble. This pink discharge emitted the brightest continuous spectrum. The discharge being very feeble, instruments of high dispersion could not be used, and photographs were therefore taken with a quartz spectrograph giving a dispersion of about 32 A per mm at λ 3800.

The continuous spectrum thus obtained, however, did not agree well with the description of the electron affinity spectrum as demanded by the theory. The present investigation, therefore, fails to throw any definite light on the cause of the emission of the continuous spectrum.

In the course of the investigation, however, a peculiarity was observed in the nature of the continuous spectrum. The spectrum, instead of continuously fading away, showed a succession of maxima and minima of intensity, due to the overlapping of strong diffuse patches. The position of the patches suggested that they might

[1] *Zeitschrift für Physik*, 5, 433, 1921.

have something to do with the second positive bands of nitrogen which occur in this region. Experiments were therefore made to observe the changes (if any) of these patches at various pressures of bromine. It was then found that by immersing the side tube in a bath at $-78°$ C., the discharge chamber looked perfectly white, owing to bromine distilling over and solidifying in the side tubes. The color of the discharge turned into gray and the spectrum (Plate V, Fig. 1*b*) showed the second positive bands of nitrogen prominently, together with a very faint continuous background due to bromine. The spectrum has been compared with that of an ordinary nitrogen tube for identification (Fig. 1*a*). On the continuous ground only one diffuse patch appeared (marked with a black dot and a pair of brackets in Fig. 1*b*), which had no correspondence with the nitrogen spectrum. This was subsequently identified with "the band with ill-defined edges due to bromine," described by Strutt and Fowler[1] in connection with their investigations of the active modification of nitrogen.

On raising the temperature of the bath to $-40°$ C., the color of the discharge tube became slightly yellow, owing to the increase in the quantity of bromine, and the discharge changed from gray to somewhat pink. The spectrum, at this stage, showed a marked change in the overlapping nitrogen bands (Fig. 2*b*). Instead of all the heads in each group being fully developed, only the less refrangible head became prominent, and this was followed by a patch of continuous spectrum. All the bands were not equally blurred, and the blurring increased on proceeding to the side of short wavelength. At the same time, some of the stronger nitrogen bands (those beyond λ 3900), which lie outside the continuous spectrum, do not appear at all. Bromine vapor has an absorption band in this region, with the maximum about at λ 4100. But, under the condition of the experiment, when there is so little bromine present in the discharge tube, it was doubtful whether absorption alone could account for the entire absence of these bands. A separate experiment on the absorption of such thin layers of bromine fully supported the doubt. That absorption is not possible, even when the tube is excited, is shown by the presence of a few faint lines of bromine in that region. In the presence of the

[1] *Proceedings of the Royal Society*, A, 86, 105, 1912.

PLATE V

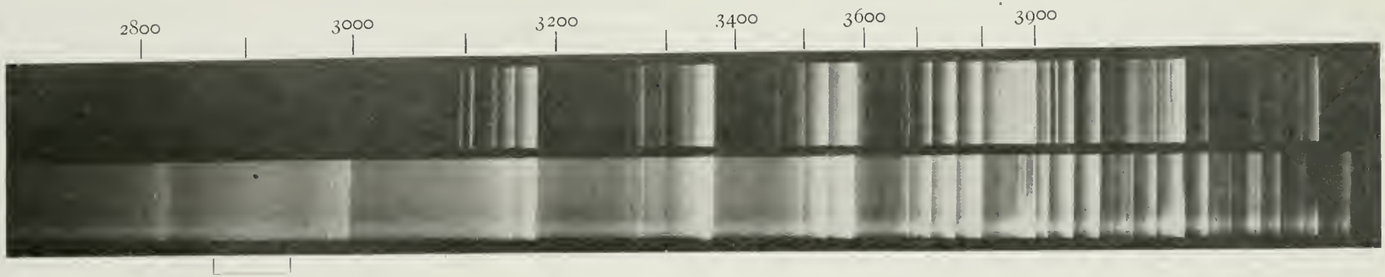

Fig. 1

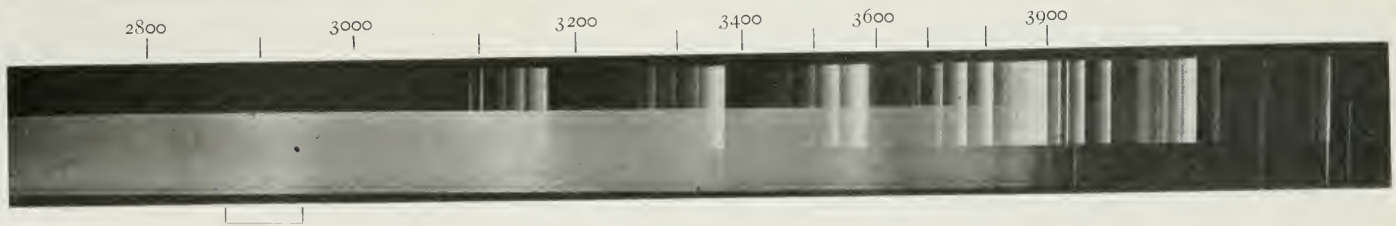

Fig. 2

Fig. 3

which possibly brings about a resolution and displacement of the nitrogen bands. As the different radiating molecules of nitrogen are at different distances from the bromine atoms, they will be subjected to electric fields of all values. Consequently there will be all degrees of resolution and displacement of the bands, the total effect of which will be to produce a complete blurring of the nitrogen bands. As such blurring depends on the chances that a radiating nitrogen molecule will be in the sphere of electrical action of the bromine atom, it is evident that the blurring would be more probable with the increase in the number of active bromine atoms. The fact that the blurring of the nitrogen bands increases with the intensity of the continuous spectrum suggests that the active bromine atoms are those that are responsible for the radiation of the continuous spectrum. Consequently it may be suggested that the mechanism of radiation of the continuous spectrum of bromine is somehow or other connected with the formation of negatively charged atoms.

The Stark effect has not been observed in the case of band spectra. In all probability, the external electric field applied in the laboratory is much smaller in magnitude than the suggested one of atomic origin. The negative result obtained in the laboratory is therefore not sufficient evidence against the suggestion made in this paper. In addition the conclusion drawn here receives support from the work of Hettner.[1] From theoretical considerations Hettner has shown that molecular rotation must be influenced by an electric field and has suggested that such influences should be looked for in the absorption bands of gases and vapors in the infra-red region, these being due to molecular rotations. The recently developed theory of band spectra ascribes the flutings of a band to molecular rotations, their position in the visible region being due to the addition of a second term giving the electronic energy of the system. Consequently the resolutions and displacements of the flutings of the nitrogen bands, resulting in a blurring of the whole system, are in agreement with Hettner's suggestions.

Attention has already been drawn to the fact that, with the emission of the continuous spectrum, some of the bands of nitrogen are not at all developed, and that among those developed the

[1] *Zeitschrift für Physik*, **2**, 4, 349–60, 1920.

blurring increases as one proceeds to the more refrangible region. For this no explanation can be given, but in the light of the theory of band spectra, as recently extended by the author, it is possible to make an attempt at an explanation.

Following the suggestion of Heuelinger and Lenz that the different bands, usually associated in a group, are due to the quantum changes of atomic vibration, it has been shown that such quantum states correspond to the different degrees of dissociated states of the molecule and that radiation takes place in a change from one state to another. A general equation has been arrived at involving two successively varying integers m and n, where m refers to the order number of the final state after radiation and n to the quantum changes in the process of radiation. On arranging the second positive bands of nitrogen into series, the more refrangible bands are found to correspond to the higher values of m, i.e., to the states of higher partial dissociation. In the presence of bromine, owing to its strong affinity for electrons and the resulting electric field, the molecules of nitrogen must be thrown into higher states of dissociation, and consequently the radiation corresponding to low values of m must be missing. This may therefore account for the absence of some of the less refrangible nitrogen bands.

We may similarly account for the increase of blurring in proceeding to the more refrangible region. These more refrangible members, as already pointed out, correspond to high states of partial dissociation. Owing to the swelling of the molecules in such states, they come in closer proximity to the bromine atoms. The electric field acting upon them is thereby increased and a greater blurring takes place.

Further, it may be remarked that the present suggestion, that bands may be displaced and resolved by an electric field of atomic origin, may possibly explain some of the hitherto obscure results as to the influence of a foreign gas on the absorption of certain gases and vapors, some of which may now be mentioned.

Dufour[1] found that the absorption bands of bromine vapor tend to shift toward the red with an admixture of other gases such as hydrogen. Clinkscales,[2] working on the banded absorption of sodium vapor, found that some of the bands shift as much as

[1] *Comptes Rendus*, 145, 757, 1907. [2] *Physical Review*, 30, 594, 1910.

0.15 A toward the violet in the presence of hydrogen. Assuming that the effect of the field due to an electro-negative atom is to produce a displacement to the short wave-length, as is to be concluded from the present experiments, hydrogen being electro-positive, the displacement observed by Dufour must be on the red side. At the same time, hydrogen being less electro-positive than sodium, the admixture of hydrogen with sodium produces the same effect as if it were an electro-negative element, and consequently the displacement will be on the violet side, as observed by Clinkscales.

It is also possible to explain why Huddleston,[1] working on "the effect of pressure on the band spectrum of nitrogen," has found that with increase of pressure there is a gradual formation of a continuous spectrum. The increase of pressure brings the molecules into sufficient proximity to produce the necessary electric field, which brings about the resolution and displacement of the bands, causing the flutings to change to continuous bands.

In the light of the experiments described in the present paper, it will be interesting for those possessing the necessary equipment to examine the effect of a very strong electric field on the bands of nitrogen. The CN bands beginning at λ 3883 also appeared in the discharge (see Plate V) and since these were not blurred at any stage of the experiment, the conclusion is that the field due to the neighboring bromine atoms is not sufficient to produce any effect on the radiating cyanogen molecule; it would be interesting to test this with an external field. Iodine and chlorine being also strongly electro-negative, their effect on nitrogen and other gases will be studied at the earliest opportunity.

In conclusion I beg to record my thanks to Professor J. Franck, director of the Physical Institute at Göttingen, for giving me all facilities and for the kind interest he took in the work which was commenced there. My thanks are also due to Professor A. Fowler for kindly allowing me to complete the work here and for the interest he has taken in it all along.

Imperial College of Science
June 30, 1922

[1] *Physical Review*, 18, 327, 1921.

A NEW SCOUTING SPECTROSCOPE FOR PROMINENCES

By OLIVER J. LEE

ABSTRACT

Compact rotating spectroscope for detecting and following solar prominences.—This instrument can be readily attached to the 40-inch telescope without removing other apparatus. It is used outside of the cone of rays coming from the large objective and is fed by a plane mirror, suspended by detachable brackets from the base of the spectroscope, which is set at an angle of 45° with the optical axis of the refractor. The combined weight of the two parts is 25 pounds. The instrument accommodates a beam of light 28 mm wide, given by a slit curved to the limb of the sun. The dispersing unit is a plane grating, by Michelson, having 15,000 lines to the inch. *Position angles* of prominences may be read directly. The observer can tell, at once, whether or not an eruption possesses size or detail which should be recorded with the spectroheliograph. The development of a prominence between photographs may be followed. Much time will be saved in periods of little solar activity.

The instrument here described was designed by the writer and constructed by Mr. Stephen Stam in the machine shop of the Yerkes Observatory. Several considerations led to its construction:

1. It takes at least $1\frac{1}{2}$ hours to remove the instrument used the night before on the 40-inch telescope, attach the Rumford spectroheliograph, and take and develop the prominence plates. When, as at present, solar activity is at or near a minimum, almost all of this time is wasted. It is, however, desirable to know when an eruption of any importance is taking place, so that photographic records may be made of it.

2. Such a spectroscope must be light of construction, must be easily attached to the telescope without removing other instruments, and when attached it must not interfere with the use of the spectroheliograph when this is desired.

3. Some means has been wanted for following the development of a prominence without photographing it continuously.

4. Its purpose is reconnaissance and not measurement. Perfection of parts and of adjustment is entirely subsidiary to ease and speed of manipulation.

All of these conditions have been fairly met.

Figure 1 shows the arrangement of the instrument and the path of a ray of light through it.

A pair of reflecting prisms having square surfaces 28×28 mm and collimating and telescope lenses having the same aperture and focal lengths of 28 cm were obtained from John B. McDowell. All these parts were "seconds." The pair of prisms cost $5.00 and the pair of lenses complete with cells $15.00. With these parts and an old straight slit, a working model of wood and paper tubes was

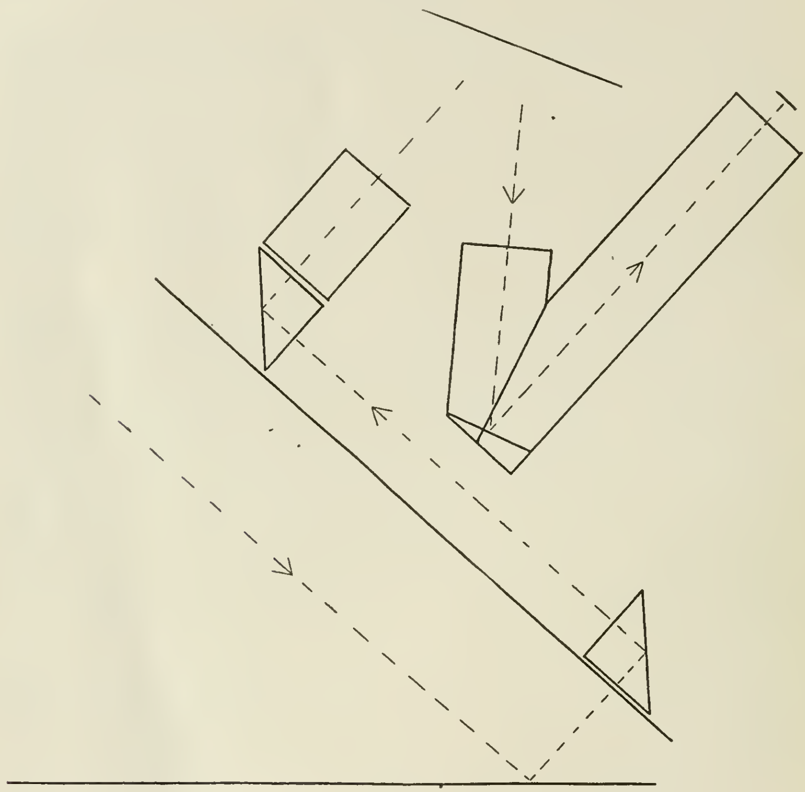

Fig. 1

made, which showed prominences well enough but was too crude to work with. Concrete improvements in construction were almost automatically suggested by this model and the permanent instrument shown in Figure 2 was the result. It is a rotating spectroscope.

It is built on a wooden board 3×32×75 cm in size. To this board is screwed a pan-shaped aluminum casting, the bottom of

which is a circular piece of steel 4 mm in thickness. This plate rotates freely and forms the base to which are attached the various parts of the spectroscope.

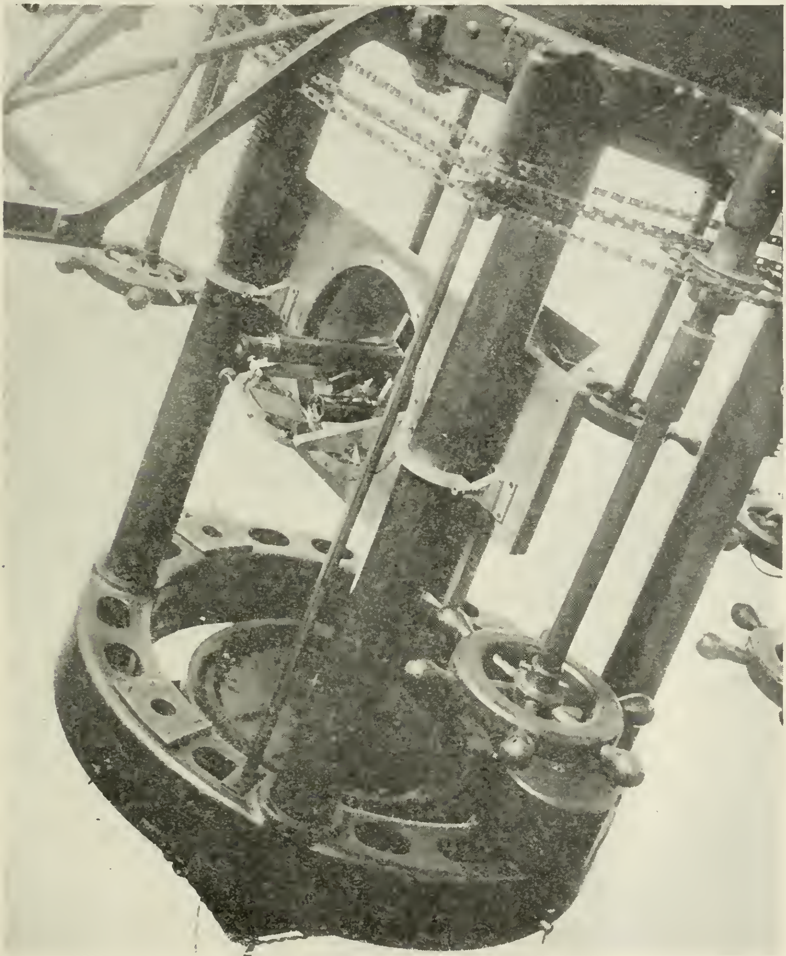

FIG. 2.—Scouting spectroscope for prominences attached to 40-inch telescope

The reflecting device, by which the solar image is turned 90 degrees and is brought to a focus outside of the cone of light of the 40-inch telescope, is separate from the rest. It consists of a mirror of plate glass 20×30 cm in size, which is partly covered by an ellip-

tical metal disk somewhat smaller than the solar image to protect it from that 95 per cent or more of the solar light which is not needed for detecting prominences on the limb. This part of the instrument is attached to the other by setting it upon four half-inch dowel pins, and it is secured in position by two spring clamps which are lifted and turned 90 degrees. The two parts taken together weigh 25 pounds.

The slit is curved to the limb of the sun and moves around it, taking about one linear inch at each step. Besides relative motion of the jaws of the slit, both may be moved together to allow for the slight variation in the apparent size of the sun.

On the observer's side of the instrument the plan of construction is easily seen. Compactness and sufficient lengths of optical path were the two opposing conditions and the compromise in the form of the spectroscope is evident. The dispersing unit is a Michelson grating, already in the possession of this observatory, having 15,000 lines to the inch. It has means of adjustment about the three axes. A spiral groove in the inner tube which carries the collimating lens engages a small set screw in the outer tube and focusing is done by simply turning this inner tube. An eyepiece giving a magnification of about fifteen diameters has been found most serviceable. A circular scale is so attached that position angles, NESW on the sun, of any prominence may be read directly.

Figure 2 shows the spectroscope in place on the 40-inch telescope. The routine of observing is simple. The instrument ring with whatever apparatus is attached to it is racked out, the spectroscope is put in place, the removal of two 15-pound weights establishes "balance," and the solar image is centered. When successive parts of the solar limb are examined, a slight pressure upon the refractor throws the chromosphere upon the slit, which flashes out red along its whole length at once if the C line is centered in the field. A prominence is quickly discovered by showing in the slit before the chromospheric arc centers. Since it takes a fairly strong prominence, visually, to show on a spectroheliogram, there is little danger of overlooking any which would be worth photographing, except, of course, a floating cloud completely detached from the solar limb. If, however, the region above every seat of activity be exam-

ined to some distance from the limb, the chance of even such an omission is almost completely eliminated. The interval of time from the moment the observer enters the dome until the survey is completed is less than 20 minutes and any subsequent examination may be made in 2 minutes.

The construction of a similar spectroscope for other telescopes would, of course, be even more simple if the base of the instrument can be placed in the focal plane of the objective. When the spectroscope can be used directly in the focal plane of the telescope and when there are not so many other conditions to be met in construction and use as existed in the case of the 40-inch telescope, it is possible to employ one of the many forms of instrument which are made by Adam Hilger, Ltd., London, for visual observation of the solar prominences.

Yerkes Observatory
October 2, 1922

Reviews

The Origin of Spectra. By Paul D. Foote and F. L. Mohler, Bureau of Standards, Washington, D.C. New York: Chemical Catalog Co., 1922. 8vo, pp. 249.

The applications of spectroscopy to the problem of atomic structure have given great stimulus to the study of spectra and to attempts to correlate spectral phenomena with phenomena in all fields which bear on the subject of atomic structure. The developments have been so rapid and varied that there has been great need of a critical, comprehensive, and suggestive survey of this field. This need has been admirably met by Dr. Foote and Dr. Mohler in their *Origin of Spectra.* This book may be considered as a supplement to Sommerfeld's *Atombau und Spektrallinien,* in that it starts with the general concepts which have been set forth by Bohr and Sommerfeld, and presents clearly the large variety of tests, interpretations, and consequences of these concepts. The subject-matter deals largely with experimental results and methods, but always with clear exposition of the theoretical and interpretative bearing of these results.

Chapters i and ii review the Bohr theory of spectra, with Sommerfeld's theory of fine structure and of the origin of the various types of spectral series in arc, spark, and X-ray spectra. Particularly useful is the critical discussion of the two prevalent systems of series notation, Fowler's and Paschen's, and the discussion of "energy diagrams," with examples.

Chapter iii deals with the ionization and resonance potentials of the elements, a subject of great value in interpretation of spectra, and one to which the authors have made contributions of great importance.

Chapters iv and v discuss the nature and peculiarities of absorption and emission spectra, the explanation of these peculiarities and their bearing on the problem of atomic structure, and the mechanism of radiation.

Chapter vi deals with "cumulative ionization," those properties of gases which result from the existence of atoms in excited or metastable states, and the passage of resonance radiation through a gas. The standpoint of "quantized energy of radiation" is boldly assumed, because it is the only standpoint from which the phenomena have been treated

with any success, and also because there is theoretical justification for quantizing energy in the case of harmonic oscillators and, therefore, in the case of special emission.

Chapter vii applies thermodynamical methods, developed by Nernst, Tolman, and others, to the problem of ionization at high temperatures, ionization being treated as a dissociation, as was suggested by Saha. Single and multiple ionization, and evidence from experiments and from solar and stellar spectra are discussed.

Chapter viii discusses thermochemical relations in cycles in which ionization is one step. If the energy changes in all other steps of the cycle are known, the work of ionization may be computed. This method is applied to distinguish between different possible ionization processes in the halogen acids.

Chapters ix, x, and xi deal, respectively, with X-ray spectra, photoelectric effects in vapors, and determinations of Planck's constant h.

Appendix I gives computational data, convenient tables of constants, and ionic or molecular velocities. Appendix II gives a clear account of the principles and results of Bohr's latest work on atomic structure—work which has not yet been published in English.

The book is well printed, unusually free from errors, contains numerous references to original sources, and is thoroughly up to date. It should be of great value to the special student of spectroscopy and related subjects as well as of interest to the reader with general scientific interest.

K. T. Compton

Princeton University

In sorrow we announce the death, on February 6, 1923, of

EDWARD EMERSON BARNARD

Senior Astronomer at the Yerkes Observatory

His valuable counsel, based upon his remarkable knowledge in the many fields of observational astronomy, was always available to the Editors, who gratefully make acknowledgment of their indebtedness to him. His contributions have especially enriched these pages.

An extended biographical sketch will appear in a future issue.

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THE MOVEMENT OF THE LINE OF NODES IN SPECTROSCOPIC BINARIES AND VARIABLES, AND ITS CONSEQUENCES

By G. SHAJN

ABSTRACT

Regression of the line of nodes due to oblateness of figure.—In the early part of the paper the mathematical analysis is given to show that one secular effect of oblateness in the primary is always a regression of the line of nodes of a satellite upon the equatorial plane of the primary.

Application of results to spectroscopic binaries.—It is shown that for reasonable values of the oblateness the rate of regression may be quite high. Tables are given, one showing the maximum range of variation of the angle of the plane of the orbit to the line of sight, and the other the maximum ratio of velocities. The stars R Canis Majoris and β Cephei are discussed by way of illustration.

Application to variable stars.—The effect of regression on the luminosity curve is discussed and the suggestion is made that the rate of regression is related to the distribution of density within the star.

Theoretical investigations and observations have shown that the figures of close pairs of stars must have considerable ellipticity. Thus the movements of a satellite about a central body of non-spherical form must be perturbed. The law of squares of distances cannot be applied to the given system.

The equations of motion are

$$\frac{d^2x_i}{dt^2}+f(m_0+m_1)\frac{x_i}{r^3}=\frac{\partial\Omega}{\partial x_i}. \quad (i=1, 2, 3) \tag{1}$$

The potential of the central star is

$$V=\frac{fm_0}{r}+\Omega\ .$$

It is known from celestial mechanics[1] that the potential of a spheroid is

$$V=\frac{fm_0}{r}\left[1-k\frac{I_1}{r^2}+l\frac{I_2}{r^4}+\ .\ .\ .\ .\ .\right], \tag{2}$$

where

$$I_1=\sin^2\delta-\tfrac{1}{3}\ ;\qquad I_2=\tfrac{35}{12}\sin^4\delta-\tfrac{5}{2}\sin^2\delta+\tfrac{1}{4}\ ;$$

δ is the declination of the satellite with respect to the equator of the central star; and k and l are constants depending on ellipticity and on the distribution of density in the body. The expressions for k and l are rather complicated. In case of homogeneity of the star, we have for k and l

$$k=\tfrac{3}{10}a^2\left(1-\frac{b^2}{a^2}\right)\ ,\qquad l=\tfrac{9}{70}a^4\left(1-\frac{b^4}{a^4}\right)\ ,$$

where a and b represent the equatorial and the polar radii.

We shall neglect the third term of (2) since it is small. Then, the perturbative function is

$$\Omega=\frac{fm_0k}{r^3}(\tfrac{1}{3}-\sin^2\delta)\ . \tag{3}$$

It is possible also to express Ω by means of the ellipticity (ϵ) and the relation of the centrifugal force to the weight on the equator (σ)

$$\Omega=fm_0(\epsilon-\tfrac{1}{2}\sigma)\frac{a^2}{r^3}(\tfrac{1}{3}-\sin^2\delta)\ . \tag{4}$$

Take the plane of the equator of the central body as fundamental and denote the inclination of the orbit to the plane of the equator by the letter γ and the angular distance of the satellite in its orbit from the ascending node on the equator by u.

Then

$$\sin\delta=\sin u\sin\gamma\ ,$$

$$\Omega=\frac{k^1}{r^3}(\tfrac{1}{3}-\tfrac{1}{2}\sin^2\gamma+\tfrac{1}{2}\sin^2\gamma\cos 2u)\ ,$$

[1] Tisserand, *Mécanique Céleste*, 2 and 4.

where

$$k^1 = fm_0\, a^2(\epsilon - \tfrac{1}{2}\sigma)\ .$$

The perturbations will have periodic and secular terms. Retaining only the secular terms, we get

$$\Omega = \frac{k^1}{r^3}(\tfrac{1}{3} - \tfrac{1}{2}\sin^2\gamma) = \frac{k^1}{A^3(1-e^2)^{\frac{3}{2}}}(\tfrac{1}{3} - \tfrac{1}{2}\sin^2\gamma)\ . \tag{5}$$

For the perturbation of the line of nodes the method of the variation of constants give

$$\frac{d\,\Omega}{dt} = \frac{1}{nA^2 \sin\gamma}\,\frac{\partial\Omega}{\partial\gamma}\ . \tag{6}$$

where A is the semi-axis of the orbit of the satellite and n is the mean motion.

On forming $\partial\Omega/\partial\gamma$ from equation (5) and substituting the values of k^1 and f, we obtain finally

$$\frac{d\,\Omega}{dt} = -n\left(\frac{a}{A}\right)^2\left(\frac{m_0}{m_0+m_1}\right)(\epsilon - \tfrac{1}{2}\sigma)\,\frac{1}{(1-e)^{\frac{3}{2}}}\cos\gamma\ ; \tag{7}$$

from which it follows that the secular motion of the line of nodes is always one of regression.

In this paper merely the consequences of the secular motion of the line of nodes in spectroscopic binaries and variables are treated. The problem of the motion of the line of apsides has been solved by Tisserand and its consequences are more or less clear. Our problem is analogous to the theory of the satellites of Jupiter, Saturn, or even better, Neptune. In general the yearly motion of the line of nodes will be quite high. For the fifth satellite of Jupiter the yearly motion is about 960°! For the fictitious system, $a/A = 5$; period, two days; ϵ, about $\frac{1}{10}$, the yearly motion $d\,\Omega/dt$ is about 160°. Even for a value of ϵ much less than $\frac{1}{10}$, the motion will be considerable.

Let us consider now the consequences of the motion of the line of nodes in the systems of close pairs. If the plane of the orbit coincides with the equator, namely $\gamma = 0$, the motion of the nodes will not be noticed in the observations. If $\gamma > 0$, which must occur rather frequently, the effect will be considerable.

The pole of the orbit of the satellite will describe a circle of radius γ. The plane of the orbit of the satellite will undergo a precessional motion.

The line of nodes (the intersection of the plane of the orbit with the equator of the primary member of the system) and consequently the plane of the orbit will not maintain the same inclination to the line of sight, though the inclination of the orbit to the equator of the primary will be invariable.

The inclination γ may be considerable, as is evident from the tables of the satellites of the large planets. In the case of spectroscopic binaries and variables the motion of the nodes will be of great importance, as the plane of the orbit varies in relation to the line of sight. Take, as principal plane, a plane perpendicular to the line of sight. The method of variation of constants for the perturbations of the plane of the orbit in relation to the principal plane gives an equation analogous to equation (6). From that equation we may get the variation of the plane of the orbit. It can be easily seen also that γ is a constant. It is easier still to get this variation from the equation:

$$\cos i = \cos \gamma \cos I + \sin \gamma \sin I \cos \Omega ,$$

where I is the inclination of the equator of the primary star with respect to the plane, which is normal to the line of sight, and i is the inclination of the orbit of the satellite relative to the same plane.

Then,

$$\sin i \frac{di}{dt} = \sin \gamma \sin I \sin \Omega \frac{d\Omega}{dt} .$$

The variation of the inclination will have different values according to the position of the line of nodes. Table I gives variations of the inclination in so far as they depend upon γ, I, and Ω.

According to this table the amplitude of the variation of the inclination may have a large value. The period of the variation depends upon the elements of the orbit, and amounts to some years if the ellipticity attains a considerable value. In case the latter is small, the period will be greater but the amplitude does not change.

The ellipticity cannot be small, owing to the short time of revolution, and probably also of rotation, and the cosmogonic conditions in close pairs.

The effect of periodic fluctuations of the plane of the orbit must lead to discovery by the observations. The component along the line of sight depends upon the inclination of the orbit to the line of sight. *A* and *B*, and all velocities lying between, simultaneously increase in the relation $\sin i_1/\sin i_2$. It can easily be seen that the amplitude increases in the same degree. It is evident that notwithstanding the variation of velocities all of the elements of the orbit remain unchanged, except $a \sin i$ and the mass function

TABLE I

γ	*I*								
	90°	80°	70°	60°	50°	40°	30°	20°	10°
0°.....	90–90	80– 80	70– 70	60–60	50–50	40–40	30–30	20–20	10–10
5......	85–95	75– 85	65– 75	55–65	45–55	35–45	25–35	15–25	5–15
10......	80–100	70– 90	60– 80	50–70	40–60	30–50	20–40	10–30	0–20
15......	75–105	65– 95	55– 85	45–75	35–65	25–55	15–45	5–35	− 5–25
20......	70–110	60–100	50– 90	40–80	30–70	20–60	10–50	0–40	−10–30
25......	65–115	55–105	45– 95	35–85	25–75	15–65	5–55	− 5–43	−15–35
30......	60–120	50–100	40–100	30–90	20–80	10–70	0–60	−10–50	−20–40

which depends on *i*. The velocities will change to such a great extent that this can be seen in the observations. In Table II we have the maximum ratio of velocities for given values of γ and *I*. Only in case $\gamma = 0$, will there be no effect.

For illustration we have taken two stars with short periods of revolution, R Canis Majoris[1] and β Cephei.[2]

For the first, the curves were drawn for 1908, 1909, 1910, 1911, and 1912, which show a decrease of the amplitude from 1908 to 1912. Then, the points of the observed velocities near minimum and maximum were reduced to the minimum and maximum by means of a table computed beforehand from the normal curve. In this table are given the variations of the velocities for the epoch just before and after the minimum or maximum at intervals of $0^{d}20$.

[1] F. C. Jordan, *Publications of the Allegheny Observatory*, 3, No. 7.

[2] A. Belopolsky, *Bulletin de l'Académie des Sciences de Russie*, 1918.

This method has been applied to give a larger weight to the points of maximum and minimum.

We give here the velocities and the amplitudes of the curve of R Canis Majoris for each year.

TABLE II

γ	I								
	90°	80°	70°	60°	50°	40°	30°	20°	10°
0......	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5......	1.00	1.03	1.07	1.11	1.16	1.23	1.36	1.63	2.97
10......	1.02	1.06	1.14	1.23	1.35	1.53	1.88	2.94	....
15......	1.04	1.10	1.22	1.40	1.58	1.94	2.73	6.58	4.85
20......	1.06	1.15	1.31	1.53	1.88	2.53	4.41	....	2.88
25......	1.10	1.22	1.41	1.74	2.29	3.50	9.40	8.11	1.88
30......	1.15	1.31	1.55	2.00	2.88	5.41	....	4.41	1.67

TABLE III

R Canis Majoris—Variation of the Amplitude

Epoch	A	B	Amplitude
	km	km	km
1908...............	88.7 (2)	14.6 (3)	74.1
1909...............	73.7 (7)	13.1 (4)	60.6
1910...............	76.4 (5)	15.8 (3)	60.6
1911...............	68.9 (3)	16.3 (2)	52.6
1912...............	67.3 (3)	17.8 (3)	49.5

The mean error of a normal place is ± 1.96 km. The diminution of the amplitude seems to be very probable.

As to β Cephei, we do not find the same effect. The components being close to each other, the probability of coincidence of the orbit of the satellite with the plane of the equator of the primary star is considerable.

I may be allowed to refer to the paper by Guthnick and Prager,[1] in which the system β Ursae Majoris is discussed. The observations of 1908–1917 (good for their precision) show a decrease of the amplitude from 5.90 km to 1.24 km. Guthnick suggests that, if

[1] *Veröffentlichungen der kgl. Sternwarte in Berlin Babelsberg*, **2**, Heft 3, 1918.

the decrease of the amplitude is confirmed, it has cosmogonic importance. It is simpler to admit here that the effect is caused by the motion of the line of nodes. Later, the amplitude must increase.

If I is nearly zero, the relation of velocities at different positions of the line of nodes is great, as can be seen from the table; but the difficulty of determining them increases, for at the same time the velocities are less.

Attention should also be called to a paper by Henroteau, "The Orbit of β Canis Majoris," which is known to me only from the abstract in the *Astronomischen Jahresbericht* for 1918. This star also shows a definite variation of the amplitude.

We have examined only a few stars. Doubtless the effect of the variation of the amplitude will be found in a considerable number of close pairs.

In variables, too, the effect caused by the motion of the line of nodes must be seen. Guthnick,[1] who has taken this into consideration justly, notes that the duration of the eclipse and the range of brightness may undergo fluctuations. Really, the conditions of eclipse depend not only on the elements and the dimension of the system, but also on the position of the visible orbit relative to the line of sight. Depending on the fluctuations of the inclination, the visual distance of the components will vary, and in one and the same phase the disk of the satellite will be projected in a different way on the disk of the central star. When $i=90°$, the path of the satellite will be projected on the diameter; and when $i\gtrless 90°$, on the chord. In one position of the line of nodes the satellite as a whole is projected on the disk of the chief star at one time, and may fall outside the disk at another. The range of brightness, as well as the duration of the eclipse, depends upon the visible distance of the stars

$$D=A(1-e^2)\frac{\sqrt{1-\cos^2 w \sin i}}{1+e\cos(w-\alpha)}\,.$$

The angle, which defines the duration of the eclipse in the general case, is a rather complicate function.

The curve of brightness varies from one epoch to another and in some cases the changes attain a considerable value. For each

[1] *Astronomische Nachrichten*, 202, 137, 1916

separate case it is possible to discount the influence of the variation of inclination on the elements of the eclipse. Indeed, photometric observations have shown that the curve of brightness of some stars varies a little from one epoch to another. It remains finally to indicate that the star R Canis Majoris as a variable shows a certain variation of the curve of brightness.[1]

The effect caused by the motion of the line of nodes may serve for the explanation of the phenomena in close pairs. If, out of a large number of observations of spectroscopic binaries and variables, we define the amplitude and period of the variations of velocities, we shall obtain some notion concerning the inclination (i), which usually cannot be found from spectral observations. In general, the curve of the variation of velocities is a sine curve and from it one can form some idea about the inclination at different epochs.

Taking the period into consideration, it is possible to form some idea of the inclination of the orbit of the satellite with respect to the equator (γ), and of the ellipticity by means of successive approximations. If the ellipticity is known from photometric observations, we can decide upon ($\epsilon - \frac{1}{2}\sigma$), and this in its turn gives the possibility of getting an idea about the *distribution of densities within the star*. If it shall be found that in rather remote epochs the amplitude of velocities does not show a variation, the coincidence of the orbits of the satellites in a close system with the plane of the equator of the central star will be proved. This fact will be, undoubtedly, of great cosmogonic importance. Otherwise, the phenomenon is not so simple as it seems to be at first sight. Indeed, if the mass of the satellite is considerable (as is often the case in close pairs), the equator and also the orbit of the primary will have precessional motions. The inclination of the equator I will be variable. But in the first approximation everything mentioned above remains unaltered.

Spectroscopic binaries of short period offer problems of great interest to students of Celestial Mechanics.

It is a pleasant duty to acknowledge my indebtedness to Professor A. Belopolsky for his interest in this paper.

PULKOVO
April 1922

[1] *Astrophysical Journal*, 41 291, 1915.

PHOTOGRAPHIC STUDIES OF NEBULAE

THIRD PAPER[1]

By JOHN CHARLES DUNCAN

ABSTRACT

Studies of the form and structure of nebulae from photographs made with the 100-inch and 60-inch reflectors and the 10-inch Cooke refractor in the years 1920–1922. Evidence of the existence of *dark nebulosity* is found in N.G.C. 1977, M 78, the Trifid nebula, the dark objects Barnard 72, 92, 93, and 133, and the America nebula. N.G.C. 4038–4039 is a bright spiral of *unique form* and with faint extensions of *extraordinary appearance.* In a field in Coma Berenices the size of the full moon, the 100-inch telescope photographs no less than *319 small nebulae.* The object N.G.C. 6822 is found to be a *mixture of stars and small nebulae*, resembling the Magellanic clouds. *Halftone reproductions* are given of photographs with the Hooker 100-inch telescope of N.G.C. 1977, N.G.C. 4038–4039, Barnard 72, the Trifid nebula, Barnard 92 and 93, Barnard 133, N.G.C. 6960, and two regions in the America nebula; and of a photograph with the 10-inch Cooke lens of the region in Cygnus that includes the America nebula.

During the past five summers, and the fall, winter, and spring of 1920–1921, it has been my privilege to participate in the program of photographic observations with the instruments of the Mount Wilson Observatory. The greater part of my observing time was spent in duplicating, as well as possible, the excellent negatives made several years ago by Mr. Ritchey and Mr. Pease with the 60-inch reflector, and in photographing some of the same objects with the 100-inch reflector. The purpose of this work was to furnish material for the study of internal motions of spiral nebulae, and several plates thus obtained have been used for this purpose by Mr. van Maanen with results which he has published.[2] In addition to this work, photographs of other interesting objects were made, as opportunity offered, with the purpose of studying their form and structure, a purpose to which the 100-inch Hooker telescope is particularly well adapted; and the present paper, like its two predecessors, gives account of some of these studies.

[1] *Contributions from the Mount Wilson Observatory*, No. 256. The first and second papers appeared as *Contributions*, Nos. 177 and 209, respectively; *Astrophysical Journal*, **51**, 4, 1920; **53**, 392, 1921.

[2] *Mt. Wilson Contr.*, Nos. 213, 214, 242, 243; *Astrophysical Journal*, **54**, 237, 347, 1921; **56**, 200, 208, 1922.

The reflectors were used at their full aperture, except in certain cases that are noted, when a diaphragm was placed in front of the mirror to improve the definition at the edge of the field. With the 60-inch reflector, the size of plates used was $3\frac{1}{2}\times 3\frac{1}{2}$ in.; with the 100-inch, either 5×7 or 8×10 in. In all cases, the plates were backed to prevent halation.

In preparing the positives for Plates VI, VIII, X, XI, XII, XV, and XVI, the contrast was increased by repeated copying on slow plates; the other reproductions were made from direct positives.

I gratefully acknowledge the benefit of Professor E. E. Barnard's counsel in selecting dark celestial objects for study; suggestions and help from Mr. Ellerman in copying the plates for reproduction; and the efficient assistance at the telescope of Mr. Hoge and Mr. Klemann, night assistants at the 60-inch and 100-inch reflectors, respectively.

The Nebula N.G.C. 1977 Orionis

$\alpha = 5^h 31^m.5, \quad \delta = -4°59' \ (1920)$

Negative Δ 125, Hooker telescope, 1921, January 6. Seed 30 plate, exposure 30^m. Aperture 84 inches. Very poor seeing, images large and round

Δ 126, Hooker telescope, 1921, January 6. Seed 30 plate, exposure $4^h 30^m$. Aperture 84 inches. Very poor seeing, images large and comate

Δ 127, Hooker telescope, 1921, January 7. Seed 23 plate, exposure $5^h 40^m$. Aperture 84 inches. Seeing fair, images round. Illustrated in Plate VI

This magnificent mass of light surrounds the fifth-magnitude star *c* Orionis, the northernmost star of the Sword of Orion, and is connected with the great nebula around θ Orionis by faint nebulosity. Excellent photographs of it have been published by Roberts,[1] Keeler,[2] and others. The dark indentation in the south side is by no means wholly without luminosity, although it seems so on the halftone reproduction. The luminous streaks that appear within it on the original negative, and its bright border on the north are suggestive of the dark "bay," Barnard 33, south of

[1] *Knowledge*, **17**, 62, 1894.

[2] *Lick Observatory Publications*, **8**, Plate 12, 1908.

PLATE VI

North

Negative Δ 127

THE NEBULA N.G.C. 1977, SURROUNDING *c* ORIONIS

Photographed with the 100-inch Hooker telescope, 1921, January 7. Exposure, 5^h40^m

Scale: 1 mm = 18″.2 (0.87 that of original negative)

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PLATE VII

North

Negative Δ 156

THE NEBULA N.G.C. 4038–4039 CRATERIS

Photographed with the 100-inch Hooker Telescope, 1921, March 7. Exposure, $3^h 15^m$

Scale: 1 mm = 10″.2 (1.55 times that of original negative)

ζ Orionis.[1] The bright ring around the star west of the nebula is of course due to halation, the backing of the plate having been lacking at that point.

The Nebulae N.G.C. 2064, 2067, 2068, 2071; M 78 Orionis

$\alpha=5^h42^m.6, \quad \delta=+0°1'$ (1920)

Negative Δ 133, 60-inch reflector, 1921, January 8. Seed 23 plate, exposure 3^h20^m. Images small and comate

This is well described by Curtis[2] and illustrated by Keeler.[3] The present plate shows faint extensions of 2071 which bring it up to about the size of 2068, and shows, more clearly than does the reproduction of Keeler's plate, the presence of dark markings in and around all four nebulae. N.G.C. 2064 and 2067 are connected by faint nebulosity and there is a faint patch 4′ southwest of 2064. A dark lake in the brightest part of 2068, with a tenth-magnitude star at its western edge, is very striking. The northern and northeastern edges of 2068 are sharply limited by an irregular line, apparently of absorbing material. A dark indentation on the western side of 2067 is reminiscent of those in M 16.[4] Near the western edge of the plate the faint stars become numerous, as if those in other parts of the field were hidden by the nebula; and it seems likely that all these nebulae are bright outcroppings near the western edge of an obscuring mass that, as is shown on plates made with wide-field instruments, occupies a large area in the constellation of Orion, including the region of θ and c Orionis and that south of ζ.

The Nebula N.G.C. 4038–4039 Crateris

$\alpha=11^h57^m.8, \quad \delta=-18°26'$ (1920)

Negative T-69-H, Cooke 10-inch lens, 1920, April 10. Seed 30 plate, exposure 1^h30^m (Hubble)

Δ 156, Hooker telescope, 1921, March 7. Seed 30 plate, exposure 3^h15^m. Images large and round. Illustrated in Plate VII

Δ 161, Hooker telescope, 1921, April 5. Seed 30 plate, exposure 3^h30^m. Images large and round

[1] *Mt. Wilson Contr.*, No. 209, Plate V; *Astrophysical Journal*, 53, Plate XI, 1921.

[2] *Lick Observatory Publications*, 13, 23, 1918.

[3] *Ibid.*, 8, Plate 14, 1908.

[4] *Mt. Wilson Contr.*, No. 177, Plate IV; *Astrophysical Journal*, 51, Plate III, 1920.

Listed in the N.G.C. as two objects with the same right ascension but differing 1′ in declination, this is really a single nebula of extraordinary form. Its interesting appearance was noted on the Franklin-Adams chart, and later on a negative made with the 10-inch Cooke lens, by Mr. Hubble, who suggested it to me for study with the 100-inch telescope. A description of this nebula, and of N.G.C. 4027 which lies near it, has recently been published by Perrine[1] from observations with the Córdoba 75-cm reflector. He describes each nebula (4038–4039 and 4027) as "a hook extending out from a ring," and this description is well confirmed by Mr. Hubble's small-scale plate, which is of excellent definition and shows both nebulae.

The Hooker plates of 4038–4039 bring out additional features of much interest. The ring is incomplete, being open where it joins the hook, which bends backward at the end, forming an S with its lower (southern) end incomplete. Both hook and ring are crowded with bright, starlike condensations similar to those seen in most bright spiral nebulae. This description, however, applies only to the brightest part of the nebula, which is surrounded by a fainter portion, of smooth, amorphous appearance, that seems to form a double bag around the ring and hook. Most remarkable of all, two faint extensions, like antennae, seem to cross at the eastern end of the bag, one reaching northward and the other southward, and both concave toward the west. The northern antenna may be traced to a distance of 6′, and the southern one to a distance of 12′ from the point where they intersect, these distances being measured along chords of the curves.[2] It will be of interest to learn what type of internal motion, if any, may be observed in this nebula from future photographs.

[1] *Monthly Notices R.A.S.*, 82, 487 ff., 1922.

[2] Since this description was written, I have seen some excellent plates of the nebula made by Lampland with the 40-inch Lowell reflector in 1917, which show the faint extensions clearly.

Region of Many Small Nebulae in Coma Berenices

$\alpha = 12^h 55^m\!.7$, $\delta = +28°25'$ (1920)

Negative Δ 171, Hooker telescope, 1921, May 7. Seed 30 plate, exposure $1^h 15^m$. Images slightly elongated

Δ 172, Hooker telescope, 1921, May 8. Seed 30 plate, exposure 4 hours. Images slightly comate

Δ 176, Hooker telescope, 1921, June 5. Seed 30 plate, exposure 1 hour. Images slightly comate

Δ 178, Hooker telescope, 1921, June 6. Seed 30 plate, exposure 1 hour. Images slightly comate

The remarkable character of this region, which is very near the north galactic pole, has been pointed out by Curtis,[1] who counted more than 300 small nebulae in an area $50' \times 40'$. They are distributed densely over the field covered by an 8×10-inch plate at the principal focus of the Hooker telescope, with perhaps some tendency

TABLE I

No.	α (1920)	δ (1920)	Remarks
1	$12^h 54^m\!.5$	$+28° 41'$	Spindle, 40″ long, with dark band making angle of 60° with axis
2	12 55.2	+28 18	Spindle, 55″ long, with dark band making angle of 60° with axis
3	12 55.7	+28 25	Globular, 30″ or more in diameter
4	12 55.8	+28 10	Spindle, 45″ long
5	12 56.2	+28 26	Elliptical, soft nebulosity, 15″×30″
6	12 56.3	+28 39	Spindle, 1′ long, bright nucleus, faint wings
7	12 56.5	+28 31	Spindle, 20″ long, with dark band making angle of 60° with axis
8	12 56.7	+28 34	Fine, two-arm spiral, 30″×40″
9	12 56.8	+28 16	Spiral, 30″×35″
10	12 57.2	+28 21	Spiral seen in plan, 1′ diameter, bright nucleus

to congregate along a northwest-southeast diagonal. On the 4-hour plate (Δ 172), in a circle of 30′ diameter—a little smaller than the full moon—centered on the position given above, I counted 319 nebulae, 115 stars, and 206 objects of doubtful character. Most of the nebulae are of the type designated by Hubble as "globular"—circular or elliptical in outline, with bright centers and ill-defined edges. There are also many of a spindle shape, lying in various position angles, and a few that are distinctly spiral.

[1] *Lick Observatory Publications*, **13**, 33, 1918.

Among the particularly interesting nebulae on the plates are those listed in Table I. The positions were derived by measuring on the negative with a millimeter scale from the globular nebula at the center. This is probably N.G.C. 4872, but it is not possible to identify it with certainty.

The dark bands of Nos. 1, 2, and 7 may possibly be spaces between two nebulae imperfectly seen; but if so, it is a coincidence that three pairs are thus similarly aligned.

The Dark Nebula Barnard 72 Ophiuchi

$\alpha = 17^h 19^m.0, \quad \delta = -23°39'$ (1920)

Negative Δ 177, Hooker telescope, 1921, June 5. Seed 30 plate, exposure 3 hours. Poor seeing, images large and elongated

Δ 189, Hooker telescope, 1921, July 4. Seed 30 plate, exposure $2^h 45^m$. Seeing fair, images fairly small and round. Illustrated in Plate VIII

This S-shaped dark marking is well known from Barnard's photographs made with the Willard and Bruce lenses. It is situated 1°.5 north and 20′ east of θ Ophiuchi, in a region of the Milky Way liberally sprinkled with faint stars. The illustration, in which the contrast of the original plate has been considerably increased, shows well the outline of the main dark body and also the two smaller objects, Barnard 68 and 69, near the lower left corner of the plate; but it fails to bring out a faintly luminous background that appears on the originals and seems to be due to multitudes of very faint stars rather than to nebulosity. An additional feature, evident on the original negatives, but barely suggested in the reproduction, is a bifurcation of the eastern part of the S, of which the fainter branch lies on the northern side.

The Trifid Nebula N.G.C. 6514, M 20 Sagittarii

$\alpha = 17^h 57^m.2, \quad \delta = -23°2'$ (1920)

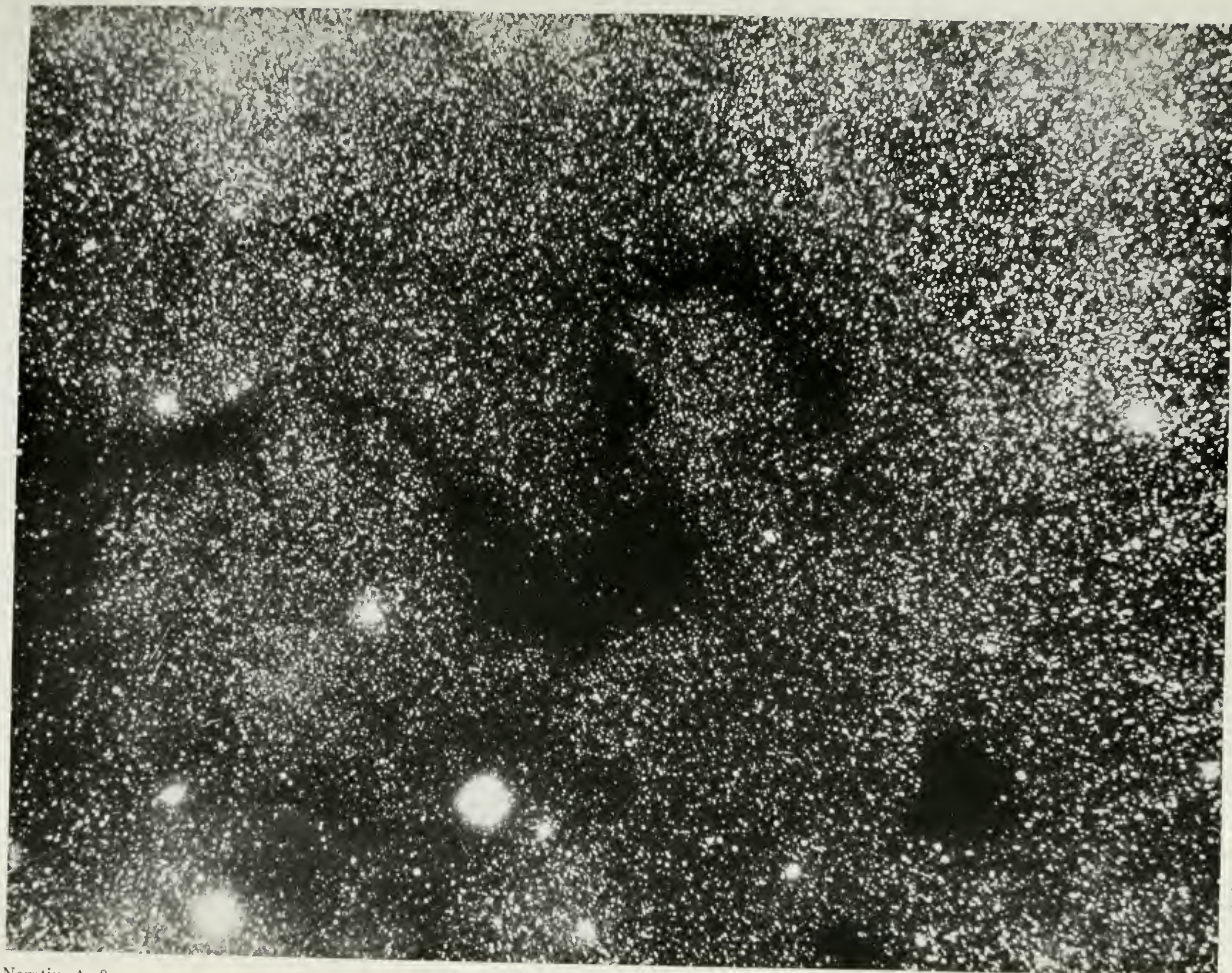
Negative Δ 175, 60-inch telescope, 1921, June 4. Aperture 54 inches. Seed 23 plate, exposure $4^h 10^m$. Seeing rather poor, images round but large

Δ 183, Hooker telescope, 1921, June 30. Seed 23 plate, exposure 25^m

Δ 184, Hooker telescope, 1921, June 30. Seed 23 plate, exposure $2^h 30^m$. Seeing fair, images elongated. Illustrated in Plate IX

PLATE VIII

North



142a

Negative Δ 189

THE DARK NEBULA BARNARD 72 OPHIUCHI

Photographed with the 100-inch Hooker telescope, 1921, July 4. Exposure, 2^h45^m

Scale: 1 mm = 18″.2 (0.87 that of original negative)

PLATE IX

North

Negative Δ 184

THE TRIFID NEBULA, N.G.C. 6514 = M20 SAGITTARII

Photographed with the 100-inch Hooker telescope, 1921, June 30. Exposure, 2^h30^m
Scale: 1 mm = 11.″5 (1.37 times that of original negative)

PLATE X

North

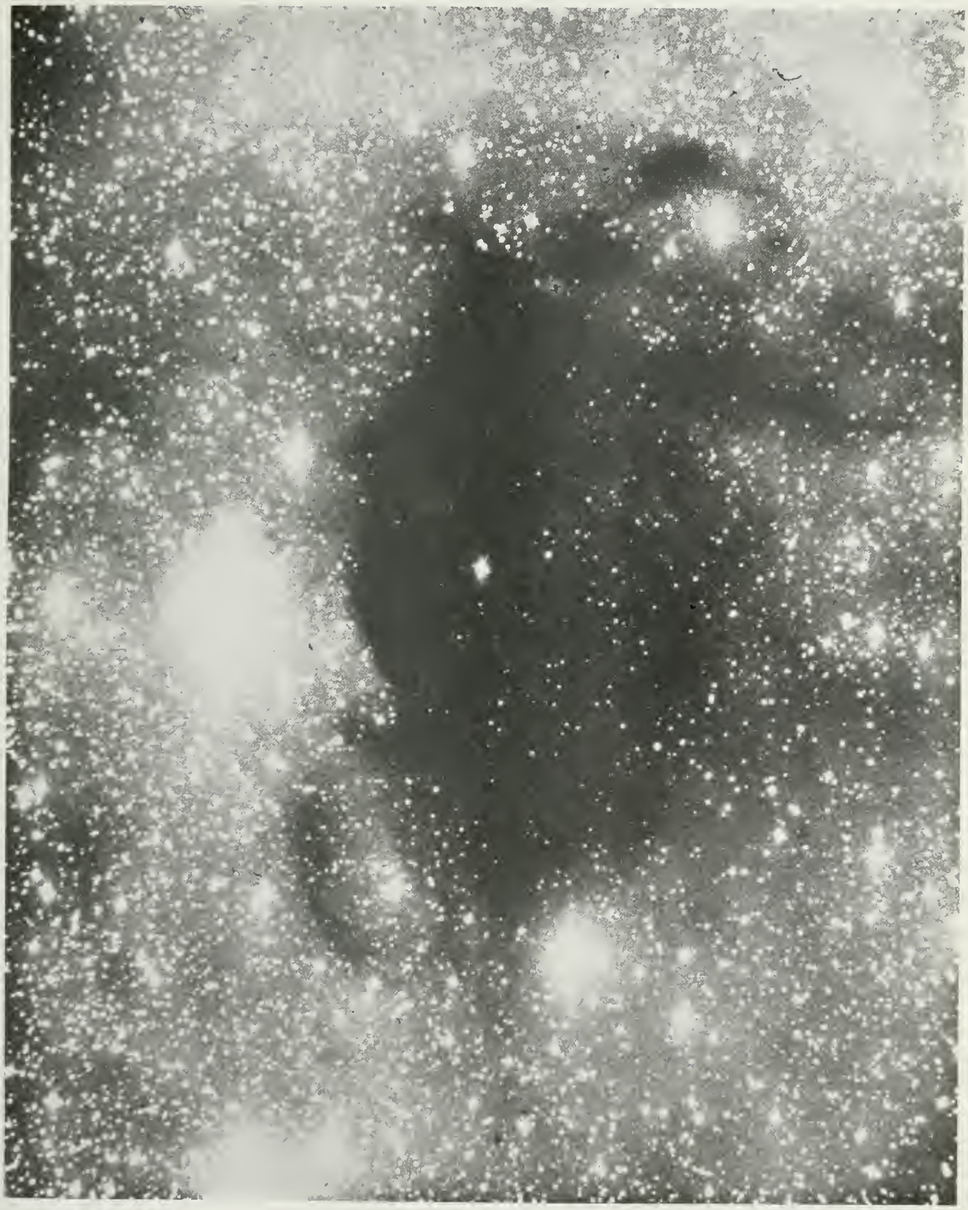
The Dark Nebulae Barnard 92 and 93 Sagittarii

Photographed with the 100-inch Hooker telescope, 1922, June 24. Exposure, 4 hours
Scale: 1 mm = 20.″3 (0.78 that of original negative)

PLATE XI

North

H 1



Negative Δ 207

THE DARK NEBULA BARNARD 92 SAGITTARII

Photographed with the 100-inch Hooker telescope, 1922, June 24. Exposure, 4 hours
Scale: 1 mm = 8″.7 (1.82 times that of original negative)

One of the most beautiful of the nebulae, which has been drawn and photographed many times. Only about half the luminous area is "trifid," but the same general character—well-defined, dark areas superposed upon bright nebulosity—prevails throughout the whole, and in the neighboring bright nebula M 8 as well. The dark lanes can best be explained by obscuration, but the obscuring material may well be dark extensions of the bright nebulosity. An interesting feature is the bright line of light, sharply convex northward, that lies near the southern edge of the brightest part and is continued on the east by a dark lane, concave northward, that ends in an irregular dark spot. This curved line of light is duplicated by a fainter one, 2′ farther south. The whole nebula is isolated from the thickly starred region about it by a border several minutes wide in which there are comparatively few faint stars, an effect shown most distinctly near the bottom of the engraving.

The Dark Nebulae Barnard 92 and 93 Sagittarii

Barnard 92; $\alpha = 18^h 11^m.1$, $\delta = -18°15'$ (1920)

Negative Δ 179, Hooker telescope, 1921, June 6. Seed 30 plate, exposure $2^h 10^m$. Dull mirror, poor seeing, images large and comate

Δ 207, Hooker telescope, 1922, June 24. Aperture 84 inches. Seed 60 plate, exposure 4 hours. Seeing good, images small and round. Illustrated in Plates X and XI

These objects have been well described by Barnard[1] from visual observations with the Yerkes 40-inch refractor and from his photographs with the Willard and Bruce lenses. The Hooker telescope, by its greater scale, light-gathering power, and resolving power, brings out many additional details.

The objects lie in one of the densest star clouds of the Milky Way. The entire field covered by an 8×10-inch plate ($54' \times 68'$), except the parts occupied by the dark objects, is filled with faintly luminous nebulosity[2] studded with myriads of faint stars. Number 92 extends about 16′ north and south and 9′ east and west. Near

[1] *Lick Observatory Publications*, 11, Plates 51, 54, 55, 57, 1913; *Astrophysical Journal*, **38**, 496 and Plates XIX, XX, 1913.

[2] The presence of this film of nebulosity was first pointed out to me by Professor Barnard, on his Bruce plates.

its center is a twelfth-magnitude star whose position was measured carefully by Barnard in 1913.[1] The eastern border is very sharply defined, especially southeast of the central star where the surrounding nebulosity is pretty bright; while on the east side the luminous nebulosity and stars encroach upon the dark spot, and the border is difficult to locate. The nebulosity is here seen in the form of curved filaments that recall the Merope nebula. This is distinctly seen in the original, although not well shown in the engraving. Three minutes of arc northeast of the central star, in the darkest part of the dark nebula, is a patch of faintly luminous nebulosity about 1′.5 in diameter. At the northern end of the main dark body are two small ones, about 2′.5 in diameter, separated from it and from each other by bridges of faintly shining nebulosity; and on the southeast is a patch, about 4′ long, which, although darker than its surroundings, contains a faint film of nebulosity and numerous stars.

Eastward from Barnard 92 extend irregular, semi-lucent lanes which, on Barnard's wide-field plates, are seen to bend to the southwest and may be traced to a distance of more than a degree, while their two principal branches are continued by rows of bright stars nearly 3° long.

In the faint nebulous background, 24′ west and 8′ north of the central star in Barnard 92, is an isolated semi-vacant spot, about 1′.5 long, which is the more sharply defined at its eastern border.

Barnard 93 consists of a head, shaped like a broad arrowhead, 3′ long, in position angle 40°, and 1′ wide, situated 21′ east and 11′ north of the central star of Barnard 92; and a tail that extends 13′ in a direction a little west of south. The head seems entirely lightless except for a close pair of stars near its base, but the tail contains a number of faint stars, and is bifurcated by a strip of faintly shining nebulosity.

On the halftone plates appear numerous small, sharp, dark defects, which are in the original negative and are examples of a fault to which the Seed 60 plates, otherwise excellent, seem especially prone.

[1] *Loc. cit.*

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PLATE XII

North

Negative Δ 188

THE DARK NEBULA BARNARD 133 AQUILAE

Photographed with the 100-inch Hooker telescope, 1921, July 3. Exposure, 4 hours
Scale: 1 mm = 13″.2 (1.20 times that of original negative)

The Dark Nebula Barnard 133 Aquilae

$\alpha = 19^h 1^m.9, \quad \delta = -6°58'$ (1920)

Negative Δ 188, Hooker telescope, 1921, July 3. Seed 30 plate, exposure 4 hours. Poor seeing, images round but rather large. Illustrated in Plate XII

This tadpole-shaped object is some 16′ long, with the head or southern end sharply defined, and the tail losing itself among the stars on the north. It is concave on the eastern side, where the dark area is encroached upon by faint stars. The southern end must be quite opaque, as in an oval area about 9′ long and 4′ wide not a single star appears on the plate. No luminous nebulosity can be seen on the plate, and the background on which the dark body is projected seems to consist entirely of stars. The form of the object is well shown, though of course on a smaller scale, in a reproduction of a Bruce photograph by Barnard.[1]

The Object N.G.C. 6822 Sagittarii

$\alpha = 19^h 40^m.4, \quad \delta = -14°58'$ (1920)

Negative Δ 190, Hooker telescope, 1921, July 4. Seed 30 plate, exposure 50^m. Images large

Δ 191, Hooker telescope, 1921, July 5. Seed 30 plate, exposure $3^h 50^m$. Images elongated

This object, which was discovered by Barnard, is described in the N.G.C. as "very faint, large, elongated, diffuse." Its study was suggested to me by Mr. Hubble, who had noted in it, from his observations with the Cooke 10-inch lens, a resemblance to the Magellanic clouds. On my plates the main portion appears to consist of a cloud of faint stars 10′ long in a north and south direction and 3′ wide. A less dense extension of about the same width and also consisting of stars can be traced 5′ farther south. At the north end is a V-shaped nebula, 25″ long, like a double-tailed comet with its apex in a star and the opening of the V directed westward. Several faint stars or starlike condensations are located in the arms of the V. At distances of 3′ on the east and 4′.5 in a direction a little north of west from this small nebula are two others of similar size and texture. Around the brightest star of each of the two last-

[1] *Astrophysical Journal*, **49**, Plate IV *b*, 1919.

named nebulae is a faint circular ring about 20″ in diameter. The eastern one of the three small nebulae is of a V-shape with the opening directed southwestward; the western one is elongated in a northwest-southeast direction. The object has been photographed with the 75-cm Córdoba reflector by Perrine,[1] who also notes its resemblance to the Magellanic clouds.

The Nebula N.G.C. 6960 Cygni

$\alpha = 20^h 43^m\!.4, \quad \delta = +30°26'(1920)$

Negative Δ 203, Hooker telescope, 1921, August 3. Aperture 84 inches. Seed 30 plate, exposure 7 hours. Images small and slightly elongated north and south. Illustrated in Plate XIII

One of the beautiful filamentous nebulae in Cygnus, which extends more than a degree in a direction approximately north and south. The fourth-magnitude star 52 Cygni is just west of the middle. No description is needed for an object so well known as this, but attention may be called to the faint outlying filaments west of the bright star and on both sides of the main nebula near the south end. The nebula is a frontier between a region of many faint stars on the east and fewer on the west.

The North America Nebula N.G.C. 7000 Cygni

$\alpha = 20^h 55^m, \quad \delta = +44\overset{\circ}{.}5$ (1920)

Negative Δ 224, Cooke 10-inch lens, 1922, August 18. Seed 60 plate, exposure 45^m

Δ 225, Cooke 10-inch lens, 1922, August 18. Seed 60 plate, exposure 4 hours

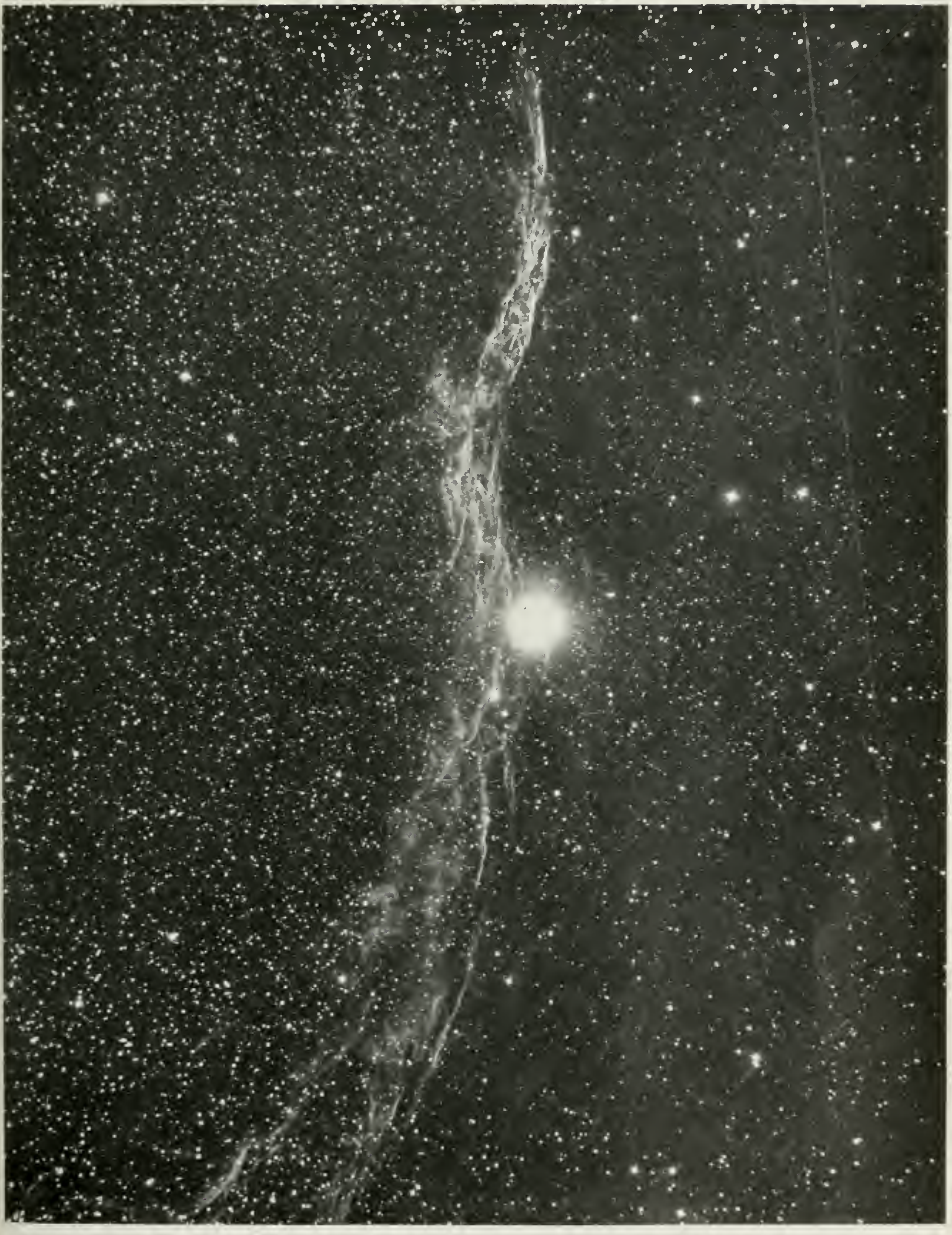
Δ 228, Cooke 10-inch lens, 1922, August 19. Seed 60 plate, exposure $5^h 45^m$. Illustrated in Plate XIV

Situated in the Milky Way about 3° east of α Cygni, this great nebula lies among dense clouds that are composed of faint stars with perhaps a very faint background of nebulosity. On the east and west the nebula is bounded by dark regions that meet just south of "Panama," presumably obscuring the whole of "South America," since that continent does not appear. West and south of the main bright nebula are faint nebulous wisps of great extent, well shown in the engraving. A group of stars in the form of a single-arm spiral,

[1] *Monthly Notices R.A.S.*, **82**, 489, 1922.

PLATE XIII

North



Negative Δ 203

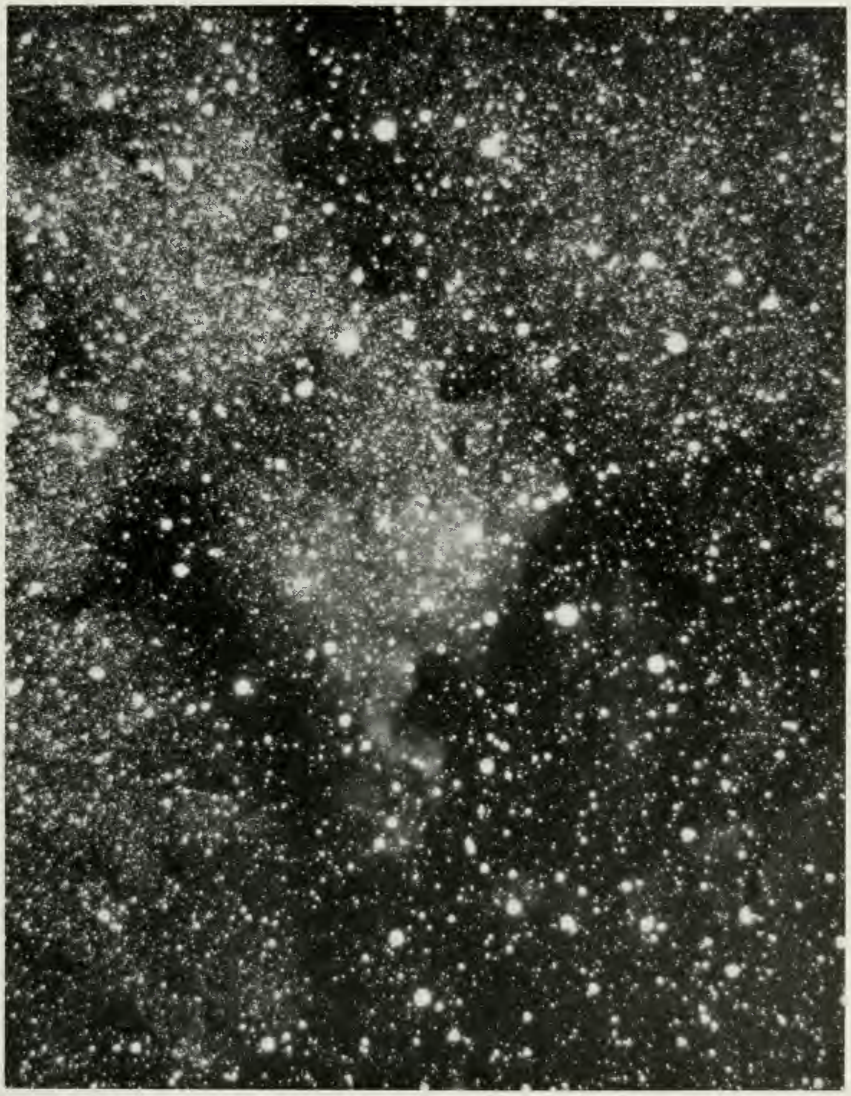
The Nebula N.G.C. 6960 Cygni, with Meteor Trail

Photographed with the 100-inch Hooker telescope, 1921, August 3. Exposure, 7 hours
Scale: 1 mm = 21.″9 (0.72 that of original negative). The line on the right is the meteor trail

PLATE XIV

North

146



Negative Δ 228

THE NORTH AMERICA NEBULA, N.G C. 7000 CYGNI

Photographed with the 10-inch Cooke lens, 1922, August 19. Exposure, 5^h45^m
Scale: 1 mm = 3′ (same as original negative)

PLATE XV

North

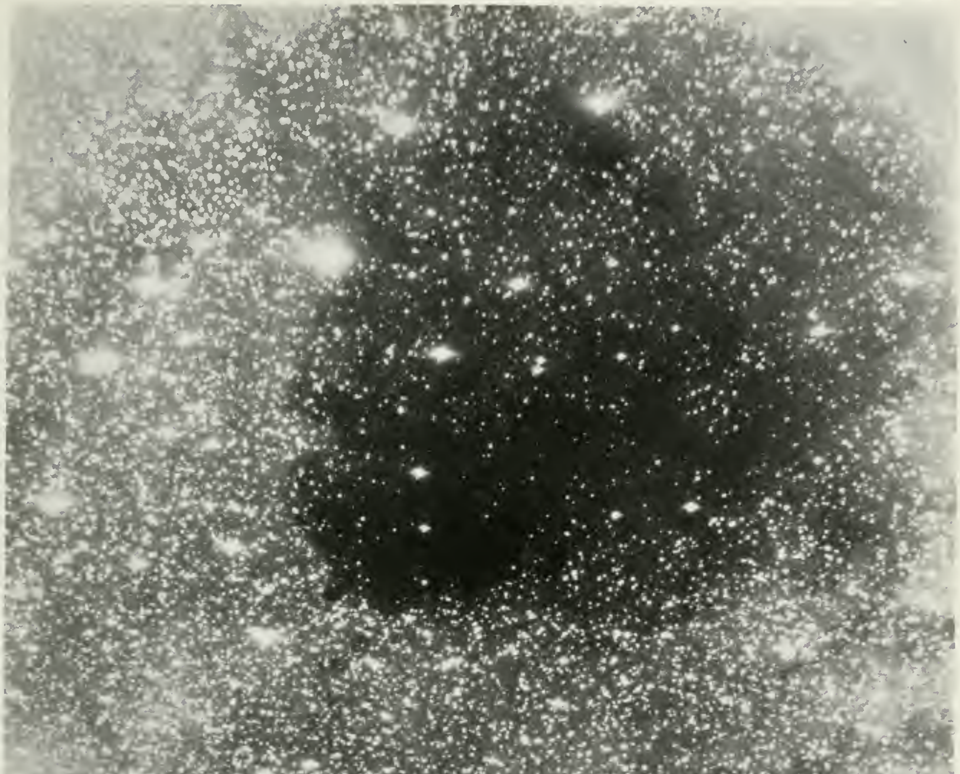
Negative Δ 217

South End of the North America Nebula

Photographed with the 100-inch Hooker telescope, 1922, July 26. Exposure, 5 hours
Scale: 1 mm = 21″9 (0.72 that of original negative)

PLATE XVI

North



Negative Δ 231

DARK AREAS AT NORTH END OF THE NORTH AMERICA NEBULA

Photographed with the 100-inch Hooker telescope, 1922, August 22. Exposure, 4 hours
Scale: 1 mm = 18″.6 (0.85 that of original negative)

which may be found 68 mm from the bottom of the engraving and 50 mm from the right edge, is a conspicuous feature of the original negative. The photograph is reproduced here mainly to show the location of the regions photographed with the Hooker telescope and reproduced in Plates XV and XVI.

The South End of the North America Nebula

$\alpha = 20^{h}55^{m}.9, \quad \delta = +43°10'$ (1920)

Negative Δ 217, Hooker telescope, 1922, July 26. Aperture 84 inches. Seed 60 plate, exposure 5 hours. Images slightly elongated. Illustrated in Plate XV

The region shown on the negative is represented on Plate XIV by an area of about 18×22 mm, with its center 56 mm from the left edge of the engraving and 49 mm from the bottom. Two defects appear in Plate XV: one a white, vertical line 3 mm long, 49 mm from the left edge and 28 mm from the top; and the other a dark, horizontal mark 2 mm long, 48 mm from the left and 40 mm from the bottom.

The plate is filled with a bewildering amount of detail, both bright and dark. The brightest part of the nebula is the irregular line or ridge that passes from the northeast corner of the plate to the middle of the south side. The brightness falls off rapidly on the east side of this ridge and slowly on the west. The area near the middle of the north side is also quite bright, but its brightness is exaggerated in the reproduction. The central part of the plate, a region some 15′ in diameter in the middle of the "Gulf of Mexico," seems to be entirely devoid of luminous nebulosity, while the star density within it is certainly less than a tenth that in the bright nebulosity east of the ridge. The border between the gulf and the continent is not sharp, and both the star density and the brightness of the nebulosity increase gradually eastward from the gulf. The main nebulosity at the southeast side of the gulf is in the form of nearly parallel streaks, which are broken by irregular transverse dark markings. Apparently associated with these streaks is the great arch of semi-obscuring matter, over a half-degree long, that is a conspicuous object just at the left of the middle of Plate XV.

Other noteworthy features are the dark area, about $3' \times 11'$, just east of the bright ridge in the upper half of the plate; the small, dark spot northeast of the bright star in the northwest corner; and the intricately curved and branched dark marking 3 cm from the bottom and 5 cm from the right edge of Plate XV. The appearance of an obscuring body in this last marking is especially convincing.

Two Dark Areas near Northern End of North America Nebula

$\alpha = 20^h 57^m.5, \quad \delta = +45°25'$ (1920)

Negative Δ 231, Hooker telescope, 1922, August 22. Aperture 84 inches. Seed 60 plate, exposure 4 hours. Images small and round. Illustrated in Plate XVI

The region shown on this negative is represented on Plate XIV by an area about 16×20 mm with its center 53 mm from the top and 52 mm from the right edge.

The luminous nebulosity in this region is of nearly uniform brightness and considerably fainter than in the southern end of the nebula. It is interrupted by two dark areas. The more northerly of the two is some $8'$ wide, well defined by an irregular border at its southeastern edge and losing itself imperceptibly in the shining nebulosity and star fields toward the northwest. It is probably nowhere without faintly luminous nebulosity, although there are considerable areas at the southern end where there are no stars.

The more southerly of the two dark areas is about $13'$ long in a north and south direction, and $5'$ wide. It is not wholly dark, and shows faint curving streaks like those in Barnard 92 and 93.

Mount Wilson Observatory and
Whitin Observatory
Autumn 1922

CORRECTIONS TO MT. WILSON CONTR., NO. 209:

Plate V: *for* 1 mm $= 14''.0$, *read* 1 mm $= 17''.7$
Plate VI: *for* 1 mm $= 12''.8$, *read* 1 mm $= 19''.6$

THE RADIAL VELOCITIES OF 1013 STARS[1]

By WALTER S. ADAMS and ALFRED H. JOY

ABSTRACT

Radial velocities of 1013 stars.—These stars, almost wholly of spectral types F to M, have been observed with a one-prism spectrograph on the 60-inch and 100-inch reflectors at Mount Wilson. The Rowland wave-lengths of the principal lines used in the measurements are given. The probable errors of the mean radial velocities have been computed and vary from 0.2 to over 3 km, with an average of about 1.35. For types F to M comparison with measurements from the Lick Observatory for 109 stars shows an average difference of 1.7 km and a systematic difference of −0.12 km, while for 83 stars measured at the Dominion Observatory the average difference is 2.4 km and the systematic difference, Mt.W−D.A.O.+0.8 km. In the case of 16 B and A stars, the systematic difference from other observers is +1.6 km. The magnitudes, spectral types, and proper motions, as far as known, are also given in the tables.

The catalogue of radial velocities, which forms the principal portion of this communication, includes many of the results obtained in this line of work at the Mount Wilson Observatory during the past few years. It is made up almost wholly of stars with spectra of types F, G, K, and M which have been observed not only for radial velocity but for determinations of absolute magnitude and spectroscopic parallax. As a result, the material is far from homogeneous as regards spectral type, proper motion, or apparent magnitude, but contains rather a selection of stars with great differences in absolute magnitude which are of especial interest in investigations of space-velocities and the relationship of velocity to absolute magnitude and mass. The results have already been used to some extent in such investigations by Strömberg, Seares, and Adams and Joy.

The spectrograms used for measurement have all been obtained at the Cassegrain focus of the 60-inch and the 100-inch reflectors, the larger instrument having been employed almost wholly for stars fainter than the eighth visual magnitude, and for a small number of stars south of −30° declination. The form of mounting of the 60-inch reflector prevents the observation of stars south of this limit. For most of the spectra, single-prism spectrographs with prisms of 64° angle and cameras of 18 inches (45 cm) focal length have been used. The linear dispersion of these instruments is

[1] *Contributions from the Mount Wilson Observatory*, No. 258.

36 A per millimeter at Hγ. The camera lenses are of the Cooke Astrographic type and give satisfactory definition from λ 4100 to Hβ. In the case of stars of advanced types of spectrum, measurements have usually been limited to the region between λ 4200 and λ 4600. A camera of 7.2 inches (18 cm) focal length has been used for a few stars which are very faint photographically, but the number of such stars included in the catalogue is very small.

The spectrograms in all cases have been measured directly on small comparators, and the wave-lengths used for reduction purposes are those of Rowland's Table of Solar Spectrum Wave-Lengths. At the present time a sufficient number of wave-lengths based upon the international system is available, especially for the lines of the iron arc which we have used as a comparison spectrum, to make a change to this system highly desirable. Accordingly, within recent months we have completed such a revision, but the results contained in the present catalogue are all based on the earlier values of Rowland. As a rule, from 10 to 15 stellar lines have been measured on each spectrogram, and so far as possible, the attempt has been made to avoid blended lines. The lines most commonly employed are the following:

Fe	4202.198	Sc, Ti, Fe	4314.820
Fe	4236.112	H	4340.634
Fe	4250	Fe	4383.720
Cr	4254.505	Fe	4404.927
Fe	4260.580	Fe	4415.293
Fe	4271	Fe	4427.482
Cr	4274.958	Fe, Mn	4461.930
Fe, Ca	4282.800	Cr, Co, Fe	4531.220
Fe	4294.301	Mg, Ti	4571.716
Fe	4308.081	H	4861.527

The stellar line at λ 4250 has been referred directly to the blended iron-comparison line, and the stellar line at λ 4271 has been used with a wave-length 0.1 A longer than the corresponding comparison line. In the course of the revision of our wave-lengths to conform to the international system, slight changes have been introduced into the relative wave-lengths of some of these lines, and the modified values will be adopted in the future.

The spectrograms in the great majority of cases have been measured by two or more observers, but when only a single measure

is available the value has been assigned half-weight in the formation of the mean. Since the results for most of the stars in the list depend upon but three or four spectrograms, the formula of Peters has been used to derive the probable error of the mean. In the case of a small number of observations, this formula probably gives a more adequate measure of the precision of the results than the more common expression, which involves the squares of the residuals. The equation as used is

$$\text{P.E.} = \pm 0.845 \frac{\Sigma d}{n\sqrt{n-1}}$$

in which d is the residual from the mean taken regardless of sign, and n the number of spectrograms. In cases where unequal weights are used, the results have been treated in the usual way through multiplication by the square roots of the weights. It is hardly necessary to state that in measurements made upon spectrograms of such low dispersion the fractional part of the kilometer has little significance, either in the mean or in its probable error.

The observations of the absolute magnitudes of stars, carried on as a part of our spectroscopic work, have resulted in the accumulation of spectrograms of most of the brighter stars of types F to M. From among these a sufficient number have been measured and included in the catalogue to make it possible to compare a considerable number of our values with those published by the Lick Observatory. Similarly, numerous stars have been observed in common with the Dominion Astrophysical Observatory. The results of this comparison for stars of types F or later are given in the accompanying summary. The differences are taken in the order, Mount Wilson *minus* Lick Observatory, and Mount Wilson *minus* Dominion Astrophysical Observatory.

Observatory	Differences			Systematic Difference	Average Difference
	No.	Positive	Negative		
				km	km
L.O.	109	51	58	−0.12	±1.69
D.A.O.	83	53	30	+0.77	±2.40

In this comparison two stars are omitted from the Lick Observatory list. These are the extreme southern star C2757 with a residual of +10 km, and Barnard's star of large proper motion with a residual of +21 km. The last-named star was observed at both observatories with spectrographs of very low dispersion, and a considerable discordance is not surprising. In the case of the southern star, it seems probable that the difference is due to errors arising from the great zenith distances at which it is observed at Mount Wilson. The most probable sources of error are atmospheric dispersion which, because of the great focal length of 135 feet, affects the distribution of light at the slit, and flexure of the 100-inch telescope, which may influence the illumination of the collimating lens. In the few cases of stars of extreme southern declination which have been measured for radial velocity, there has appeared to be a tendency for the spectrograms obtained with the 100-inch telescope to give too large positive velocities, but the effect is not always present.

The comparison with the results obtained at the Dominion Astrophysical Observatory shows a fairly definite systematic difference of about +0.8 km. The effect appears to be largest in the case of stars of the M-type of spectrum, and it is possible that at least a portion of it may be due to the difference in the method of measurement, the Victoria observers having used the spectrocomparator for this purpose. In view of the low dispersion employed at both observatories, however, the discordance does not appear to be excessive, except in the case of a few individual stars.

The number of B- and A-type stars in the catalogue is small, and only a few comparisons can be made with the results of other observers. For a total of sixteen stars with velocities measured at several different observatories, the mean difference, Mount Wilson *minus* others, is +1.6 km. The stars Boss 892, Boss 3518, and β G.C. 12274 show differences of +11.6, −10, and +10.5 km, respectively.

In the table of results, the stars are listed according to their numbers in the *Catalogue* of Boss, unless an abbreviation appears. In the latter case, C is used to indicate *Cincinnati Publication*,

TABLE I

RADIAL VELOCITIES OF 1013 STARS

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
3 Br..	0^h 1^m.0	+57°53′	6.4	G4	0″.266	3	− 11.2	±1.8	....	
3 Ft..	0 1.0	+57 53	7.5	G7	0.266	3	− 15.9	2.6	....	
12....	0 3.8	+58 36	2.4	F2	0.559	3	+ 11.2	2.1	+ 12.8	L
C 9....	0 4.0	+64 31	7.0	K0	0.30	3	+ 8.2	1.4	....	
19....	0 5.1	+45 31	5.1	F2	0.004	3	− 5.6	0.2	....	
Lal. 10....	0 5.4	+32 34	7.2	K5	0.052	3	− 13.2	0.2	....	
B.D.+66°7	0 5.4	+66 35	9.2	G8	0.138	3	+ 19.3	0.8	....	
33....	0 9.6	−19 29	4.7	Ma	0.068	4	− 21.2	1.2	− 22.1	L
β G.C. 104....	0 11.5	+35 55	7.7	F6	.....	3	+ 6.6	1.4	....	
62....	0 17.2	+12 56	6.4	K2	0.063	3	+ 3.8	1.7	....	
63....	0 17.7	−12 46	6.4	G2	0.396	3	− 7.3	1.5	....	
Lal. 442....	0 18.4	+44 32	7.7	G6	.....	3	+ 36.2	1.5	....	
Lal. 510....	0 20.3	+33 34	8.3	K0	0.019	3	− 35.2	2.3	....	
Lal. 584....	0 22.3	+33 29	8.0	K0	0.075	3	+ 1.5	1.4	....	
84....	0 23.0	+15 54	6.5	K5	0.018	4	− 1.5	0.6	....	
C 49....	0 23.2	+ 9 39	6.0	F3	0.212	3	− 10.5	1.3	....	
86....	0 24.5	+76 28	6.4	K0	0.335	3	+ 19.5	0.3	....	
89....	0 24.8	+29 12	5.3	F2	0.066	3	− 9.6	1.0	− 11.2	V
96....	0 26.2	+52 17	5.7	K0	0.056	3	− 48.3	0.5	....	
106....	0 27.5	+27 44	6.4	G6	0.014	3	− 15.1	1.3	....	
121....	0 31.3	+43 56	5.4	K2	0.035	4	− 31.2	1.3	....	
128....	0 32.4	+ 2 35	6.6	K3	0.122	4	+ 4.6	1.1	....	
134....	0 34.7	+20 53	5.6	K1	0.047	3	− 16.1	0.7	− 18.1	V
C 87....	0 35.5	−24 21	6.2	G2	0.711	5	− 53.4	1.8	....	
137....	0 35.6	− 4 54	6.1	G8	0.023	4	+ 35.0	1.7	....	
β G.C. 374 Br..	0 37.2	+ 3 37	7.8	F7	.....	3	+ 7.8	2.0	....	
H.D. 4042....	0 37.8	+70 17	6.9	G7	0.035	3	− 2.5	1.3	....	
154....	0 39.6	+54 40	5.5	A2	0.027	7	− 7.6	1.5	....	
160....	0 41.2	−23 4	5.6	G7	0.189	4	− 14.2	1.9	....	
161....	0 41.3	+14 56	5.6	Mb	0.064	5	− 27.7	1.2	....	
166....	0 42.3	+50 54	6.8	A2	0.006	3	− 4.7	1.8	....	
168 Br..	0 43.0	+57 17	3.7	F8	1.242	3	+ 7.4	0.5	+ 10.0	L, B
168 Ft..	0 43.0	+57 17	7.4	K5	1.242	4	+ 13.4	2.3	....	
170....	0 43.1	+ 6 45	6.1	G9	0.106	3	− 0.2	2.0	....	
C 106....	0 44.5	−23 46	7.2	G7	0.517	3	+ 5.5	1.0	....	
178^1....	0 44.5	+27 10	6.3	F0	0.092	4	+ 4.5	1.2	....	
178^2....	0 44.5	+27 10	6.3	F0	0.092	3	+ 6.3	0.7	....	
181....	0 45.1	−11 11	5.2	F8	0.325	4	+ 7.1	1.2	....	
193....	0 49.1	+58 26	5.0	K2	0.053	3	− 22.3	0.8	− 22.4	L
197....	0 49.6	+23 5	5.6	K0	0.142	3	+ 2.0	1.5	....	
233....	0 59.7	+ 5 7	6.2	K4	0.031	3	− 14.5	1.4	....	
C 142....	1 0.4	+63 24	8.7	K7	1.55	3	− 2.9	1.4	....	
243....	1 1.3	+12 25	6.2	G7	0.040	3	+ 8.0	1.6	....	
C 146....	1 2.2	+22 26	8.6	K5	0.525	3	− 2.9	0.6	....	
C 149....	1 3.3	+61 1	7.8	F5	0.638	3	−325.1	±0.5	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
C 150....	1h 4m1	+67°15′	6.6	F7	0″.236	3	− 9.5	±0.4	......	
266....	1 5.2	− 9 26	6.6	K0	0.039	4	− 19.9	0.9	......	
274....	1 6.8	+44 48	6.6	Ma	0.042	4	+ 22.0	1.1	......	
285 Ft..	1 9.4	− 8 28	7.8	G9	0.29	4	+ 17.0	2.5	......	
286....	1 9.5	+ 6 28	6.2	K0	0.029	6	− 8.1	2.2	......	
287....	1 9.7	− 1 31	5.8	F4	0.221	7	+ 25.0	1.6	......	
β G.C. 690 Br..	1 13.5	− 1 23	8.1	K1	0.489	5	+ 13.7	1.3	......	
C 167....	1 14.0	− 9 27	8.9	G1	0.528	3	− 4.9	1.3	......	
C 174....	1 15.8	+30 49	8.8	K2	0.50	3	+ 20.7	1.7	......	
C 185....	1 19.5	+17 59	9.1	K2	0.606	5	+ 9.0	1.1	......	
318....	1 20.9	+18 39	5.3	F0	0.041	3	− 7.2	1.8	− 9.3	V
332....	1 24.9	+ 5 38	5.1	K5	0.292	3	+ 37.5	1.4	+ 35.4	V
333....	1 25.2	+67 54	7.0	K3	0.122	3	− 14.2	0.9	......	
C 204....	1 27.3	+68 26	6.7	G3	0.402	4	− 30.8	1.3	......	
H.D. 9454....	1 27.8	+71 54	7.8	F6	0.037	3	− 18.6	0.9	......	
346....	1 30.3	+48 13	6.2	G8	0.014	3	− 40.9	0.6	− 44.6	V
350....	1 30.9	+40 54	4.2	F8	0.418	3	− 27.9	0.7	− 27.3	L, B
C 226....	1 32.7	+29 4	8.7	K1	0.482	3	− 2.4	0.5	......	
C 238....	1 36.8	+63 20	8.2	K5	0.70	5	− 50.0	2.9	......	
381 Br..	1 36.8	−11 49	6.1	F2	0.410	3	− 10.0	2.0	......	
C 240....	1 37.4	−18 24	7.4	G0	0.543	3	− 5.5	0.5	......	
391....	1 39.4	−16 28	3.6	G7	1.922	3	− 15.0	0.4	− 16.0	L, C
392....	1 39.5	+19 35	6.2	G3	0.111	3	− 44.0	1.3	......	
405....	1 43.0	+32 11	5.8	F7	0.354	3	− 26.8	2.0	......	
H.D. 11635....	1 49.1	+33 15	8.7	G7	.....	3	− 6.1	1.3	......	
B.D.+32°356....	1 52.3	+32 28	8.8	F8	.....	3	− 29.4	1.4	......	
441 Ft..	1 52.4	+23 7	7.4	G0	0.122	3	− 8.5	1.8	......	
Lal. 3619....	1 53.1	+33 51	7.6	G9	0.033	3	− 40.5	1.4	......	
450....	1 54.9	+ 2 37	5.8	G0	0.336	3	− 17.5	1.5	......	
451....	1 55.1	−21 19	5.7	Ma	0.014	5	− 14.5	1.7	......	
H.D. 12350....	1 56.0	+70 43	7.6	F1	0.039	3	− 9.1	2.8	......	
Lal. 3705....	1 56.8	+54 13	7.7	F6	0.026	3	− 14.0	1.3	......	
468 Br..	1 57.8	+41 51	2.3	K2	0.070	3	− 9.1	0.6	− 10.9	L, B
477....	2 1.5	+22 59	2.2	K2	0.239	5	− 15.0	0.7	− 14.3	9
H.D. 12953....	2 1.7	+57 57	5.9	A2	.....	6	− 41.1	1.7	......	
486....	2 4.1	+73 33	6.2	G6	0.071	3	− 36.5	1.5	......	
H.D. 13267....	2 4.6	+57 11	6.4	B8	.....	5	− 36.3	1.3	......	
H.D. 13561....	2 7.3	+56 2	9.0	B3	.....	3	− 44.5	2.6	......	
C 286....	2 7.5	+67 13	7.8	K2	0.603	3	− 13.2	0.5	......	
β G.C. 1144....	2 7.6	+47 1	6.0	F1	0.112	3	− 8.4	1.7	......	
504....	2 7.7	− 2 52	5.7	F9	0.374	3	− 5.1	0.5	......	
H.D. 13716....	2 8.6	+57 18	8.5	B3	.....	3	− 60.1	1.6	......	
C 288....	2 9.2	+64 30	8.4	G2	0.50	4	− 31.5	0.8	......	
C 289....	2 9.5	− 1 40	9.2	F8	1.034	6	+ 18.7	1.7	......	
H.D. 13841....	2 9.8	+56 34	7.2	B2	.....	5	− 40.2	±0.8	......	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
507....	2h 9m.9	+56°35′	6.4	B2	0″.015	6	− 40.6	±1.6	....	
510....	2 10.0	+25 19	5.8	F5	0.194	3	+ 23.5	1.7	+ 26.0	V
H.D. 13866....	2 10.0	+56 15	7.7	B2	.....	3	− 48.8	2.7	....	
513....	2 10.9	+57 26	6.1	K2	0.073	4	+ 3.2	0.7	....	
515....	2 11.0	+57 3	6.2	G4	0.007	5	− 10.3	1.1	....	
Lal. 4181....	2 11.4	+42 8	7.9	F1	0.022	3	− 44.1	2.4	....	
518....	2 12.0	− 6 53	5.7	G8	0.141	3	+ 7.1	1.4	....	
519....	2 12.0	+56 40	6.7	B4	0.013	4	− 43.1	2.3	....	
β G.C. 1188 Br..	2 12.2	+56 42	8.6	B1	.....	4	− 44.6	0.3	....	
523....	2 12.8	+ 1 17	5.8	F7	0.527	3	+ 21.0	1.9	....	
528....	2 13.6	+19 14	6.8	F9	0.116	3	+ 1.7	1.6	....	
C 298....	2 14.0	+70 43	8.5	K2	0.62	3	− 25.2	1.7	....	
532....	2 14.5	−26 25	6.4	G7	0.505	3	+ 5.9	1.1	....	
C 300....	2 14.5	+68 18	7.4	K0	0.13	4	+ 12.0	2.0	....	
533....	2 14.9	+56 47	6.5	A1	0.011	5	− 46.4	0.6	− 48.3	V
534....	2 15.4	+55 23	5.2	A2	0.005	3	− 14.8	0.7	....	
535....	2 15.9	+56 56	7.0	B9	0.009	3	− 51.4	0.5	....	
Lal. 4358....	2 16.4	+22 25	6.6	G2	.....	3	+ 22.2	0.9	....	
H.D. 14662....	2 16.9	+54 55	6.5	F9p	.....	7	− 26.5	1.0	....	
Lal. 4391....	2 17.9	+33 24	7.6	K3	0.056	3	− 1.8	2.3	....	
544....	2 18.2	+56 9	6.2	B2	0.012	5	− 47.0	1.3	....	
H.D. 14899....	2 19.2	+56 47	7.4	B9	.....	3	− 42.0	2.2	....	
556....	2 22.1	+ 9 45	6.5	F5	0.355	4	− 41.3	1.6	....	
C 329....	2 27.7	+42 21	7.6	G4	0.45	3	+ 15.2	0.4	....	
577....	2 28.5	+72 23	5.3	G4	0.031	3	+ 0.4	0.4	....	
582....	2 29.7	+34 15	5.6	Ma	0.061	3	− 6.6	1.4	....	
583....	2 29.8	+ 7 2	6.2	G9	0.107	3	− 24.6	0.8	....	
591....	2 31.2	+12 1	5.7	F4	0.297	3	+ 6.7	1.6	....	
592....	2 31.2	+24 13	7.4	F5	0.152	4	+ 16.7	0.9	....	
C 340....	2 32.6	+30 24	7.2	G0	0.625	4	− 99.8	1.2	....	
612....	2 36.1	− 1 7	5.7	F6	0.250	3	+ 7.3	1.1	....	
617....	2 37.4	+48 48	4.2	F6	0.351	5	+ 22.9	0.8	+ 26.0	L, B
622 Ft..	2 38.1	+ 2 49	6.2	F3	0.210	3	− 13.0	1.6	....	
β G.C. 1427 Br .	2 41.8	+18 57	7.4	G1	0.176	4	+ 4.0	1.6	....	
639....	2 43.4	+55 29	3.9	K4	0.030	3	+ 0.5	0.5	− 1.6	L
C 364....	2 44.9	+45 34	9.2	G4	0.63	3	+ 16.0	0.4	....	
β G.C. 1490 Br..	2 49.7	+26 28	7.5	K1	0.333	3	+ 33.2	1.1	....	
661....	2 50.8	+17 37	5.6	F5	0.340	6	+ 14.1	1.5	....	
667....	2 52.4	+20 16	5.8	F4	0.222	3	+ 27.0	0.7	+ 27.4	V
669....	2 52.8	+79 1	5.7	Ma	0.039	3	− 33.9	0.7	− 40.0	V
672....	2 53.0	+46 49	5.6	G3	0.035	3	+ 8.0	1.2	....	
686....	2 56.0	+26 4	5.9	A4	0.014	3	− 8.1	1.9	....	
699....	2 58.9	+63 40	5.8	A0	0.006	5	− 2.2	2.4	....	
710....	3 1.8	+49 14	4.2	G1	1.269	3	+ 49.9	1.5	+ 49.6	L, B
712....	3 2.7	+18 25	6.5	K6	0.042	3	+ 43.8	±1.1	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
722....	3h 7m7	− 1°34′	5.1	F9	0″.209	3	+ 17.3	±1.1	....	
β G.C. 1623....	3 8.9	+ 0 23	8.1	F9	.....	3	+ 23.3	1.1	....	
749....	3 13.3	− 1 18	5.6	G9	0.252	8	+ 28.3	0.9	....	
756....	3 14.5	+25 18	6.4	K3	0.095	3	+ 26.4	2.0	....	
758....	3 14.8	+48 43	6.2	F6	0.209	3	+ 23.5	1.6	+ 24.0	V
764....	3 15.9	−43 27	4.3	G5	3.168	3	+ 88.3	1.4	+ 87.4	L, C
765....	3 16.0	+64 14	5.6	K2	0.019	3	− 20.5	0.7	....	
770....	3 17.0	+20 23	5.2	K4	0.050	3	+ 5.9	0.7	+ 1.3	V
C 453....	3 20.1	− 5 42	8.1	K5	0.816	3	− 11.3	1.1	....	
807....	3 26.3	+35 7	5.8	B5	0.011	5	+ 24.9	2.6	....	
814....	3 28.2	− 9 48	3.8	K1	0.972	3	+ 16.0	0.8	+ 15.6	L, C
C 468....	3 28.4	+17 30	6.4	G6	0.331	3	+ 11.8	1.4	....	
Lal. 6590....	3 29.6	+18 34	7.9	Mb	0.012	3	− 8.0	1.1	....	
825....	3 31.8	+ 0 5	4.4	F9	0.536	4	+ 26.2	0.2	+ 28.6	L, B
829....	3 33.8	+16 13	6.3	G6	0.058	3	+ 14.6	0.7	....	
831....	3 34.1	− 5 57	6.0	K0	0.204	4	+ 40.2	0.2	....	
C 494....	3 36.9	+42 18	7.4	G0	0.417	3	+ 31.2	1.0	....	
843....	3 38.0	+19 21	6.3	G5	0.137	3	+ 74.9	1.0	+ 79.8	V
848....	3 38.5	−10 6	3.7	G9	0.749	3	− 7.5	0.9	− 5.4	L
852....	3 38.9	+23 48	3.8	B7	0.053	3	+ 10.5	3.2	+ 14.6	Y
856....	3 39.3	+24 9	4.4	B7	0.049	3	+ 3.6	2.1	+ 3	Y
860....	3 39.9	+24 3	4.0	B9	0.054	3	+ 9.4	1.1	+ 6.4	D
862 Br..	3 40.2	+41 9	8.6	K0	1.372	3	+ 50.1	1.8	....	
H.D. 23654....	3 41.7	+23 18	8.3	K1	0.089	4	− 31.9	2.0	....	
C 513....	3 44.4	+ 1 4	8.6	K1	0.67	4	− 15.9	2.4	....	
C 518....	3 46.2	+22 23	7.8	G5	0.388	3	+ 8.4	0.4	....	
889....	3 46.4	+48 21	5.9	K0	0.058	4	+ 8.5	0.9	....	
C 521....	3 46.4	+60 53	7.8	K0	0.479	3	+ 48.4	2.0	....	
892....	3 47.4	+17 2	6.0	A7	0.147	4	+ 37.6	1.0	+ 26	B
895....	3 48.4	+75 53	8.3	K6	0.646	3	+ 20.6	2.2	....	
897....	3 48.6	+60 49	5.3	K4	0.016	3	+ 1.9	1.5	....	
901 Br..	3 49.3	− 3 15	5.0	G4	0.034	3	+ 28.6	0.6	+ 27:	L
Lal. 7255....	3 51.8	+38 33	6.4	K0	.....	3	+ 22.5	0.4	....	
Lal. 7356....	3 53.2	+ 1 10	7.9	K1	0.008	3	+ 35.1	0.7	....	
H.D. 25056....	3 53.8	+53 35	7.4	Gop	.....	3	− 5.8	1.0	....	
Lal. 7286....	3 54.5	+58 40	8.0	B4	0.026	3	− 25.4	1.7	....	
C 540....	3 55.0	+74 54	7.3	F8	0.345	3	+ 35.2	1.1	....	
927....	3 56.5	+35 2	8.6	K0	2.200	3	− 29.5	0.8	....	
β G.C. 2007 Br..	3 57.4	+39 14	7.4	G4	0.201	3	+ 23.9	1.6	....	
937....	3 58.9	+ 2 33	5.4	F7	0.190	3	− 19.2	0.6	− 18.8	V
939....	3 59.4	+21 44	6.0	G1	0.222	3	+ 25.9	1.3	....	
945 Br..	4 0.9	+37 49	7.1	K0	0.289	3	+ 27.0	1.8	....	
H.D. 25878....	4 0.9	+53 6	7.1	G2p	.....	4	+ 16.4	1.3	....	
949....	4 1.6	+37 28	6.2	K1	0.214	3	− 39.7	0.8	....	
950....	4 1.9	+37 47	5.6	F7	0.260	3	+ 25.6	±0.5	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
951....	$4^h\ 2^m.3$	+17° 4′	6.1	K5	0″.024	3	− 30.1	±0.3	....	
952....	4 3.3	+19 21	5.7	K1	0.114	3	+ 31.1	0.7	+ 21.6	B
955....	4 4.7	+26 13	5.6	F1	0.046	3	+ 18.1	0.9	+ 19.6	L
961....	4 6.0	+ 5 16	5.7	F1	0.143	4	+ 34.5	1.0	....	
C 560....	4 8.6	+22 6	9.1	A8	0.54	4	+338.2	2.7	....	
β G.C. 2093....	4 9.6	+31 27	7.4	F9	.....	3	− 26.0	0.9	....	
984....	4 10.7	− 7 49	4.5	K1	4.085	4	− 42.0	1.6	− 41.6	L
β G.C. 2134 Br.:	4 14.2	+16 17	7.1	F7	0.107	7	+ 36.3	0.9	....	
1017....	4 17.2	+17 18	3.9	G9	0.115	3	+ 41.0	0.8	+ 37.8	L, C
1021....	4 18.1	+33 44	5.8	F3	0.093	3	− 35.4	1.0	....	
1023....	4 18.4	+ 9 14	5.1	A3	0.029	3	− 4.8	1.7	....	
H.D. 27848....	4 18.6	+16 51	7.8	F5	0.102	3	+ 42.7	0.6	....	
H.D. 27859....	4 18.7	+16 40	8.0	G0	0.135	3	+ 43.5	0.7	....	
1025....	4 19.1	+18 49	6.0	A6	0.123	4	+ 36.1	0.4	....	
β G.C. 2187 Br..	4 20.0	+18 39	7.7	G4.	.....	4	+ 42.0	1.7	....	
C 585....	4 20.1	+46 38	6.7	G4	0.317	3	+ 38.1	0.8	....	
H.D. 28099....	4 20.9	+16 31	8.0	G5	.....	3	+ 40.3	1.2	....	
H.D. 28205....	4 21.9	+15 22	8.0	G0	.....	3	+ 40.0	0.5	....	
1055....	4 24.9	+15 28	5.5	F0	0.110	4	+ 36.1	1.4	+ 38.4	V
H.D. 28568....	4 25.1	+15 55	6.7	F2	.....	3	+ 43.1	0.6	....	
H.D. 28805....	4 27.3	+15 36	8.5	G7	.....	3	+ 39.9	1.1	....	
H.D. 29103....	4 29.8	+19 46	7.2	F8	0.033	3	+ 11.5	0.5	....	
C 594....	4 29.8	+52 42	8.5	K6	0.53	6	+ 36.7	1.6	....	
1086....	4 32.4	+15 50	5.8	A5	0.094	4	+ 40.7	2.5	+ 35.6	V
H.D. 29400....	4 32.7	+66 33	8.9	G4	0.375	3	− 51.5	1.0	....	
H.D. 29581....	4 34.4	+30 7	8.1	F6	.....	3	+ 8.7	1.6	....	
1098....	4 34.7	−14 33	5.6	K0	0.186	4	+ 56.6	2.5	....	
1106....	4 36.2	+22 45	7.8	A0	0.047	3	+ 14.5	1.6	....	
1120....	4 40.4	+18 33	6.1	K3	0.091	3	+ 38.8	0.8	....	
1132....	4 43.1	−17 7	5.6	G2	0.217	4	+ 16.3	1.3	....	
1149....	4 46.9	+14 5	5.2	Ma	0.059	3	− 5.7	1.2	+ 3.5	V
1150....	4 46.9	+55 6	5.6	A0	0.007	4	− 0.7	1.0	....	
1154....	4 48.2	+ 2 21	5.7	Ma	0.034	3	+ 13.1	0.4	....	
1164....	4 49.6	+74 7	6.2	K3	0.050	3	− 51.3	1.4	....	
1167....	4 50.5	+33 0	2.9	K3	0.028	3	+ 17.7	1.1	+ 18.5	L, Y, B
1168....	4 50.6	−16 54	5.8	G8	0.001	3	+ 10.3	0.5	....	
C 634....	4 51.3	+34 7	8.0	G9	0.60	3	+ 40.8	1.1	....	
1175....	4 51.7	+23 48	6.0	K0	0.025	3	+ 4.4	0.7	....	
1179....	4 52.7	+66 41	6.3	F7	0.355	3	+ 16.2	1.3	....	
1184....	4 54.0	+15 46	6.7	F6	0.104	3	+ 43.6	1.0	....	
1196....	4 57.5	+58 53	6.4	G5	0.039	3	− 7.5	0.7	....	
C 654....	5 0.0	+64 48	6.4	F2	0.190	3	+ 4.1	1.1	....	
1212....	5 1.5	+18 31	5.0	G0	0.540	3	+ 19.8	0.7	....	
1217....	5 2.2	+ 9 21	6.3	G0	0.381	3	− 22.6	1.5	....	
1222....	5 3.3	+46 50	5.6	F3	0.170	4	+ 36.3	±1.3	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
1256....	5h 11m.1	+42° 41′	5.9	Mb	0″.049	3	− 37.5	±0.8	....	
1259....	5 12.1	+40 1	4.8	G1	0.843	3	+ 64.8	0.7	+ 65.4	L, B
1267....	5 13.3	+20 2	6.2	K0	0.048	3	− 46.9	0.6	....	
C 683....	5 14.1	− 3 11	8.6	K5	0.722	3	+ 85.7	0.6	....	
1299....	5 19.1	− 7 54	4.2	G6	0.042	3	− 17.0	1.5	− 17.8	L
β G.C. 2745 Br..	5 23.3	+54 35	7.8	G1	0.409	4	+ 25.9	0.6	....	
C 699....	5 23.5	− 3 34	8.6	K6	0.841	3	− 57.7	1.3	....	
1324....	5 24.0	+57 9	6.5	G0	0.255	3	+ 36.2	1.2	....	
C 705....	5 26.4	− 3 42	8.8	Ma	2.222	7	+ 11.4	1.3	....	
1360....	5 29.9	+85 9	6.4	K4	0.020	6	− 45.0	1.8	....	
1373....	5 31.4	+ 9 14	4.4	G7	0.321	3	+ 99.2	0.9	+ 98.6	L, B, C
Lal. 10522....	5 32.3	+42 37	7.4	K4	0.038	3	+ 17.8	1.4	....	
C 714....	5 33.3	+74 34	7.3	G4	0.240	3	+ 24.5	0.7	....	
1393....	5 34.5	+56 32	6.2	K0	0.037	3	− 28.4	1.4	....	
H.D. 37937....	5 37.0	+43 31	7.7	K0	0.033	3	+ 6.3	0.7	....	
1411....	5 38.2	+49 47	5.5	A2	0.009	3	− 12.1	1.4	....	
H.D. 38130....	5 38.4	+42 49	8.0	K2	0.011	3	+ 12.1	1.1	....	
1419....	5 40.3	−22 27	6.4	K5	0.461	3	− 9.2	0.8	....	
1434....	5 42.9	+24 32	5.0	K0	0.036	3	+ 15.7	1.5	+ 19.4	V
β G.C. 2977....	5 42.9	+29 41	7.8	F1	.....	4	− 11.7	0.4	....	
β G.C. 3037....	5 50.3	+13 55	6.9	G5	0.629	3	− 1.6	1.2	....	
1483....	5 53.0	+44 35	6.4	K2	0.051	3	+ 2.2	0.9	....	
1498....	5 56.1	+42 55	6.1	G9	0.188	3	+ 38.1	2.0	....	
C 756....	5 57.3	+19 23	9.0	F9	0.93	4	−190.8	2.6	....	
1511....	5 59.2	−26 17	5.2	K2	0.104	3	+178.9	3.0	+182.8	L, C
C 758....	5 59.3	+14 24	6.7	F7	0.212	3	+ 36.0	0.5	....	
C 760....	5 59.5	+35 24	6.1	G1	0.332	3	− 11.8	0.6	....	
C 761....	5 59.7	+26 34	8.9	K4	0.42	3	− 91.0	0.8	....	
1517...	6 0.6	−32 10	5.6	B4	0.123	3	+ 99.	2.3	+102	L
H.D. 41497....	6 0.5	+76 31	8.0	F5	0.004	3	− 17.0	0.4	....	
H.D. 42474....	6 5.8	+23 14	7.4	Mdp	.....	3	+ 18.2	1.2	....	
1549....	6 6.3	+22 56	6.3	Ma	0.027	4	+ 22.0	1.0	....	
1564....	6 9.0	+19 11	5.2	F6	0.224	3	+ 35.5	0.6	+ 33.6	V
1565....	6 9.0	+29 32	4.4	G8	0.274	3	+ 20.5	0.7	+ 20.3	L, B
C 780....	6 9.6	+44 45	8.6	G9	0.41	3	− 34.7	2.3	....	
1574....	6 10.5	− 0 28	5.7	F6	0.262	3	− 36.1	2.5	....	
1576....	6 10.8	+46 24	6.5	K1	0.133	3	+ 0.5	0.8	....	
1577....	6 10.8	+12 18	5.1	F5	0.212	3	+ 7.2	1.2	+ 9.6	B
1582....	6 12.0	+ 5 8	5.8	G0	0.282	3	+ 13.3	1.7	....	
C 801....	6 22.0	+36 33	7.1	G1	0.394	3	− 4.6	1.8	....	
1653....	6 27.0	+ 4 56	6.0	G8	0.027	3	+ 21.5	1.0	....	
1665....	6 28.6	+61 34	6.0	G8	0.341	3	− 45.9	0.2	....	
β G.C. 3474....	6 30.2	+27 22	7.0	G0	0.061	3	+ 28.9	0.9	....	
1687....	6 31.7	+39 29	5.7	K3	0.119	3	+ 33.3	0.8	....	
β G.C. 3499C...	6 32.0	+12 14	8.3	F1	.....	3	+ 9.6	±1.2	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
C 821....	$6^h35^m.1$	+24° 3′	8.0	K6	0″.34	3	− 44.3	±1.2	....	
β G.C. 3562 Br..	6 37.3	+40 44	6.9	Mb	0.171	3	+ 41.0	0.7	....	
1724....	6 39.5	+43 41	5.3	F9	0.158	3	− 24.0	0.8	....	
C 834....	6 47.4	− 5 3	6.8	K5	0.582	6	− 9.8	1.7	....	
1783....	6 49.5	−11 55	4.2	K4	0.138	3	+ 99.4	1.3	+ 97.7	L, C
C 837....	6 49.5	+40 13	8.3	K5	0.43	5	+ 49.9	1.0	....	
H.D. 51349....	6 51.9	+69 21	7.5	Ma	0.005	4	+ 13.6	1.9	....	
1801....	6 53.7	+87 12	5.3	Ma	0.051	3	− 23.6	1.7	....	
Lal. 13500....	6 55.2	+33 49	7.3	F0	.....	3	+ 6.3	1.7	....	
1806....	6 56.3	+24 21	5.2	G1	0.015	3	− 8.5	1.1	− 9.3	B
1818....	6 59.2	− 5 11	5.9	K2	0.006	5	+ 40.8	1.1	....	
1822....	6 59.6	+34 38	5.6	G4	0.087	3	+ 4.6	0.5	....	
β G.C. 3876....	7 6.6	+27 24	6.4	F8	.....	3	− 13.8	2.4	....	
1856....	7 7.6	+16 20	5.3	Mb	0.051	3	− 7.8	0.7	....	
1861....	7 8.6	+25 4	6.0	K7	0.105	4	+ 47.1	1.5	....	
1880....	7 11.1	+41 4	5.8	B9	0.011	6	− 15.0	2.1	....	
1898 Ft..	7 14.2	+22 10	8.0	K4	0.025	3	+ 2.7	1.5	....	
1919....	7 17.2	+40 52	5.3	K0	0.024	3	+ 22.4	1.0	....	
C 884....	7 17.7	+46 18	8.9	K0	0.45	3	+ 8.5	1.1	....	
1937....	7 20.5	+68 40	5.8	K0	0.044	3	+ 56.9	0.5	....	
1940....	7 20.9	+21 44	6.4	F5	0.309	3	+ 51.1	0.1	....	
1963....	7 24.3	− 1 42	5.8	K3	0.011	4	− 4.4	1.3	....	
1971....	7 26.0	+17 18	5.6	K1	0.089	3	− 39.7	1.4	....	
H.D. 59720....	7 26.2	+66 41	7.5	Mb	0.017	3	+ 19.6	0.8	....	
1988....	7 29.8	−22 5	4.5	F7	0.081	4	+ 58.8	1.7	+ 61.0	L, C
2000....	7 32.6	+57 19	6.2	K4	0.018	3	− 12.7	1.9	....	
2001....	7 32.6	+34 49	4.9	F0	0.125	3	+ 5.5	2.0	+ 6.5	L
2005....	7 33.7	+17 54	5.2	K8	0.004	3	+ 29.7	1.7	....	
2006....	7 33.8	+48 22	5.8	G7	0.144	3	+ 40.3	0.3	....	
2024....	7 37.1	+65 42	6.0	K2	0.046	3	− 28.0	0.7	....	
C 922....	7 38.1	+39 49	6.8	F6	0.68	4	− 3.8	0.5	....	
2031....	7 39.2	+28 16	1.2	G9	0.625	4	+ 4.8	1.0	+ 3.6	9
2049....	7 41.1	+33 40	5.3	Ma	0.040	3	− 9.5	0.6	....	
2053....	7 41.9	−33 59	5.4	F9	1.710	3	+ 99.9	2.2	+100	L
2075....	7 47.1	−13 38	5.7	G0	0.344	3	− 18.1	1.2	....	
2079....	7 47.4	+47 49	5.7	K4	0.043	3	+ 17.6	0.8	....	
β G.C. 4320....	7 47.8	−13 36	6.9	K0	0.101	3	− 28.6	0.5	....	
2084....	7 48.2	+74 11	5.6	K3	0.034	3	+ 36.4	1.2	....	
C 940....	7 49.1	+19 31	7.9	K4	0.471	6	− 18.2	1.8	....	
2098....	7 51.3	+16 3	6.0	K3	0.056	3	+ 10.7	2.0	....	
β G.C. 4355....	7 52.1	+ 1 25	6.4	F6	0.178	3	− 0.7	0.5	....	
2099....	7 52.6	−22 37	4.4	G1p	0.041	3	+ 12.2	0.8	+ 13.4	L, C
C 947....	7 53.7	+21 8	8.6	G9	0.573	4	− 27.7	0.9	....	
2117....	7 54.9	+25 40	5.9	G8	0.014	3	+ 2.4	1.8	....	
H.D. 65734....	7 55.0	+54 25	7.5	A6	0.006	5	− 27.2	±2.0	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
2118....	7h 55m.1	+17° 35′	5.8	K4	0″.015	3	+ 41.9	±0.7	....	
2130....	7 57.1	+ 2 37	4.5	K2	0.106	4	+ 72.1	0.8	+ 71.4	L, B
C 952....	7 57.2	+72 13	8.0	G0	0.496	3	+ 36.6	1.0	....	
2131....	7 57.4	+28 4	5.0	K3	0.056	3	− 3.7	1.3	....	
β G.C. 4414....	7 58.8	+12 35	7.9	G9	0.141	3	− 11.3	1.3	....	
H.D. 66637....	7 59.4	+12 29	8.5	K0	.....	5	+ 34.9	0.3	....	
2157....	8 4.4	+25 49	5.8	G5	0.361	3	− 39.8	0.4	− 44.7	V
2169....	8 6.5	+17 57	6.0	G0	0.155	3	− 5.1	0.9	....	
2170....	8 6.6	−12 38	4.7	G8	0.027	4	+ 36.2	1.8	+ 37.3	L
2174....	8 7.0	+76 4	5.7	G6	0.027	3	+ 7.2	1.5	....	
C 977....	8 12.0	+30 56	8.6	K5	0.873	5	+ 12.1	1.3	....	
2202....	8 14.0	+27 32	5.2	F7	0.388	3	+ 33.8	0.9	....	
C 982....	8 16.3	+66 48	8.9	K4	0.52	3	+ 18.5	1.9	....	
2218....	8 17.6	+18 39	5.9	F1	0.064	3	+ 35.3	0.8	....	
2223....	8 18.5	+10 57	6.3	Ma	0.021	3	+ 3.8	1.1	....	
2224....	8 18.6	−26 2	5.9	F5p	0.017	3	+ 64.0	0.9	....	
2229....	8 20.2	+17 23	6.2	F4	0.246	3	+ 36.9	0.4	+ 36.6	V
2231....	8 20.3	+67 38	6.0	G7	0.056	3	− 2.3	1.0	....	
2234....	8 20.5	+ 7 53	5.2	G5	0.043	4	+ 12.2	1.8	+ 15.1	V
2268....	8 26.4	+38 22	6.0	K3	0.204	6	+ 15.3	1.8	....	
2275....	8 27.1	+24 25	6.4	G9	0.091	3	+ 75.2	0.8	....	
H.D. 72522....	8 28.3	+54 4	8.7	G8	0.93	3	+ 12.8	1.7	....	
2278....	8 28.6	+73 59	6.3	G8	0.106	7	+ 1.1	1.5	....	
C 1000....	8 28.8	+42 6	8.6	K4	0.66	5	+ 58.9	1.0	....	
2282....	8 29.6	+19 56	6.6	F5	0.045	3	+ 35.5	2.9	....	
2289....	8 30.9	+53 45	5.7	G4	0.072	6	− 42.6	1.7	....	
2294....	8 32.1	+33 9	6.1	K2	0.023	3	+ 4.5	1.1	....	
C 1008....	8 32.2	+26 24	7.6	G1	0.237	3	+ 23.4	1.2	....	
C 1014....	8 34.4	+11 53	7.9	K2	0.531	5	− 12.9	1.2	....	
β G.C. 4709....	8 34.4	+ 6 8	7.8	G1	0.339	3	− 17.1	0.6	....	
2308....	8 34.4	+20 22	6.5	G8	0.044	3	+ 36.9	1.7	+ 33.9	V
2309....	8 34.4	+20 19	6.5	A2	0.036	3	+ 33.1	1.2	+ 33.4	V
2310....	8 34.6	+20 1	6.4	G8	0.041	3	+ 33.3	1.7	+ 36.4	V
2312....	8 34.8	−22 19	5.4	G6	0.478	3	+ 43.6	0.8	....	
Lal. 17072....	8 35.8	+33 44	8.5	K2	.....	3	+ 24.7	2.0	....	
2328....	8 37.7	+13 2	5.7	A6	0.011	3	− 18.8	1.2	....	
2333....	8 38.6	+42 3	8.2	K4	0.702	3	− 25.4	1.4	....	
H.D. 74484....	8 39.2	+33 37	8.0	K2	0.011	3	− 27.0	0.6	....	
2348....	8 40.6	+29 8	4.2	G5	0.054	3	+ 17.4	0.2	+ 16.2	L, B, C
2368....	8 45.2	+44 6	5.2	G6	0.037	3	+ 15.1	0.3	+ 20	B
β G.C. 4815 S...	8 46.0	+71 11	8.7	K6	1.390	3	+ 48.0	0.6	....	
β G.C. 4815 N..	8 46.0	+71 11	9.1	K9	1.390	3	+ 47.8	2.2	....	
β G.C. 4844....	8 49.0	+26 35	6.7	G1	0.44	3	+ 35.6	0.2	....	
2394....	8 50.5	+12 0	5.7	K4	0.021	3	+ 24.8	1.2	....	
2412....	8 54.0	−15 45	5.9	F5	0.324	4	+121.8	±1.8	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	*m*	Sp.	μ	No.	*v*	P.E.	Other Determinations	
									v	Auth.
							km	km	km	
2417....	8^h55^m3	+32° 39′	5.8	A2	0″007	4	− 12.8	±2.0	....	
2441....	9 1.6	+67 32	4.9	F5	0.070	3	− 0.1	1.1	− 2.1	L, B
2442 Br..	9 1.7	+23 23	6.7	F3	0.161	3	+ 28.6	0.6	....	
2442 Ft..	9 1.7	+23 23	7.1	F4	0.161	3	+ 31.8	0.2	....	
C 1083....	9 3.8	−14 44	7.3	G0	0.557	4	+ 59.2	1.8	....	
β G.C. 4968 Br..	9 6.9	+47 24	7.8	F7	0.038	3	− 20.2	1.4	....	
2478....	9 9.1	+35 3	6.0	G1	0.149	3	+ 55.9	0.8	....	
2501....	9 13.4	+18 8	6.6	F6	0.180	5	− 15.3	2.0	....	
H.D. 81299....	9 19.7	+33 12	7.9	G9	0.037	3	+ 17.9	1.4	....	
2528....	9 20.0	+17 1	6.3	K0	0.079	3	+ 13.5	1.5	....	
2529....	9 20.4	− 4 41	5.8	K4	0.023	4	+ 5.8	1.7	....	
2536....	9 22.9	+81 46	4.6	K4	0.027	3	− 0.4	0.7	− 5.9	L
2538....	9 23.1	+ 9 30	5.5	G0	0.055	3	− 7.4	1.3	....	
C 1126....	9 24.7	+ 6 5	7.6	K4	0.533	3	+ 27.2	1.4	....	
2552....	9 26.2	+52 8	3.3	F5	1.092	3	+ 16.9	1.8	+ 15.8	L, B
2555....	9 26.6	+11 45	5.1	G9	0.130	3	+ 31.7	0.6	....	
2556....	9 26.6	+10 9	5.3	K3	0.024	3	+ 14.4	0.2	+ 20.8	V
C 1131....	9 26.9	+27 26	7.1	G8	0.279	3	+ 14.3	1.9	....	
2580....	9 31.5	+16 53	5.9	K1	0.021	3	+ 6.4	1.4	....	
C 1143....	9 31.5	+72 12	7.8	F6	0.26	5	− 27.5	2.4	....	
2589....	9 33.2	+ 5 6	4.8	K2	0.176	3	+ 46.2	1.1	+ 45.8	L, B
2603....	9 35.9	+26 22	6.4	K2	0.049	3	− 25.7	1.3	....	
2609....	9 37.7	−23 28	5.0	F8	0.474	3	+ 33.6	1.8	....	
2618....	9 40.2	+24 14	3.1	G0	0.048	3	+ 4.1	0.7	+ 5.1	L, Y, B, C
2621....	9 40.9	+ 7 10	6.0	Ma	0.040	3	+ 5.2	0.9	+ 0.9	V
2622....	9 41.0	+12 16	5.9	K5	0.016	3	+ 30.5	2.2	....	
C 1163....	9 43.5	+14 14	8.1	A2p	0.829	3	− 22.6	1.5	....	
2639....	9 45.6	+13 32	6.7	K7	0.037	3	− 8.7	1.2	....	
2656....	9 49.5	+73 21	6.0	K2	0.079	4	+ 5.4	0.7	....	
2658....	9 50.2	−18 32	5.2	Ma	0.050	3	+ 43.0	1.6	....	
2660....	9 50.3	+57 54	6.0	G4	0.068	7	− 42.4	0.6	− 46.3	V
2663....	9 51.1	+ 9 24	5.9	K1	0.090	3	+ 9.5	0.9	....	
2671....	9 52.8	+ 8 47	6.3	K4	0.035	3	− 21.1	1.5	− 17.4	V
C 1189....	9 54.9	+56 5	8.3	G7	0.498	4	+ 23.9	1.2	....	
C 1192....	9 56.1	+71 21	8.3	G1	0.24	3	− 38.2	1.0	....	
β G.C. 5291....	9 57.9	+38 30	6.8	F6	0.170	3	+ 33.8	1.1	....	
2694....	10 1.9	+17 15	3.6	A2	0.012	3	+ 1.0	1.1	+ 2.6	Y, L
2698 Ft..	10 2.9	+12 29	7.6	K2	0.247	3	+ 1.1	1.1	....	
β G.C. 5334 Br..	10 3.6	−19 15	7.2	G2	0.355	3	+ 11.9	0.6	....	
C 1222....	10 6.3	+24 15	8.6	G4	0.414	4	+ 82.7	1.5	....	
B.D.+51°1586....	10 7.5	+50 48	9.0	G6	.....	4	+ 9.9	1.7	....	
C 1225....	10 7.5	+53 1	9.2	K3	0.74	3	− 25.1	2.2	....	
2720....	10 9.4	+31 58	6.6	G3	0.047	3	+ 14.6	1.9	....	
β G.C. 5365....	10 10.8	+18 14	6.6	F1	.....	3	− 6.1	1.7	....	
2728....	10 11.0	+24 0	5.9	G1	0.209	3	− 29.5	±1.1	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
2740....	10h 14m.1	+54° 43′	6.2	Ko	0″.023	3	+ 11.3	±0.5	+ 8.1	V
C 1244....	10 14.2	+20 22	9.2	Mdp	0.490	4	+ 9.8	2.3	....	
2741....	10 14.3	+19 59	5.0	F5	0.329	3	+ 4.9	0.9	+ 6.8	L, B
2742....	10 14.5	+20 21	2.6	G8	0.338	3	− 36.6	0.7	−36	L, B
2743....	10 14.5	+20 21	3.8	G5	0.353	3	− 36.0	2.8		
2744....	10 15.1	+42 21	6.7	Ao	0.001	4	− 7.3	2.8	....	
2748....	10 15.9	+ 2 48	6.5	B4	0.008	3	+ 17.2	2.5	....	
β G.C. 5409 Br..	10 17.4	+15 52	7.4	G2	0.292	3	+ 18.9	0.8	....	
2762....	10 18.9	+83 4	5.3	Fo	0.084	3	+ 8.7	1.4	....	
2776....	10 22.1	+37 13	4.4	G6	0.161	3	+ 4.7	0.6	+ 6.2	L, B
2777....	10 22.4	+10 16	5.9	A2	0.014	3	− 10.0	1.8	....	
2780....	10 22.8	+66 8	6.4	K2	0.045	3	− 25.1	1.7	− 25.1	V
C 1264....	10 23.2	− 6 5	8.1	G9	0.478	5	+ 28.7	1.6	....	
2799....	10 26.6	+76 14	5.0	G7	0.030	3	+ 18.7	0.8	+ 16.2	L
2800....	10 26.9	+14 39	5.7	Ma	0.041	3	+ 34.6	1.1	+ 34.0	V
2816....	10 29.6	+ 7 28	5.2	G5	0.119	4	+ 5.9	1.9	....	
β G.C. 5515....	10 34.5	+ 9 22	8.1	F6	.....	3	+ 13.8	0.5	....	
2846....	10 36.3	− 1 13	6.4	Ko	0.191	4	+ 43.6	0.5	....	
H.D. 92706....	10 37.2	+ 1 23	7.6	Ko	0.008	3	+ 18.5	1.4	....	
2853....	10 37.7	+46 44	5.3	A8	0.289	5	+ 6.1	1.4	....	
2865....	10 40.1	+57 54	6.5	Ma	0.081	3	− 2.8	0.6	− 1.0	V
2868....	10 40.9	+ 6 54	6.3	K1	0.039	5	− 8.4	1.6	....	
2881....	10 43.4	+29 57	6.3	G9	0.103	3	+ 10.2	1.0	....	
C 1307....	10 46.1	+20 49	8.1	F1	0.502	4	+ 61.4	1.6	....	
B.D.+33°2049....	10 47.1	+33 31	7.6	G9	0.049	3	+ 11.8	1.0	....	
2899....	10 47.7	+34 45	3.9	K1	0.304	7	+ 18.2	0.5	+ 16.5	L, B
2912....	10 50.5	+42 33	6.1	K1	0.101	3	− 52.4	0.3	− 55.7	V
C 1325....	10 50.9	+28 17	8.6	G6	0.472	3	+ 6.7	1.9	....	
2918....	10 52.0	+78 18	6.3	G7	0.081	4	− 49.2	1.4	− 50.4	V
2920....	10 53.9	+40 58	5.1	G2	0.320	3	+ 11.7	1.0	....	
2922....	10 54.5	+46 4	5.7	K5	0.016	3	+ 9.5	0.3	....	
2925....	10 54.9	−17 46	4.2	Ko	0.481	3	+ 46.0	0.1	+ 47.2	L, Y, C
2927....	10 55.4	+ 4 9	5.0	K1	0.023	3	+ 8.7	1.7	....	
2938....	10 58.5	+ 0 32	6.2	K3	0.066	6	− 7.8	1.6	....	
2950....	11 1.8	+ 2 30	5.7	G9	0.386	3	+ 55.8	1.1	....	
2956....	11 3.5	+25 12	5.6	A2	0.004	8	− 5.6	1.7	....	
β G.C. 5695 B...	11 5.5	+31 0	9.8	Ma	0.623	5	− 27.6	0.8	....	
β G.C. 5695 A...	11 5.6	+31 0	8.8	K7	0.623	4	− 13.5	0.6	....	
β G.C. 5695 C...	11 5.6	+31 2	9.0	G6	.....	5	− 33.8	1.6	....	
2971 Br..	11 8.6	+74 1	7.8	K5	0.405	3	+ 8.6	0.6	....	
2978....	11 10.6	+13 51	5.5	K2	0.028	4	+ 11.1	1.6	+ 11.7	V
2986....	11 13.2	− 4 31	7.3	G8	0.808	5	+ 10.6	1.5	....	
β G.C. 5744 Br..	11 14.3	− 1 6	7.0	F6	0.268	3	+ 19.0	1.4	....	
C 1383....	11 14.8	+66 23	9.2	Ma	2.986	3	+ 47.4	1.7	....	
β G.C. 5757 Br..	11 16.6	+18 45	8.1	K1	0.182	3	− 3.6	±1.8	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	*m*	Sp.	μ	No.	*v*	P.E.	Other Determinations *v*	Other Determinations Auth.
							km	km	km	
Lal. 21683....	11h18m.5	+37°46′	6.9	A4	0″.063	3	− 12.3	±0.6	....	
3022....	11 23.4	+62 19	5.9	F3	0.267	5	− 8.6	1.8	....	
3025....	11 24.5	+15 58	6.0	K3	0.066	3	− 28.1	0.2	....	
3030....	11 25.3	+18 58	5.7	K0	0.078	3	+ 25.5	1.5	....	
3032....	11 26.6	+14 55	6.3	F9	0.387	4	− 4.7	0.4	....	
3033....	11 26.7	+61 38	5.8	F4	0.079	3	− 44.9	0.6	....	
C 1426....	11 28.4	+65 48	7.2	F5	0.204	3	− 31.4	0.2	....	
3046....	11 29.6	+55 20	5.8	G5	0.012	3	+ 18.4	2.2	....	
3047....	11 29.6	−32 18	6.1	K2	1.058	3	− 22.1	2.6	− 24	L
3075....	11 35.8	+34 46	5.5	G5	0.390	3	− 5.1	0.5	....	
3078....	11 36.4	+32 18	5.7	F2	0.347	4	+ 31.2	0.3	....	
3081....	11 36.9	+67 18	5.5	K2	0.058	3	+ 5.8	1.3	....	
3093....	11 41.6	+56 11	5.4	K4	0.043	3	+ 5.0	0.6	....	
3095....	11 41.7	−39 57	5.0	G3	1.574	3	+ 13.6	0.7	+ 17.5	L, C
3100....	11 43.7	−26 12	5.4	Mc	0.028	3	+ 5.3	1.1	....	
B.D.+14°2447....	11 48.7	+14 35	8.1	F1	0.032	3	+ 5.9	1.1	....	
β G.C. 5960 S...	11 49.4	+19 58	8.4	G4	0.445	4	+ 8.2	1.6	....	
β G.C. 5960 N..	11 49.5	+19 59	8.4	G6	0.436	4	+ 4.5	0.5	....	
3141....	11 56.5	+36 36	5.6	G9	0.136	3	+ 31.1	0.7	....	
C 1497....	12 0.1	− 0 57	8.4	G8	0.531	3	+ 14.6	1.3	....	
3155....	12 0.1	+ 9 17	4.2	G6	0.221	4	− 29.6	0.9	− 29.2	L, C
3157....	12 0.6	+63 30	6.2	K1	0.087	3	− 23.5	1.4	− 27.2	V
β G.C. 6028....	12 1.0	+69 15	7.1	F4	.....	3	− 12.6	0.8	....	
3166....	12 3.3	−24 10	4.2	F1	0.093	3	+ 5.6	2.1	+ 3.6	L, C
3169....	12 4.6	+ 2 28	6.1	K2	0.190	4	+ 3.3	0.9	....	
3179....	12 6.8	+57 37	6.5	K4	0.020	3	+ 35.3	1.0	....	
C 1523....	12 7.8	+10 36	8.0	K0	0.430	4	− 8.3	0.9	....	
3186....	12 9.8	+53 59	6.3	G9	0.023	3	+ 0.6	0.2	....	
β G.C. 6090 Ft..	12 10.0	− 6 42	8.3	G9	0.317	4	+ 18.6	1.0	....	
C 1533....	12 10.0	− 9 44	6.1	F3	1.024	5	+ 5.9	1.5	....	
Lal. 23040....	12 13.0	+19 0	7.5	F6	0.026	3	− 27.7	2.3	....	
C 1542....	12 14.1	+17 6	7.0	G2	0.237	4	+ 5.7	1.0	....	
3208....	12 14.4	+88 15	6.3	A8	0.067	3	− 2.5	2.2	....	
3209....	12 14.5	+28 43	6.3	F8	0.245	6	− 7.6	1.2	....	
3213....	12 15.3	+ 3 52	5.1	K0	0.307	3	+ 35.9	0.2	....	
3216....	12 15.7	+18 21	4.9	G6	0.139	3	+ 47.4	0.6	+ 42.7	L, B
C 1551....	12 16.9	+42 42	9.1	Ma	0.57	4	+ 15.2	1.8	....	
C 1564....	12 19.9	+38 52	8.1	F6	0.62	3	− 1.8	0.8	....	
3233....	12 20.2	+24 29	6.1	G9	0.072	3	− 4.4	0.9	....	
C 1579....	12 24.6	− 2 46	8.6	G3	0.721	4	− 3.6	0.8	....	
3258....	12 24.9	−12 50	6.4	G0	0.251	5	− 0.2	1.0	....	
3261....	12 25.3	+52 5	6.2	F6	0.275	3	+ 18.6	1.2	....	
Lal. 23398....	12 26.1	+17 10	7.5	K1	0.025	3	+ 11.4	0.6	....	
3274....	12 28.0	+10 51	6.5	G8	0.063	3	+ 1.0	0.8	....	
3279....	12 29.0	+41 54	4.3	G0	0.758	3	+ 7.0	±0.6	+ 7.9	L, B

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
C 1604....	12h 36m.0	+69° 21′	8.2	G1	0″.46	3	− 5.5	±0.6	....	
3307 S...	12 36.6	− 0 54	3.7	F0	0.564	3	− 20.3	1.6	− 20.0	L
3307 N..	12 36.6	− 0 54	3.7	F1	0.564	4	− 18.4	1.8		
3321....	12 40.3	+39 49	6.0	F6	0.375	3	+ 79.3	1.9	....	
C 1627....	12 43.9	+25 23	6.4	G6	0.367	3	− 7.3	0.7	....	
C 1628....	12 44.3	+60 52	5.9	F5	0.096	5	− 9.5	2.1	− 10:	L
C 1630....	12 44.6	+ 1 45	8.1	G8	0.665	3	− 3.4	0.5	....	
3344....	12 46.2	− 9 48	6.5	G8	0.021	5	− 16.8	2.0	....	
3346....	12 46.5	+ 3 36	6.1	K3	0.035	3	+ 3.0	0.7	+ 2.9	V
3347....	12 46.8	+28 5	5.1	F6	0.031	5	− 2.8	0.5	....	
C 1640....	12 47.9	−17 57	8.3	F3	0.877	3	+143.9	0.9	....	
3353....	12 48.1	− 3 1	6.2	F6	0.262	3	− 7.6	1.9	....	
C 1657....	12 53.9	− 9 18	7.7	K2	0.850	3	− 3.6	1.3	....	
C 1664....	12 55.7	+18 55	6.1	F5	0.245	3	+ 1.0	0.7	....	
C 1666....	12 56.1	−26 50	8.2	F4	0.545	3	+226.4	2.1	....	
3380....	12 56.1	+67 8	5.5	G8	0.150	4	− 28.1	1.1	− 31.4	V
3381....	12 56.2	+17 40	6.0	G8	0.033	3	− 5.8	0.6	....	
3398....	13 1.5	+23 9	5.9	Mc	0.066	4	− 4.3	1.0	....	
3401....	13 2.4	+28 10	4.9	K4	0.089	3	− 15.9	1.8	− 14.5	L
3402....	13 2.4	+62 35	6.3	G8	0.045	4	+ 15.6	1.5	+ 13.9	V
3412....	13 5.1	+18 3	5.2	F4	0.450	3	− 18.9	0.2	− 18.8	L, Y
C 1692....	13 6.4	+10 9	8.5	G2	0.559	3	+ 18.6	0.9	....	
3424....	13 7.2	+28 23	4.3	G1	1.184	3	+ 5.6	0.4	+ 6.1	L, B, C
C 1695....	13 7.5	+18 3	7.8	F6	0.578	3	+ 49.3	0.3	....	
3438....	13 10.6	−19 25	5.3	K1	0.322	3	+ 31.7	1.2	....	
3443....	13 11.8	+ 9 57	5.2	G0	0.382	3	− 24.3	1.7	....	
β G.C. 6442 Br..	13 11.9	+17 33	6.6	K3	0.691	3	+ 4.8	0.7	....	
3446....	13 12.6	+ 6 0	5.0	Ma	0.016	3	− 28.2	1.6	− 26.0	L
3448....	13 13.2	−17 45	4.8	G8	1.529	3	− 10.9	0.8	− 6.6	L
3449....	13 13.5	−22 39	3.3	G7	0.083	3	− 6.6	1.7	− 5.1	L, C
β G.C. 6452....	13 14.9	+35 39	9.6	Ma	0.884	3	− 2.4	0.9	....	
C 1722....	13 15.7	+ 4 39	8.8	K3	0.575	5	− 23.0	1.2	....	
C 1723....	13 16.1	+43 38	8.2	K1	0.438	3	− 38.8	2.2	....	
β G.C. 6476....	13 18.9	+29 45	8.9	K5	0.535	4	− 36.1	1.9	....	
3483....	13 22.6	+63 46	6.5	G7	0.444	3	− 30.1	0.8	....	
3487....	13 23.5	+14 19	5.2	G3	0.634	3	+ 5.3	1.1	....	
3488....	13 23.6	+72 55	6.1	K4	0.030	4	− 47.1	0.5	....	
H.D. 117262....	13 24.1	+34 11	8.2	K0	0.026	3	− 9.6	1.7	....	
β G.C. 6500 Br..	13 24.2	+12 0	8.1	G8	.....	3	+ 3.4	1.7	....	
H.D. 117317....	13 24.4	+70 51	7.5	F1	0.040	3	− 40.1	2.4	....	
β G.C. 6524 Br..	13 28.3	+35 25	7.0	A5	.....	4	− 29.1	1.2	....	
3518....	13 33.0	+36 48	4.9	A2	0.104	3	− 16	1.8	− 6	A
H.D. 118936....	13 35.1	+76 26	8.0	G4	0.026	4	− 42.6	2.0	....	
Lal. 25249....	13 35.1	+48 24	7.9	F3	0.026	4	− 5.8	1.1	....	
Lal. 25265....	13 36.0	+33 21	7.8	F8	0.059	3	− 0.3	±1.1	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
β G.C. 6589 Br..	13h36m.3	+51° 2′	6.3	G0	0″.157	4	− 8.5	±2.2	....	
3537....	13 38.0	+ 4 3	5.6	K3	0.302	3	− 41.9	1.4	....	
3541....	13 39.0	+23 12	6.4	K5	0.072	3	+ 9.6	0.3	....	
C 1786....	13 40.7	+15 26	8.5	Mb	2.298	4	+ 15.1	1.3	....	
C 1789....	13 41.5	+56 23	6.4	F6	0.372	3	− 1.0	1.7	....	
3551....	13 41.9	− 9 13	6.2	K4	0.042	3	+ 7.8	0.3	....	
3554....	13 42.0	+ 6 51	6.3	F9	0.499	4	− 31.8	1.2	....	
3557....	13 42.2	+78 34	6.1	G8	0.080	3	− 6.2	1.4	− 8.2	V
3558....	13 42.5	+17 57	4.5	F7	0.487	5	− 16.9	0.9	− 16.2	L, B
3571....	13 44.4	−17 38	5.1	K0	0.108	4	− 41.6	0.8	....	
β G.C. 6641 Br..	13 44.5	+27 29	7.9	K6	0.455	3	− 20.0	0.4	....	
β G.C. 6641 Ft..	13 44.5	+27 29	8.2	K6	0.455	3	− 21.2	1.2	....	
C 1804....	13 45.8	−23 53	6.5	G2	0.658	3	+ 2.6	0.5	....	
3581....	13 46.7	+35 10	6.0	Ma	0.072	4	− 37.0	1.3	− 41.9	V
3594....	13 49.6	− 1 1	5.3	K1	0.091	3	− 3.9	0.2	....	
3595....	13 49.7	− 7 34	6.2	F8	0.167	5	− 19.5	2.0	....	
3605....	13 53.8	+15 8	6.0	K4	0.087	4	− 40.2	0.8	....	
3616....	13 59.0	−14 29	6.4	K1	0.048	4	− 14.5	1.9	....	
3630....	14 3.9	+44 20	5.4	Mb	0.037	3	− 34.7	0.2	− 38.4	V
3631....	14 4.6	+49 56	5.4	Ma	0.068	4	− 11.9	1.3	− 13.6	V
3634....	14 5.7	+59 49	6.5	G7	0.117	3	+ 11.6	2.3	....	
3641....	14 7.5	−26 9	6.3	G8	0.004	3	− 9.2	2.2	....	
3647....	14 9.1	− 5 29	6.3	F8	0.317	3	− 33.6	0.8	....	
3650....	14 9.3	+13 26	5.5	F7	0.267	3	− 38.1	1.4	− 38.6	L
3656....	14 10.2	+69 54	5.4	Ma	0.068	3	− 22.1	1.9	....	
3658....	14 10.4	+41 59	6.2	K3	0.122	3	− 7.2	1.5	....	
3669....	14 13.1	−18 15	5.7	A3	0.070	5	− 9.5	1.8	....	
3671....	14 13.3	−25 22	5.9	F5	0.538	3	− 21.5	1.6	....	
C 1885....	14 17.6	+30 6	8.6	K6	0.727	4	− 39.7	2.1	....	
3691....	14 18.0	−11 15	6.3	G8	0.066	3	− 0.7	0.8	....	
3710....	14 23.0	− 1 47	5.0	F9	0.134	3	− 10.7	1.0	− 8.8	L
3715....	14 25.2	+50 18	5.6	G3	0.312	3	− 4.6	0.7	....	
β G.C. 6896 Br..	14 25.8	−15 11	8.4	G2	0.433	4	+ 28.7	1.7	....	
C 1913....	14 28.7	+ 9 47	8.9	K3	0.55	4	+ 28.1	1.6	....	
3736....	14 33.6	+18 44	6.0	K0	0.096	3	− 13.2	0.6	....	
3754....	14 36.9	+12 5	5.6	G5	0.205	9	− 23.2	1.3	− 23.1	V
3757....	14 37.5	−34 45	4.1	K3	0.207	3	− 33.0	0.7	− 39.2	L, C
β G.C. 6977 Br..	14 38.2	+58 23	7.4	K0	0.244	3	− 8.5	0.8	....	
3761....	14 39.0	+26 57	4.9	Ma	0.024	3	+ 7.7	0.8	+ 8	L
3771 Br..	14 40.6	+27 30	2.7	G8	0.049	4	− 13.5	1.5	− 16.4	m, L, B, C
Lal. 26928....	14 41.1	+34 48	7.8	Ma	0.044	3	− 32.8	1.6	....	
β G.C. 6999 Br..	14 41.4	+10 4	7.5	G3	0.272	3	+ 24.9	1.2	....	
β G.C. 7001....	14 41.7	+42 48	7.2	F5	0.129	3	− 21.6	0.7	....	
Lal. 26994....	14 44.2	+18 37	7.4	K0	0.026	3	− 27.3	0.2	....	
3785....	14 45.2	+38 13	6.0	F2	0.276	3	− 33.4	±1.8	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
C 1962....	14h46m.0	−23° 53′	7.7	K5	1″.022	3	− 64.5	±1.3	....	
3797....	14 46.6	−30 10	6.4	G0	0.342	3	− 25.3	0.7	....	
3798 Ft..	14 46.8	+19 31	6.8	K4	0.168	3	+ 6.9	2.1	....	
β G.C. 7040 Br..	14 47.9	+19 9	8.1	F9	0.185	3	− 48.1	1.0	....	
H.D. 131444....	14 48.6	+66 4	7.2	Ma	0.007	3	− 27.4	0.9	....	
C 1972....	14 48.9	+19 33	6.0	K1	0.485	3	− 24.8	1.2	....	
C 1973....	14 49.3	+23 45	8.7	K4	0.817	3	− 32.3	1.4	....	
3810....	14 51.3	−11 0	5.6	K2	0.002	5	+ 15.1	2.0	....	
3812....	14 51.6	−20 58	8.9	Ma	1.916	3	+ 25.9	3.9	....	
3816....	14 52.4	+ 0 14	5.7	K1	0.069	3	+ 18.5	0.7	+ 19.5	V
3817....	14 52.5	+16 47	5.8	G3	0.028	3	− 16.1	0.4	− 16.0	V
3822....	14 53.1	+50 2	5.7	F8	0.257	3	− 15.1	1.7	....	
β G.C. 7079 Br..	14 53.7	− 4 35	6.0	F8	0.384	3	− 29.3	0.9	....	
C 1988....	14 54.2	−21 36	8.5	F4	0.785	3	+158.0	1.9	....	
3833....	14 57.1	+82 55	5.7	F8	0.294	3	− 39.9	0.8	....	
3843....	15 0.3	+72 9	6.7	F9	0.398	3	− 44.6	1.2	....	
3847....	15 0.5	+48 3	5.3	G0	0.387	3	− 24.3	0.5	− 23.7	L, B
3855....	15 2.9	+25 16	5.0	F4	0.262	3	− 10.4	0.8	− 7	L
C 2011....	15 3.0	+ 9 16	8.7	F6	0.534	3	− 60.6	2.2	....	
C 2018....	15 4.8	−15 59	9.9	G9	3.684	3	+306.4	1.7	....	
β G.C. 7162 Ft..	15 8.3	+19 40	7.3	G7	0.654	5	− 41.1	1.5	....	
3882....	15 10.2	+ 5 19	5.4	G9	0.023	3	− 33.9	1.0	....	
3887....	15 11.5	+33 41	3.5	G5	0.155	3	− 10.7	2.2	− 12.6	L, B
3895....	15 14.2	+ 2 9	5.2	F6	0.644	3	+ 50.6	1.6	+ 53.8	L, B
C 2043....	15 14.6	− 8 18	7.9	F9	0.211	3	+ 28.1	1.4	....	
C 2044....	15 14.7	+26 4	8.1	G9	0.568	3	− 32.9	2.2	....	
3900....	15 15.2	−17 48	6.2	G8	0.069	4	+ 3.8	1.2	....	
3907....	15 15.9	+ 1 5	5.5	K2	0.125	3	+ 10.5	0.2	....	
β G.C. 7226 Br..	15 16.1	+31 3	9.9	F5	.....	2	− 42.2	1.0	....	
C 2055....	15 17.7	+ 1 47	8.7	K4	0.507	3	− 29.9	0.2	....	
3927....	15 20.7	+37 42	7.2	G0	0.172	4	− 8.8	0.2	....	
3931....	15 21.2	+15 47	5.5	K6	0.033	3	− 18.6	0.8	....	
β G.C. 7263....	15 21.6	+18 31	7.8	F7	0.023	4	− 6.8	1.9	....	
β G.C. 7268 Br..	15 22.7	− 8 59	6.8	G8	0.349	8	+ 2.9	2.3	....	
β G.C. 7268 Ft..	15 22.7	− 8 59	8.1	K2	0.349	7	+ 7.8	1.6	....	
3938....	15 23.3	+25 27	6.3	Ma	0.039	3	− 6.3	1.0	....	
3952....	15 28.7	− 9 43	4.8	K1	0.389	3	+ 45.3	0.4	+ 48.6	L, C
3960 Ft..	15 30.0	+10 52	5.2	A6	0.068	3	− 34.3	1.1	....	
3963....	15 31.0	+17 59	6.1	G7	0.081	10	− 21.8	0.7	....	
3983....	15 35.0	+80 47	6.5	G3	0.246	3	− 16.3	0.3	....	
4001....	15 39.3	+ 6 44	2.8	K2	0.139	3	+ 5.7	1.2	+ 3.3	m, L, B, C
C 2112....	15 41.7	+53 18	7.3	G2	0.261	3	− 34.2	1.1	....	
4029....	15 46.1	−13 50	6.2	G5	0.044	4	− 21.4	2.4	....	
4048....	15 50.2	+20 36	5.8	K4	0.083	3	− 60.8	1.3	....	
4054....	15 51.3	+43 26	5.5	Ma	0.075	3	− 9.8	±1.0	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
C 2134....	15h54m.5	+28° 1′	8.1	G8	0″.85	3	− 71.2	±1.0	....	
4075....	15 56.7	+18 6	5.3	G7	0.153	3	− 16.9	0.2	− 16.6	V
4077....	15 57.2	+33 36	5.4	F9	0.810	3	+ 20.1	1.7	....	
4083....	15 58.9	−11 6	7.2	G7	0.080	4	− 33.7	2.1	....	
4087 C...	15 59.6	−19 32	5.1	B3	0.04	4	− 7.6	2.9	....	
C 2149....	15 59.9	+25 31	7.1	K0	0.862	3	− 36.1	0.2	....	
C 2154....	16 1.5	+39 26	6.8	G9	0.571	3	− 59.6	0.4	....	
C 2160....	16 2.9	+38 55	8.6	K5	0.591	3	+ 24.8	2.7	....	
B.D.+70°861....	16 3.3	+70 23	8.2	Mb	0.020	4	− 17.0	2.7	....	
4101....	16 3.6	+17 19	5.3	G5	0.046	4	− 10.0	0.7	− 9.4	V
4130....	16 8.3	+ 5 17	5.6	K4	0.046	3	− 2.8	1.6	....	
4132 N..	16 8.6	+13 48	7.6	G9	0.459	5	+ 21.3	1.2	....	
H.D. 146025....	16 9.0	+32 52	7.9	K1	0.027	3	− 6.6	0.3	....	
H.D. 146470....	16 11.4	+32 25	8.5	K4	0.035	3	−164.3	0.6	....	
4147....	16 13.0	− 4 27	3.3	G6	0.086	3	− 11.2	0.5	− 9.2	L, C
Lal. 29744....	16 13.2	+38 53	8.0	K3	0.026	3	− 33.6	1.9	....	
4152....	16 14.0	+66 37	8.2	K2	0.011	3	+ 11.7	2.3	....	
4157....	16 15.0	+75 28	6.5	K2	0.041	3	− 24.1	1.2	....	
C 2184....	16 16.5	+67 29	8.9	K6	0.505	3	− 13.8	2.7	....	
4165....	16 17.5	+19 23	3.8	A2	0.062	3	− 39.3	4.7	− 39	L
4173....	16 18.6	+34 2	5.4	Ma	0.049	3	− 9.6	2.2	....	
4175....	16 18.7	+33 56	5.3	K6	0.055	3	− 36.6	0.5	....	
Lal. 29893...	16 19.2	+32 52	7.9	K4	0.011	3	− 0.4	0.1	....	
Lal. 29932....	16 20.7	+32 42	8.1	K4	0.038	3	− 44.3	0.9	....	
β G.C. 7627 Br..	16 21.0	+17 32	7.9	G0	0.019	4	− 31.7	2.0	....	
4192....	16 22.6	+61 44	2.9	G6	0.062	3	− 14.5	2.0	− 13.9	L, B
4195....	16 23.5	+ 0 53	5.5	K5	0.076	6	+ 7.6	1.7	....	
C 2196....	16 23.6	+ 3 29	9.0	G9	0.532	6	+ 22.7	2.3	....	
4207....	16 26.2	+20 42	5.3	G3	0.095	3	+ 17.1	1.7	+ 17.4	V
C 2202....	16 27.4	+48 11	7.0	F7	0.301	3	− 46.9	1.4	....	
4222....	16 31.1	− 2 7	5.9	K0	0.546	3	− 14.4	1.9	....	
B.D.+51°2121....	16 34.2	+51 45	9.6	G3	0.119	3	+ 25.7	0.7	....	
4234....	16 34.9	+77 39	6.4	G8	0.286	3	− 31.5	1.2	....	
4242....	16 36.0	+49 7	5.1	Ma	0.044	3	− 54.0	0.4	− 55.8	V
4257....	16 40.1	+ 6 17	6.7	G8	0.352	8	− 5.0	1.3	− 6.3	V
4260....	16 40.4	+ 1 12	6.0	B9	0.003	3	− 9.4	3.5	....	
4262....	16 40.8	+15 56	5.8	Mb	0.054	3	− 17.7	1.4	....	
4264....	16 41.0	+ 8 46	5.4	K5	0.016	4	− 22.3	2.3	....	
C 2238....	16 41.4	+33 41	8.6	K8	0.37	3	− 30.0	1.6	....	
4285....	16 46.3	+ 1 23	5.5	A1	0.029	8	− 26.3	1.4	....	
β G.C. 7778 Br..	16 46.4	+ 9 34	7.0	F6	.....	3	− 35.1	0.6	....	
β G.C. 7783 Br..	16 47.9	+28 49	6.7	F4	.....	3	− 24.9	0.8	....	
C 2251....	16 50.1	− 8 9	9.2	Md	1.234	5	+ 11.9	1.8	....	
4305....	16 50.4	+43 0	6.7	F9	0.349	4	+ 7.9	1.7	+ 5.8	V
4307....	16 50.6	+21 7	5.5	G7	0.059	3	− 0.7	±1.4	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
4310....	$16^h 50^m.9$	+25° 53′	6.3	G5	0″.031	3	− 0.1	±2.0	− 0.6	V
4311....	16 51.0	+18 36	5.6	K6	0.106	3	+ 13.4	0.8	+ 11.5	V
4318....	16 53.8	−24 56	5.9	Ma	0.022	5	− 31.6	1.8	......	
4326....	16 56.0	−18 44	6.4	K0	0.052	3	+ 43.7	1.2	......	
4336....	16 58.5	+14 14	5.1	Mb	0.070	3	+ 46.6	1.3	+ 41.0	V
4343....	16 59.9	+35 33	6.8	Mb	0.058	3	− 11.4	1.9	......	
C 2279....	17 0.2	+ 0 51	5.9	G0	0.321	7	− 18.6	1.5	......	
β G.C. 7872 Br..	17 2.0	+59 42	9.1	K3	0.427	4	− 73.2	1.3	......	
4351....	17 2.2	+48 57	6.3	K0	0.089	5	+ 12.1	1.3	......	
C 2284....	17 3.4	+ 4 34	7.2	G0	0.208	3	− 18.4	0.8	......	
H.D. 154928....	17 3.4	+73 27	7.7	K5	0.007	3	− 4.9	0.1	......	
4364....	17 6.3	+40 54	5.1	K3	0.053	3	− 52.2	0.4	− 59.4	V
β G.C. 7898 Br..	17 7.8	+21 20	7.5	K1	.....	3	− 48.3	1.7	......	
4373....	17 10.1	+14 30	3.5	Mb	0.030	5	− 31.2	1.2	− 32.4	L, B, C
4386....	17 12.1	−34 53	5.9	K5	1.194	4	+ 3.4	1.3	− 4:	L
β G.C. 7936....	17 12.2	+56 15	7.9	F5	.....	3	− 3.4	1.4	......	
H.D. 156824....	17 14.6	+ 2 40	8.6	F2	.....	5	+ 25.6	2.3	......	
4393....	17 14.9	+28 56	5.8	G8	0.045	4	− 14.0	0.7	......	
4396....	17 15.3	+60 47	6.4	G9	0.046	3	+ 17.7	2.3	......	
C 2310....	17 15.4	+ 9 34	8.2	G0	0.313	4	− 12.8	1.1	......	
4413....	17 18.7	−21 21	6.0	G8	0.045	4	− 55.4	1.2	......	
4414....	17 19.0	−24 9	6.3	K1	0.004	4	+ 20.5	1.7	......	
4424....	17 21.4	−12 25	6.3	F6	0.075	3	− 40.2	1.4	......	
4433....	17 25.2	− 0 59	6.0	G6	0.215	3	− 80.4	0.3	− 70.7	C
B.D.+68°930....	17 26.2	+68 27	9.1	G7	.....	3	− 33.8	1.9	......	
C 2339....	17 29.1	+63 56	7.4	G1	0.21	3	− 29.2	0.9	......	
4447....	17 29.2	+16 23	5.7	G9	0.058	3	− 21.0	0.4	......	
4449....	17 29.3	−21 59	6.6	B9	0.015	6	− 12.4	1.6	......	
β G.C. 8068 A...	17 30.0	+ 6 6	7.9	F8	0.070	4	+ 2.6	2.0	......	
C 2347....	17 33.4	+18 37	9.1	Ma	1.39	3	− 9.3	0.9	......	
C 2348....	17 33.9	+18 37	9.1	F0	0.28	3	−240.0	2.5	......	
4470....	17 34.0	+61 57	5.3	G0	0.563	4	− 11.0	1.8	......	
B.D.+18°3424....	17 34.3	+18 37	9.4	K2	1.20	3	+ 1.6	1.3	......	
4473....	17 35.4	+74 17	7.1	G8	0.081	3	− 8.4	0.6	......	
C 2351....	17 35.8	+68 52	8.5	K0	0.150	3	− 55.1	1.5	......	
C 2353....	17 36.2	+37 16	8.5	F7	0.945	4	+ 39.2	1.2	......	
C 2354....	17 37.0	+68 26	9.5	Mb	1.334	4	− 16.5	3.2	......	
4480 Br..	17 37.0	+24 34	6.5	K2	0.052	4	− 31.7	0.6	......	
4482....	17 37.5	+16 0	5.6	F2	0.096	4	− 44.2	1.0	......	
4487....	17 38.5	+ 4 37	2.9	K2	0.158	7	− 11.4	1.2	− 11.5	7
C 2358....	17 39.0	+21 40	7.4	G8	0.623	3	+ 20.6	2.2	......	
4488....	17 39.3	+24 22	5.7	G2	0.132	4	− 25.0	1.0	......	
4497....	17 42.5	+27 47	3.5	G5	0.817	3	− 16.1	1.2	− 15.7	L, B, C
4504....	17 43.7	+72 12	4.9	F3	0.267	3	− 9.6	1.3	− 10.4	L
4505....	17 43.7	+72 12	6.1	F7	0.279	4	− 11.4	±1.3	......	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
4508....	17h44m8	+25° 39′	5.3	K1	0.″046	3	− 23.9	±0.5	....	
β G.C. 8209 Br..	17 47.6	− 7 53	7.6	G0	0.246	4	−124.8	2.2	....	
Lal. 32882....	17 50.9	+42 40	7.6	K0	0.004	3	− 0.7	0.6	....	
4527....	17 51.2	+ 0 41	5.7	B3	0.006	7	− 17.5	2.0	....	
Barnard's Star....	17 52.9	+ 4 25	9.7	Mb	10.5	5	−107	4.5	−128	L
4542....	17 54.7	+30 12	4.5	F2p	0.004	3	− 21.2	0.7	− 21.6	L
4553....	17 56.7	−24 17	5.5	F4	0.016	3	− 8.9	1.6	....	
4555....	17 57.1	+45 30	5.9	K6	0.044	3	− 9.3	1.5	....	
β G.C. 8316 Br..	17 58.4	+26 22	7.1	K1	0.72	3	+ 23.3	0.5	....	
4571 Br..	18 0.4	+ 2 31	4.3	K0	1.131	3	− 3.9	2.0	− 7.4	L
C 2392....	18 0.7	+ 4 39	6.8	G1	0.296	3	−124.1	1.4	....	
4576....	18 1.2	−21 27	6.2	B2	0.014	3	− 5.8	1.1	....	
Lal. 33290....	18 1.4	+42 51	7.5	K2	0.035	3	− 13.1	0.9	....	
C 2396....	18 2.1	+ 8 52	7.7	F7	0.152	3	+ 17.1	2.2	....	
4582....	18 3.2	+30 33	5.2	F5	0.113	3	+ 0.9	1.0	+ 0.4	L
4593....	18 4.6	+36 23	5.7	K3	0.213	4	− 6.4	0.6	− 6.9	V
4595....	18 4.9	+ 3 6	5.7	F4	0.205	3	− 16.6	1.9	− 14.2	V
Lal. 23516....	18 5.9	+43 33	8.1	K1	0.017	4	− 23.1	1.1	....	
4606....	18 8.1	+31 23	5.0	Ma	0.018	3	+ 3.7	1.1	− 1.4	V
H.D. 167063....	18 8.5	+33 15	7.1	K6	0.020	3	0.0	0.7	....	
4609....	18 8.5	+54 15	5.9	G9	0.274	4	− 14.6	1.3	− 16.8	V
4618....	18 11.6	− 3 2	6.1	G2	0.274	8	+ 1.5	1.0	....	
4623....	18 13.3	+64 22	5.0	F2	0.346	3	− 35.5	0.7	− 35.7	L
β G.C. 8485 Br..	18 14.7	− 8 2	6.6	F2	.....	3	− 49.6	0.4	....	
4638....	18 16.1	− 2 55	3.4	G8	0.898	3	+ 10.2	1.4	+ 9.6	L, B
β G.C. 8507 Br..	18 16.9	+27 29	7.1	G1	.....	3	− 19.0	1.3	....	
4647....	18 17.6	+51 18	6.2	K1	0.062	3	− 10.4	2.4	....	
Lal. 33888....	18 18.0	+ 7 9	7.6	K2	0.012	3	− 31.3	1.9	....	
4656....	18 19.4	+21 43	3.9	K1	0.324	3	− 55.6	2.0	− 57.0	L, B, C
Lal. 34058....	18 19.9	+43 53	8.2	G8	0.001	4	+ 1.5	1.0	....	
Lal. 33996....	18 20.3	+ 9 41	7.9	K3	0.016	3	− 16.4	1.6	....	
C 2421....	18 21.6	+ 8 34	8.5	G9	0.47	3	− 24.9	2.5	....	
H.D. 169957....	18 22.0	+ 8 2	8.9	G2	0.151	3	+ 41.2	2.9	....	
β G.C. 8568 Br..	18 22.7	+ 6 29	9.0	A5	0.110	3	− 35.0	1.6	....	
C 2423....	18 23.9	+46 1	8.3	G0	0.398	4	− 86.5	1.7	....	
H.D. 170615....	18 25.3	+44 11	7.7	K3	0.014	3	− 29.3	0.5	....	
H.D. 170780....	18 26.1	+ 8 1	7.6	Ma	0.029	3	− 22.4	1.2	....	
4688....	18 26.3	+59 29	6.5	G8	0.061	3	− 9.1	0.4	....	
4701....	18 28.6	+23 33	6.0	K4	0.011	4	− 3.8	1.2	....	
4711....	18 31.7	+52 16	5.4	G7	0.008	3	− 19.9	2.0	....	
4721....	18 33.0	+33 23	5.5	B9	0.024	7	− 27.6	1.7	− 29.3	O
β G.C. 8679....	18 33.2	− 3 17	6.5	G0	.....	8	− 21.4	1.5	....	
C 2448....	18 34.4	+28 51	8.2	G5	0.47	3	+ 28.4	1.2	....	
4723 Br..	18 34.5	+63 37	8.1	F7	0.254	7	− 11.5	2.1	....	
β G.C. 8734 Br..	18 37.1	+31 28	8.8	K3	0.82	3	+ 33.9	±3.3	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Auth.
							km	km	km	
4751....	18h41m3	− 1° 4′	5.7	A4	0″031	7	+ 18.5	±3.4	....	
β G.C. 8798 Br..	18 41.7	+59 29	8.8	Mb	2.307	5	+ 1.0	2.1	....	
β G.C. 8798 Ft..	18 41.7	+59 29	9.3	Mb	2.307	3	+ 8.0	3.2	....	
Lal. 34958....	18 44.1	−10 29	7.9	K2	0.05	5	− 53.4	1.5	....	
4787....	18 49.3	+52 51	5.6	G8	0.288	3	+ 2.8	1.2	....	
4811....	18 52.1	+48 44	5.9	F2	0.143	3	− 11.0	1.8	− 12.2	V
C 2475....	18 53.1	+ 5 48	9.8	Ma	1.247	3	+ 19.3	1.7	....	
Lal. 35518....	18 56.2	+18 20	8.0	G7	0.017	4	− 23.6	1.9	....	
4840 B...	18 57.6	− 0 51	8.5	G5	.....	3	− 25.9	2.2	....	
4852 Ft..	18 59.7	− 4 11	7.2	K5	0.037	6	− 58.5	2.8	....	
4860....	19 1.2	+31 36	5.8	K5	0.085	5	+ 6.8	1.1	....	
C 2492....	19 2.2	+ 7 29	9.5	K5	0.812	5	+ 12.2	2.7	....	
4863....	19 2.2	+76 54	6.5	F3	0.080	3	− 27.8	1.3	− 28.5	V
β G.C. 9043 Br..	19 2.6	+22 1	7.5	F3	.....	3	+ 11.5	1.2	....	
β G.C. 9116 AB.	19 8.0	+16 41	6.4	B9	0.030	5	− 18.3	2.0	....	
β G.C. 9116 C...	19 8.0	+16 41	7.6	G6	0.243	5	+ 37.6	0.9	....	
β G.C. 9136 Br..	19 10.0	+18 54	7.9	F7	0.031	3	− 8.3	1.7	....	
4915....	19 13.3	−15 43	6.3	K3	0.285	4	− 18.0	1.4	....	
4920....	19 14.0	+46 49	6.0	F4	0.285	4	− 44.6	1.3	− 45.0	V
B.D.+41°3306....	19 15.7	+41 28	7.6	G8	0.66	3	−125.0	2.6	....	
4937....	19 17.2	− 0 27	6.0	G8	0.051	4	− 10.4	1.3	....	
Lal. 36491....	19 17.6	−10 54	7.0	Ro	.....	3	− 44.6	2.6	− 43.8	D
4958....	19 20.8	+43 12	6.0	G5	0.046	3	+ 3.1	1.7	− 2.0	V
β G.C. 9319....	19 22.5	+27 7	7.8	F9	.....	3	− 7.6	1.8	....	
4985....	19 26.4	−28 13	7.0	G6	0.741	4	− 42.5	2.2	....	
4987....	19 26.7	+27 45	5.4	B8	0.014	4	− 13.6	1.4	....	
β G.C. 9381....	19 27.2	+17 34	8.4	G1	0.127	3	− 58.2	0.7	....	
β G.C. 9401 Br..	19 28.9	+20 12	7.4	A7	0.057	8	− 52.1	2.2	....	
5005....	19 31.7	+51 1	5.8	F6	0.205	3	+ 0.7	1.1	....	
5009....	19 32.6	+69 29	4.8	G9	1.840	4	+ 29.8	1.4	+ 26.2	L, B
5014....	19 33.8	+49 59	4.6	F2	0.249	4	− 26.4	0.7	− 28.3	L, B
5019....	19 35.0	−16 31	5.4	K1	0.085	3	− 56.0	0.7	....	
5031....	19 37.8	+45 17	5.0	F4	0.142	5	− 20.2	1.4	....	
5033....	19 37.9	+11 35	5.6	F5	0.013	3	− 27.0	2.3	....	
5038....	19 39.2	+50 17	6.4	G3	0.204	3	− 28.0	0.9	....	
β G.C. 9569 Br..	19 39.8	+26 54	6.6	G0	0.062	3	− 8.4	1.5	....	
5043....	19 40.4	+41 32	6.0	K6	0.017	3	− 40.4	1.2	....	
5045....	19 40.7	+37 7	5.0	G7	0.076	3	− 26.1	0.8	− 23.9	V
β G.C. 9602....	19 41.8	+33 22	8.5	K4	0.433	4	+ 6.5	1.5	....	
5049....	19 42.1	+34 46	6.2	K1	0.012	3	− 18.1	0.7	− 20.2	V
5051 Br..	19 42.6	+33 30	5.0	F6	0.452	3	+ 1.4	0.8	+ 5.5	B
5051 Ft..	19 42.6	+33 30	8.1	K4	0.452	4	+ 1.2	1.4		
β G.C. 9650 Br..	19 45.0	+35 4	6.8	F4	0.119	6	− 27.2	2.2	....	
5075....	19 48.1	+52 44	5.2	K5	0.072	3	− 16.5	1.8	− 20.2	V
5082....	19 48.7	− 8 50	6.0	K4	0.022	3	− 49.0	±1.5	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
5083....	19ʰ49ᵐ0	+46°46′	5.5	Oe5	0″015	4	− 5.4	±2.1	....	
5093....	19 50.4	+ 6 9	3.9	G8	0.484	5	− 41.1	1.0	− 39.5	L, B, C
5102....	19 52.3	+38 13	4.9	B6	0.012	6	− 30.8	1.2	− 33.8	L, O
5119....	19 54.4	−10 13	5.9	F8	0.488	4	+ 22.6	1.8	....	
C 2607....	19 55.5	−12 31	7.6	F6	0.504	3	− 15.2	1.3	....	
5143....	19 59.3	+ 7 0	5.6	G8	0.025	3	− 27.5	1.3	....	
5149....	20 0.4	+64 32	5.4	Ma	0.015	3	− 34.6	0.9	....	
5156....	20 2.6	+23 20	5.1	B3	0.008	6	− 4.6	2.5	− 12.0	V
5157....	20 2.6	+35 42	5.5	G8	0.496	3	− 35.1	0.9	....	
B.D.+36°3883....	20 3.7	+36 17	7.1	Ma	.....	4	− 15.4	1.0	....	
5166....	20 4.6	−36 21	5.3	K3	1.626	3	−131.6	1.3	−130	L
5168....	20 5.5	+20 37	8.7	G8	0.122	3	− 42.5	1.7	....	
5169....	20 5.5	+20 37	6.3	F2	0.111	6	− 40.5	1.1	....	
5176....	20 6.9	−12 55	5.9	F4	0.269	3	+ 22.1	1.4	....	
β G.C. 9979 Br..	20 6.9	+43 38	7.5	G5	.....	3	− 40.8	1.7	....	
B.D.+15°4089....	20 8.2	+15 47	7.6	F6	0.016	3	+ 4.7	1.3	....	
5184....	20 9.9	+61 47	5.7	F6	0.155	3	− 15.1	0.3	− 17.9	V
β G.C. 10044 Br..	20 11.0	+52 49	7.2	F5	0.179	4	− 33.5	1.3	....	
5196....	20 11.9	+21 17	6.2	K1	0.031	3	− 3.6	1.1	....	
5201....	20 12.5	+24 22	5.4	G7	0.027	3	+ 17.9	1.6	....	
5229....	20 18.6	+39 56	2.3	Gop	0.003	3	− 5.9	1.0	− 5.6	P, m, L, B
5231....	20 19.2	+40 42	6.1	K6	0.054	3	+ 1.1	2.0	....	
5255....	20 25.3	+30 2	4.1	F6	0.010	4	− 18.0	1.5	− 19.2	L, B
5256....	20 25.5	−15 23	6.2	G2	0.073	3	+ 30.9	1.7	....	
C 2662....	20 26.7	+45 35	6.5	K2	0.173	3	− 30.7	0.9	....	
5263....	20 26.9	−10 12	5.8	G3	0.317	6	+ 8.7	1.1	....	
5276....	20 29.3	+56 26	6.3	K4	0.021	6	− 14.1	1.8	....	
5280....	20 30.4	+72 12	6.4	K4	0.025	4	− 42.3	1.0	− 43.9	V
B.D.+5°4570	20 33.7	+ 5 18	9.0	F4	0.90	4	− 15.0	2.4	....	
5299....	20 34.0	+12 58	6.1	K2	0.020	4	− 17.1	1.4	− 13.1	V
C 2672....	20 34.2	−24 8	6.3	G8	0.658	3	− 50.0	3.2	....	
5304....	20 34.3	+ 9 44	5.2	G2	0.315	10	− 50.4	1.1	− 52.5	B
C 2673....	20 34.4	+42 29	7.1	F6	0.197	5	+ 1.1	0.9	....	
5306....	20 34.4	−18 29	5.3	Ma	0.035	3	− 12.3	0.7	....	
C 2676....	20 34.6	+ 4 37	8.4	K5	0.844	8	− 42.9	1.7	....	
C 2682....	20 36.2	+19 34	6.4	G3	0.337	3	− 37.5	1.5	....	
B.D.+19°4489....	20 37.6	+19 30	7.5	K1	0.009	3	+ 12.9	0.8	....	
Lal. 40126....	20 41.1	+31 25	8.0	K1	0.015	3	− 12.1	1.1	....	
β G.C. 10504 Br..	20 41.5	+15 32	7.5	G8	0.121	4	− 25.1	0.8	....	
β G.C. 10504 Ft..	20 41.5	+15 32	8.2	G6	0.100	3	− 28.6	1.3	....	
5334....	20 42.0	+15 46	5.5	F6	0.195	4	− 7.3	1.0	− 6.3	L, B, C
5335....	20 42.0	+15 46	4.5	K1	0.207	3	− 5.6	1.3		
5344....	20 42.9	+57 13	4.6	F8	0.241	3	− 27.8	0.8	− 31.2	L, B
5364....	20 46.1	− 6 0	6.0	F5	0.090	3	− 26.6	1.3	....	
5379....	20 50.3	+27 41	5.2	K5	0.009	3	+ 8.7	±1.2	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
5385....	20h 50m.9	+13° 20′	5.4	K0	0″.023	3	− 8.0	±0.8	− 11.2	V
C 2707....	20 51.3	+61 48	8.6	Ma	0.77	3	− 8.4	2.3	....	
5388....	20 52.1	+80 11	5.6	K0	0.040	3	− 27.2	1.0	− 26.3	V
C 2712....	20 53.1	+42 30	7.9	G9	0.284	3	− 18.9	0.6	....	
Lal. 40627....	20 53.1	+48 48	6.0	G9	.....	4	− 14.2	1.8	....	
B.D.+69°1136....	20 53.6	+69 34	7.8	G9	0.020	3	+ 10.1	0.9	....	
5399 C...	20 54.1	+ 3 55	7.4	F2	0.191	3	+ 9.0	2.0	....	
5409....	20 55.9	+18 56	6.2	Ma	0.075	3	− 14.4	1.5	....	
Lal. 40811....	20 58.8	+31 57	7.2	K4	0.014	3	− 8.2	1.4	....	
C 2727....	20 59.1	+45 29	7.9	K2	0.403	3	− 11.6	2.9	....	
C 2732....	21 0.4	+ 6 41	8.9	K7	0.553	3	− 65.1	2.4	....	
5433....	21 2.4	+38 15	5.6	K7	5.267	5	− 65.5	0.3	} −63:	L
5434....	21 2.4	+38 15	6.3	K8	5.153	5	− 62.3	1.5		
Lal. 41058....	21 4.9	+32 22	8.1	K0	0.023	3	− 24.0	2.7	....	
β G.C. 10792 N..	21 5.9	+19 33	8.5	A9	0.038	3	− 18.9	2.7	....	
β G.C. 10792 S...	21 5.9	+19 33	8.1	F2	0.038	3	− 21.6	1.2	....	
C 2753....	21 8.8	+73 18	8.6	K0	0.518	3	+ 10.3	1.6	....	
5457....	21 10.2	−15 35	5.5	Mb	0.019	7	− 38.3	1.6	....	
C 2757....	21 11.4	−39 15	6.8	Ma	3.532	3	+ 23	2.2	+ 13	L
Lal. 41378....	21 13.3	+17 12	7.7	F3	0.037	3	− 1.8	2.6	....	
Lal. 41391....	21 13.8	+17 18	7.6	K1	0.022	3	+ 5.3	1.6	....	
Lal. 41405....	21 13.9	+17 34	7.2	F4	0.016	3	+ 8.5	0.4	....	
5472....	21 14.2	+55 23	6.2	K2	0.024	3	− 20.0	0.9	− 18.4	V
5476....	21 15.8	− 4 59	6.0	G7	0.026	3	− 5.5	0.8	....	
5482....	21 16.6	+23 26	5.8	G9	0.253	3	− 88.3	1.6	....	
5490....	21 17.6	− 9 45	6.2	K4	0.050	3	+ 13.4	0.5	....	
Lal. 41576....	21 18.4	+16 4	7.6	K4	0.022	3	− 67.4	0.9	....	
C 2783....	21 21.4	+ 0 41	6.4	F4	0.186	3	+ 10.4	2.2	....	
5519....	21 23.9	+31 47	5.7	F0	0.134	3	− 25.2	1.1	− 25.2	V
C 2790....	21 24.5	−12 56	9.4	K6	1.052	3	− 86.2	2.0	....	
C 2794....	21 25.3	+11 50	7.7	F6	0.180	5	− 23.5	2.1	....	
5523....	21 25.8	+46 6	5.3	G9	0.114	4	− 17.4	1.1	....	
C 2797....	21 26.0	+45 27	7.9	G8	0.56	3	− 83.2	0.7	....	
B.D.+45°3566....	21 26.8	+46 6	8.3	F0	.....	4	− 11.4	0.6	....	
β G.C. 11051 Br..	21 28.4	+20 16	7.1	F6	.....	3	+ 15.0	1.4	....	
β G.C. 11051 Ft..	21 28.4	+20 16	8.0	F7	.....	3	+ 10.6	2.6	....	
B.D.+67°1324....	21 30.0	+67 56	8.7	K2	0.027	3	− 19.9	1.6	....	
5543....	21 30.2	+45 9	4.2	G5	0.099	3	+ 7.0	0.9	+ 7.0	L, B
Lal. 42193....	21 33.8	+32 41	7.6	G8	0.045	3	+ 9.5	0.9	....	
5560....	21 34.5	+ 1 48	5.3	K0	0.088	3	− 34.1	1.2	− 34.9	V
B.D.+30°4496....	21 35.1	+31 5	8.2	F8	0.047	3	− 27.8	0.7	....	
C 2816....	21 36.6	+26 18	7.4	G4	0.355	4	− 41.0	3.4	....	
5569....	21 37.1	+ 0 50	5.8	K2	0.015	5	+ 10.7	1.7	....	
5584....	21 39.3	+ 9 25	2.5	K0	0.025	3	+ 6.1	0.2	+ 5.3	7
C 2823....	21 39.7	+24 53	9.1	K0	0.65	3	− 50.7	±0.9	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
5587....	21h39m7	+28°17′	4.7	F5	0″370	3	+ 17.2	±0.8	+ 18.8	L
5595....	21 40.9	− 9 44	6.2	Mb	0.016	4	+ 21.3	1.5	....	
5602....	21 41.8	+25 6	6.5	K1	0.148	4	− 44.4	1.9	....	
Lal. 42626....	21 46.5	+32 12	8.6	K0	.....	3	− 19.6	2.1	....	
B.D.+64°1596....	21 47.2	+64 46	7.9	Mb	0.010	4	+ 4.9	1.9	....	
Lal. 42655....	21 47.3	+31 27	7.5	K4	0.029	3	− 11.3	1.5	....	
5632....	21 49.7	+55 44	6.9	G4	0.028	4	− 14.4	2.9	....	
5638....	21 51.4	+53 28	6.9	F6	0.172	4	− 3.7	1.1	....	
5639....	21 51.5	+56 8	6.0	B8	0.009	5	− 19.8	1.3	....	
5645....	21 53.2	−21 40	6.2	Mb	0.015	3	+ 3.7	1.6	....	
C 2857....	21 53.4	+ 3 18	7.?	G0	0.308	4	+ 24.1	1.0	....	
5658....	21 56.2	+12 38	5.7	F3	0.077	5	+ 6.5	1.6	....	
5674....	22 0.6	+ 4 34	4.9	K6	0.144	4	− 19.5	1.5	− 15.7	L
5677....	22 0.9	+64 8	6.5	F7	0.226	3	− 6.3	1.8	− 4.5	L
5682....	22 1.8	+82 23	7.1	F6	0.141	5	− 22.7	0.9	....	
5691....	22 2.7	+18 59	5.8	F1	0.126	3	− 14.3	1.6	....	
5694....	22 3.8	+58 21	6.3	G6	0.030	3	− 10.9	2.5	....	
5707....	22 5.3	− 4 46	6.1	G9	0.063	4	− 17.6	1.3	....	
5711....	22 7.0	+15 33	6.1	K0	0.022	3	+ 11.4	1.3	....	
5721....	22 8.2	+56 21	5.4	F6	0.270	3	− 22.1	1.5	− 18.8	V
5723....	22 8.4	+69 38	5.5	F2	0.058	4	+ 1.2	1.1	− 0.4	V
5724....	22 8.4	+34 7	5.4	K2	0.047	3	− 5.1	0.1	− 8.7	V
5730....	22 9.1	+28 7	6.0	K3	0.045	4	− 18.4	0.9	......	
5741....	22 11.1	+72 49	6.1	G8	0.031	3	+ 1.5	1.1	....	
5745....	22 11.6	− 9 32	6.1	K3	0.053	3	+ 12.6	0.6	....	
5746....	22 11.6	+37 15	4.2	K3	0.020	3	− 8.5	0.7	− 7.4	L, B
5752....	22 13.6	−13 48	6.1	G9	0.074	3	+ 30.5	1.5	....	
5772....	22 18.8	+20 21	6.1	F4	0.338	7	− 24.1	1.0	....	
5781....	22 21.1	−17 15	6.6	G1	.....	4	− 3.8	1.6	....	
5782....	22 21.1	−17 15	6.4	G1	0.225	4	− 6.7	0.9	....	
5786....	22 21.5	+ 3 53	5.8	F7	0.295	5	− 18.2	1.2	....	
5790....	22 22.8	+ 4 12	4.9	K0	0.325	4	+ 53.7	1.6	+ 54.7	L
Lal. 43876....	22 23.4	+16 45	7.5	K0	0.017	3	− 39.8	0.7	....	
5796....	22 23.9	+64 37	5.7	B1	0.011	6	− 14.1	1.4	....	
β G.C. 11761 Br..	22 24.4	+57 11	9.3	Mb	0.87	4	− 23.6	2.4	....	
5799....	22 24.7	−13 26	6.2	F2	0.181	3	− 11.2	0.9	....	
5800....	22 24.9	+ 3 55	5.7	F1	0.141	4	+ 0.5	2.0	− 0.9	V
5804....	22 25.4	+47 12	4.6	K2p	0.021	3	− 10.6	1.6	− 11.6	L, B
B.D.+75°832....	22 27.1	+75 43	7.9	K1	0.009	3	− 12.9	0.8	....	
5822....	22 30.1	−24 30	6.0	K0	0.016	4	− 2.5	2.1	....	
C 2937....	22 30.5	+ 4 52	8.4	G1	0.45	4	− 7.4	1.7	....	
B.D.+60°2414....	22 31.4	+60 19	7.1	F2	.....	3	− 0.1	1.2	....	
5838....	22 33.3	+73 7	5.2	F2	0.171	7	+ 3.0	0.8	....	
β G.C. 11873....	22 34.4	+43 47	6.9	F9	0.241	4	− 19.6	1.4	....	
5847....	22 34.9	+19 10	6.1	A2	0.019	3	− 12.3	±2.6	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Auth.
							km	km	km	
5851....	22h 35m.9	+14° 1′	6.6	G4	0″.302	4	− 10.5	±0.3	....	
5868 Ft..	22 39.6	+38 56	8.3	G8	0.016	3	− 22.4	1.1	....	
5872....	22 40.6	+18 50	6.5	G8	0.060	4	− 21.0	1.4	− 22.6	V
5874....	22 41.7	+11 40	4.3	F5	0.545	3	− 8.8	1.2	− 5.3	L, B, C
5875....	22 41.7	+23 2	4.1	G7	0.060	4	− 3.2	0.7	− 3.8	L, B, C
5881 A...	22 42.7	− 4 45	7.1	G3	0.354	5	− 23.1	1.8	....	
5881 B...	22 42.7	− 4 45	7.6	G4	0.354	5	− 25.4	2.2	....	
5892....	22 46.4	+13 26	8.0	K2	0.474	3	− 1.2	0.4	....	
5905....	22 49.4	− 7 44	6.3	K3	0.041	3	+ 9.5	2.2	....	
5907....	22 49.5	−16 48	5.7	K3	0.234	5	− 35.8	2.0	....	
5909....	22 50.0	− 5 31	6.0	G8	0.018	4	− 8.3	1.9	....	
5914....	22 52.0	+49 12	5.1	K2	0.012	5	− 7.8	1.2	− 7.8	L
5915....	22 52.1	− 5 21	6.4	G6	0.023	4	− 8.2	1.7	....	
5917....	22 52.6	+20 14	5.6	G4	0.210	3	− 32.8	1.2	− 31.6	V
C 3001....	22 55.0	−23 4	7.7	K4	0.899	4	+ 17.1	1.7		
C 3002....	22 55.1	+68 29	8.4	K0	0.66	3	− 25.9	2.2	....	
5929....	22 55.5	− 0 21	6.4	G3	0.038	3	− 16.3	1.0	....	
5931....	22 55.9	+56 25	5.5	G2p	0.009	3	− 59.2	0.9	....	
5932....	22 56.2	− 7 36	6.4	K5	0.027	3	− 1.6	1.6	....	
5958....	23 3.2	+48 45	5.8	F6	0.194	3	− 1.3	1.9	....	
C 3028....	23 4.0	− 2 48	8.3	K4	0.587	4	− 40.4	0.8	....	
5974....	23 7.0	+26 18	6.4	G7	0.228	3	− 11.3	0.9	− 9.8	V
5977 Br..	23 8.9	− 9 28	8.3	F5	0.557	4	− 32.3	1.2	....	
β G.C. 12274....	23 11.9	−14 22	8.2	A5	1.301	4	+ 5.4	2.4	− 5.5	L
5989....	23 12.1	+52 40	5.6	F8	0.270	5	− 24.8	1.1	....	
β G.C. 12990 Br..	23 13.8	+ 4 52	9.0	K2	0.492	3	− 13.8	1.2	....	
5998 Ft..	23 13.9	−14 0	7.6	K2	0.314	5	+ 8.5	1.5	....	
6000....	23 14.5	+67 34	5.0	G7	0.066	3	− 20.4	0.8	− 17.8	L, B
C 3054....	23 15.0	+28 19	8.8	K1	0.66	3	− 50.4	2.2	....	
6002....	23 15.0	+47 50	6.4	K1	0.206	3	+ 23.1	1.2	....	
C 3059....	23 16.8	+43 33	7.6	K0	0.671	3	+ 2.1	0.9	....	
6013....	23 17.8	+20 1	6.6	G1	0.320	3	− 22.9	0.7	....	
6020....	23 18.9	+31 59	6.5	F0	0.233	6	+ 9.7	0.4	....	
6023....	23 20.0	+31 50	5.5	B9	0.005	4	+ 19.7	0.9	....	
Lal. 45883....	23 21.0	+52 26	6.9	Mb	.....	3	+ 4.2	0.8	....	
6031....	23 21.8	+ 0 42	4.9	A3	0.124	3	− 4.1	2.7	− 3.7	L
C 3074....	23 21.8	+44 47	7.4	G1	0.467	3	− 1.8	0.9	....	
6033....	23 22.1	+ 0 34	6.4	G7	0.055	3	− 12.8	0.5	− 4.4	V
B.D.+43°4462....	23 22.6	+43 19	8.0	K2	0.018	3	− 13.8	0.6	....	
6042....	23 24.4	− 5 5	6.4	K5	0.283	5	− 24.5	1.9	....	
Lal. 46051....	23 25.8	+43 25	8.1	K0	0.009	3	+ 8.8	1.4	....	
6048....	23 26.4	− 4 38	6.5	F8	0.247	4	− 11.6	1.9	....	
6049....	23 26.4	+38 41	5.3	G9	0.294	3	− 57.1	0.3	− 59.3	V
B.D.+73°1042....	23 29.3	+73 40	8.0	G8	0.013	3	− 23.3	0.7	....	
C 3093....	23 30.4	+30 27	6.7	G1	0.589	3	−103.4	±2.5	....	

TABLE I—*Continued*

Star	α 1900	δ 1900	m	Sp.	μ	No.	v	P.E.	Other Determinations v	Other Determinations Auth.
							km	km	km	
C 3096....	23h31m.0	+17°53′	8.0	G5	0″.702	5	− 24.7	±1.5	....	
6077....	23 34.8	+ 5 5	4.3	F4	0.574	3	+ 2.1	0.5	+ 5.6	L, B
6078....	23 35.2	+77 4	3.4	K1	0.169	3	− 40.4	0.9	− 41.6	5
B.D.+74°1033....	23 35.6	+75 12	7.2	F2	0.007	3	+ 0.1	2.4	....	
B.D.+74°1034....	23 36.9	+75 2	8.0	G9	0.028	3	− 4.0	1.8	....	
6097....	23 39.9	+55 15	6.6	G6	0.029	3	+ 8.2	0.3	....	
C 3117....	23 40.0	+29 0	8.9	K1	0.91	4	+ 52.0	1.8	....	
6109....	23 43.4	− 6 56	6.3	K1	0.026	3	− 24.7	0.4	....	
C 3124....	23 44.0	+ 1 52	8.7	Ma	1.393	4	− 63.1	2.5	....	
C 3126....	23 44.9	+ 2 19	8.4	G8	0.499	4	− 26.3	2.0	....	
6124....	23 47.2	+77 3	6.5	F2	0.278	3	+ 0.3	1.0	....	
6125....	23 47.3	+21 7	6.3	Ma	0.055	3	− 4.4	1.3	....	
6127....	23 47.4	+18 34	5.2	Ma	0.047	4	− 7.3	3.1	....	
B.D.+73°1066....	23 51.4	+73 35	8.2	K2	0.020	4	− 49.5	1.3	....	
6150....	23 52.7	+24 35	4.8	Ma	0.056	4	− 2.9	1.3	− 4.6	L
6151....	23 53.0	+49 53	6.8	G4	0.245	5	+ 2.7	2.2	....	
C 3143....	23 53.5	+46 10	9.5	Ma	0.64	3	+ 5.0	1.5	....	
Lal. 47019....	23 54.2	+26 43	8.3	G9	.....	3	− 30.4	0.6	....	
C 3146....	23 54.3	−20 35	7.4	G5	0.608	4	+ 21.8	1.1	....	
6158^1....	23 54.4	+33 10	6.6	G0	0.106	3	− 8.3	1.2	− 8.6	V
6158^2....	23 54.4	+33 10	6.6	F9	0.106	3	− 5.2	2.2	− 5.7	V, O
6172....	23 56.9	+26 33	5.8	G3	1.291	5	− 32.6	2.1	− 34	L, Y
C 3162....	23 59.7	+34 6	6.2	G1	0.766	3	+ 3.8	±0.8	....	

No. 18; H.D., the *Henry Draper Catalogue,* and β G.C., *Burnham's General Catalogue of Double Stars.* The magnitudes are Harvard visual magnitudes and the spectral types are from the Mount Wilson determinations. Successive columns in the table then give the total proper motion, the number of spectrograms, the mean radial velocity, and the probable error of the mean as derived from the formula referred to above. The abbreviations used to indicate the results obtained at other observatories conform to those adopted by Voûte in his *First Catalogue of Radial Velocities* and are as follows:

A Allegheny	m McMillan
B Bonn	O Ottawa
C Cape	P Potsdam
D Detroit	V Victoria
L Lick	Y Yerkes

Many observers have shared in the work of securing spectrograms of the long list of stars in the catalogue, and we are indebted to numerous members of the Computing Division for assistance in measuring and reducing the negatives. We wish to express our appreciation especially to Messrs. Strömberg, Monk, and Hoge, and to Miss Burwell, Miss Brayton, and Miss Shumway, of the Computing Division.

MOUNT WILSON OBSERVATORY
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ON THE VISUAL AND PHOTOGRAPHIC ALBEDO OF THE EARTH

By K. R. RAMANATHAN

ABSTRACT

The visual and photographic albedo of the earth.—This subject, previously treated by H. N. Russell and C. G. Abbot, is here discussed afresh in the light of Luckiesh's recent measurements of the visual brightness of earth-areas as seen from an aeroplane and Raman's theory of the color of the sea as originating from the molecular scattering of light in deep water. The effects of molecular scattering by the atmosphere and the sea are considered in detail with reference to the spectral distribution of the scattered light. The computed visual albedo comes out to be 0.46, and the photographic 0.58, in close agreement with the astronomical values 0.45 and 0.6 respectively.

INTRODUCTION

A valuable discussion of the photometric measures of the albedo of the earth was put forward by H. N. Russell,[1] who came to the conclusion that the visual albedo of the earth according to Bond's definition, as deduced from Very's observations of earth-shine on the moon, was 0.45 and the photographic albedo was 0.6. Further, Abbot[2] from radiation measurements and a consideration of the physical characteristics of the earth, has computed the total energy albedo to be 0.37. The difference between the visual and photographic albedoes is considerable and it has been suggested that this is due to the fact that a part of the light diffused out by the earth arises from the molecular scattering of light in the earth's atmosphere. Since the publication of the papers referred to, two investigations have appeared which make it desirable to consider the problem of the earth's albedo afresh. Luckiesh[3] has published observations made during aeroplane flights of the visual albedo of different kinds of landscape and also inland and oceanic waters. Then again, Abbot, in his discussion of the albedo of water-covered areas, considered only the reflection of light at the *surface* of the liquid. In reality, we have also to consider the important part arising from the diffusion of light entering the water. A theory of the color of the sea as being due to the molecular scatter-

[1] *Astrophysical Journal*, **43**, 173, 1916.

[2] *Annals of the Smithsonian Astrophysical Observatory*, **2**, 161–163.

[3] *Astrophysical Journal*, **49**, 108, 1919.

ing of light in deep ocean water has been put forward by C. V. Raman,[1] who has also pointed out the importance of the contribution which the scattered light from the sea would make to the albedo of the earth. In this paper, it is proposed to discuss the problem of the albedo, taking into account the spectral distribution of the molecular scattered light from the atmosphere and the sea.

We may divide the light going out of the earth into space as arising in the following ways:

1. Light molecularly scattered from the atmosphere above the region of dust;

2. Light scattered by the clouds;

3. Light scattered from the unclouded portions of the earth in the lower dusty regions of the atmosphere;

4. Light diffusely reflected from the earth's surface including the oceans.

Of these, the light molecularly scattered from the atmosphere and the sea would be richer in the shorter wave-lengths, and hence, on the aggregate, the returning light would be bluer than the incident.

LIGHT MOLECULARLY SCATTERED FROM THE ATMOSPHERE ABOVE THE REGION OF DUST

The discussions by F. E. Fowle[2] and L. V. King[3] of the extensive measurements of atmospheric transparency made by the Smithsonian Observatory have shown that in the region of wave-lengths shorter than 0.69 μ the loss of light on a clear day above the Mount Wilson level (1730 meters) could be accounted for almost entirely by the molecular scattering of light by the atmosphere.[4] The selective absorption exercised by the oxygen and water-vapor lies entirely in the region of wave-lengths longer than 0.70 μ and hence outside the visual and photographic regions. We shall take Mount Wilson level to be the upper limit of the region of dust. Abbot has given a table of intensities of energy in the normal solar spectrum outside the earth's atmosphere. From these and the values of the yearly mean dry-air transmission coefficients at Mount

[1] *Proceedings of the Royal Society*, A, **101**, 64, 1922. See also *Molecular Diffraction of Light*, published by the Calcutta University Press, 1922.

[2] *Astrophysical Journal*, 38, 393, 1913.

[3] *Philosophical Transactions of the Royal Society*, A, **212**, 375, 1913.

[4] Except for a possible small selective absorption between 0.51 μ and 0.65 μ.

Wilson,[1] we can determine graphically, from the areas of the energy wave-lengths curve, the percentage of light lost in transmission in different regions of the spectrum. Between 0.45 μ and 0.68 μ which we may take to be the effective visual region, I find the percentage of energy lost on normal transmission to be 10, and, in the region between 0.35 μ and 0.45 μ which we may take to be the effective photographic region, the percentage is 24. We have now to consider the actual loss that occurs, taking into account the facts that at a large portion of the earth's surface the light is incident obliquely and that, on the average, a portion of the atmosphere above Mount Wilson level is always clouded. At the place where the sun's rays meet the earth at an angle i, the transmission coefficient is $a^{\sec i}$ where a denotes the same quantity for normal incidence. The quantity of light falling on the surface of a sphere of radius r between the angles i and $i+di$ is

$$2\pi r^2 I \sin i \cos i \, di \, ,$$

where I is the intensity of the incident light. The amount of light transmitted in the same range is this quantity multiplied by $a^{\sec i}$, and this, taken over the whole sphere is

$$2\pi r^2 I \int_0^{\frac{\pi}{2}} \sin i \cos i \, a^{\sec i} \, di \, . \qquad (1)$$

The integral does not seem to be capable of being easily evaluated; instead, it was done by graphical integration and the total loss in the visual region was found to be 17 per cent and in the photographic region 38 per cent. Assuming that half this amount would be reflected back to space, we get 8.5 per cent to be the visual, and 19 per cent to be the photographic albedo of the atmosphere above Mount Wilson, if it were entirely unclouded. Actually, however, according to Abbot,[1] about 60 per cent of the earth's cloudiness is above Mount Wilson level and since, on the average, 52 per cent of the earth's surface is always clouded, 31 per cent of the skies above Mount Wilson will be always covered by clouds. The distribution of clouds with height as given in Humphreys' *Physics of the Air*[2] is as follows:

1. Cirrus level—height above surface about 10 km
2. Cirro-stratus level—height above surface about 8 km

[1] Fowle, *Smithsonian Miscellaneous Collections*, 69, No. 3. [2] P. 309.

3. Alto-cumulus level—height above surface about 4 km
4. Cumulus level—height above surface about 1.5 km
5. Fog-level—ground

Of these, the first two would be comparatively thin and would transmit a large portion of the light. We may take the average cloud-height above Mount Wilson to be $5\frac{1}{2}$ km, corresponding to a level at which the pressure is half the sea-level pressure. Thus, the albedo due to the molecular scattering of the atmosphere above the dust-level would be as is shown in Table I.

TABLE I

	Visual	Photographic
From cloudless portion..........	0.69×8.5	0.69×19
From clouded portion...........	$0.31 \times \frac{38}{63} \times 8.5$	$0.31 \times \frac{38}{63} \times 19$
Total.....................	7.5 per cent	17 per cent

We have assumed the fraction scattered from the clouded portion to be less than that from the unclouded, in the ratio of the pressure above the average higher cloud-level to the pressure at Mount Wilson. We have also neglected the scattering due to the moisture in the dustless region.

LIGHT REFLECTED FROM THE CLOUDS

According to Abbot's energy-measurements and Luckiesh's photometric observations, we shall take the average reflecting power of clouds to be 65 per cent. Since the average cloudiness of the earth is 52 per cent, we get, for the percentage of light reflected back from the clouds, the values given in Table II.

TABLE II

	Visual		Photographic	
	Percentage Incident	Percentage Reflected	Percentage Incident	Percentage Reflected
From higher clouds (average level 5.5 km)	$100 - \frac{38}{63} \times 8.5 = 95$	$0.31 \times 95 \times 0.65 = 19.1$	$100 - \frac{38}{63} \times 19 = 88.6$	$0.31 \times 88.6 \times 0.65 = 17.9$
From lower clouds (average level 1.7 km)	$100 - 8.5 = 91.5$	$0.21 \times 91.5 \times 0.65 = 12.5$	$100 - 19 = 81$	$0.21 \times 81 \times 0.65 = 11.1$
Total..........	....	31.6	....	29.0

Very little need be allowed from this for loss in transmission upward, for the diminution is due to scattering, and the scattered light would either go up or again get reflected from the clouds.

LIGHT SCATTERED FROM THE LOWER ATMOSPHERE

The light reflected from the lower atmosphere below the Mount Wilson level can be calculated as follows. From Abbot's data[1] for the mean coefficients of normal atmospheric transmission at Mount Wilson and Washington, the percentage of energy lost between the levels of the two stations is calculated for different wave-lengths, and an energy wave-length curve constructed with these data. From the area of the curve, it is found that in the region $0.45\ \mu$–$0.68\ \mu$, 14 per cent of the energy is lost, and practically the same percentage between $0.35\ \mu$ and $0.45\ \mu$. Assuming 20 per cent of this to be absorbed and half the remainder to be sent to earth and the other half back, we get 6 per cent to be the ratio of the light reflected back at normal incidence. Taking into account the fact that at a considerable portion of the earth the light is incident obliquely and that only 48 per cent of the earth is unclouded, the contribution of the lower atmosphere comes out to be nearly 4.5 per cent.

The above estimates agree very well with the careful visual photometric measurements of Kimball,[2] who found a mean transmission coefficient of 0.77 in clear weather, corresponding to the estimate here made of a loss of 10 per cent above Mount Wilson and 14 per cent below it.

LIGHT REFLECTED FROM THE EARTH'S SURFACE

Since more than three-fourths of the earth's surface is covered by seas, the most important contribution of the albedo from the surface of the earth would come from them. The deep blue color of the ocean waters has been explained by C. V. Raman[3] as originating from the molecular scattering of light within the water. The fraction of the incident energy scattered by unit volume of a homo-

[1] *Astrophysical Journal*, 34, 203, 1911.

[2] *Monthly Weather Review*, 43, 650, 1914; also H. N. Russell's article, *loc. cit.*

[3] *Proceedings of the Royal Society*, A, 101, 64, 1922.

geneous fluid like water is, for the most part, given by the Einstein-Smoluchowski expression

$$\frac{8\pi^3}{27}\frac{RT\beta}{N\lambda^4}(\mu^2-1)^2(\mu^2+2)^2\,, \qquad (2)$$

where β is the isothermal compressibility of the medium, μ its refractive index, N the Avogadro constant and λ the wave-length of the incident light. R and T have their usual meanings in kinetic theory. Since the amount of scattered light varies inversely as the fourth power of the wave-length, we can easily see why the blue end of the spectrum should predominate in the light returning from the water. Two other factors go to make the scattered light even a richer blue: one is the greater absorption in water of the red and yellow regions of the spectrum, and the other is an addition, to the light scattered in accordance with (2), of some unpolarized light which is greater in the violet end of the spectrum.[1] In his paper, Raman has given a table of the luminosity of deep ocean water for different wave-lengths in terms of the luminosity of a layer of dust-free air one kilometer thick when viewed transversely to the incident light. In calculating these values, he made use of the absorption coefficients of water obtained by Count Aufsess. Recently, W. H. Martin[2] has obtained fresh values for the absorption coefficients with pure, dust-free water, and Table III gives Raman's revised values for the luminosity of ocean water in terms of that of an atmosphere of dust-free air which would give an equal effect by lateral scattering.

TABLE III

Luminosity of Ocean Water

	Wave-Length in μ						
	0.658	0.602	0.590	0.578	0.546	0.499	0.436
Equivalent atmospheres of dust-free air....................	0.06	0.09	0.23	0.35	0.65	0.90	2.0

Confining ourselves to the region 0.45 to 0.68 μ, the fraction of incident light scattered comes out to be 0.06. Luckiesh's direct

[1] C. V. Raman, *Molecular Scattering of Light*, p. 55.

[2] *Journal of Physical Chemistry*, May, 1922.

determination of the "reflecting" power of the Atlantic, as measured from an aeroplane, gave an average value of 3.5 per cent. The difference is no doubt to be attributed to the fact that, in the above calculation, the absorption coefficient has been assumed to be equal to that of dust-free distilled water. Luckiesh makes it clear that 90 per cent of the light returning from the sea as viewed normally, is due to light diffused within it. The foregoing estimate refers to direct overhead observation. In an oblique direction, the molecular scattering would give rise to a greater luminosity, as a more extensive surface layer of water would come into operation, and, indeed Luckiesh has noticed that the brightness increases more than twice when the sea-surface is viewed at an angle of 45°.

Besides the molecular scattering, there is also the specular reflection to be taken into account, which would give rise to a reflection factor of about 3 per cent.

On the whole, we may put 8 per cent to be the fraction coming back from the ocean waters. As regards land areas, Luckiesh has obtained the following reflection factors for some typical surfaces: fields, 7.2 per cent; barren lands, 12.0 per cent; woods, 4.3 per cent. A snow-covered surface is known to reflect about 75 per cent of the incident light, but almost the whole of the snow-covered areas of the earth are near the poles, where only a small fraction of the sun's energy is received and where, moreover, a large part of the incident light would have been scattered by the atmosphere before reaching the earth. We may take, without much error, 12 per cent to be the average reflecting power of the land areas of the earth. Thus, the percentage of light scattered from the earth's surface in the visual region would come to be nearly $0.80(\frac{3}{4}\times 8+\frac{1}{4}\times 12)$, i.e., 7.2. Of this, allowing 30 per cent for loss on transmission through the atmosphere and remembering that only 48 per cent of the earth's surface is unclouded, we get 2.5 per cent to be the contribution of the surface of the earth to the visual albedo.

As regards the photographic albedo, the scattered light from the sea would contribute much more. An examination of Table III would show the enormous concentration of energy in the violet end of the spectrum. In the near ultra-violet, the light scattered from the sea is likely to be even stronger, since the absorption coefficient

of water gets smaller and smaller as we go to the region of shorter wave-lengths. But against this, it has to be said that the presence of motes would tend to increase the absorption. We have previously estimated the photographic albedo of the sky above Mount Wilson to be 0.19, and, from Table II, it is evident that we shall not be far wrong if we take the fraction of the incident energy scattered by the sea to be 0.30. With the usual allowance for the loss of energy of the incident and reflected rays on transmission through the atmosphere and the fact that a portion of the earth's surface is clouded, we get the photographic albedo due to the sea to be nearly 6 per cent. The land areas would contribute practically nothing to the photographic albedo except the snowy regions near the poles, whose effect may come to about 1 per cent. Thus, the aggregate photographic albedo due to the surface of the earth would come to nearly 7 per cent.

SUMMARY

TABLE IV

	Visual	Photographic
	Per Cent	Per Cent
Light scattered from the gases of the atmosphere above the dust level......	7.5	17
Light reflected from the clouds.........	31.6	29.0
Light scattered from the lower atmosphere............................	4.5	4.5
Light reflected from the surface of the earth including the oceans...........	2.5	7.0
Total............................	46	57.5

The visual and photographic albedoes obtained above agree very well with the astronomical value 0.45 and 0.6. Considering the uncertainties in the estimation of the earth's albedo factors and the difficulties of photometric measurement of the earth-shine on the moon, the agreement is actually better than could have been anticipated.

In conclusion, I have great pleasure in expressing my thanks to Professor C. V. Raman for his suggestion of the problem and interest in the progress of the work.

University College, Rangoon
August 5, 1922

SERIES OF MULTIPLE LINES WITH FOURFOLD RYDBERG CONSTANT IN THE SPECTRUM OF POTASSIUM

By KNUD AAGE NISSEN

ABSTRACT

Series of multiple lines with fourfold Rydberg constant in the spectrum of potassium.—Of the lines measured by McLennan, Schillinger, and Nelthorpe, sixty-four between λ 1873 and λ 4608 have been arranged in one diffuse subordinate series, $(2p)-(md)$, and in several sharp subordinate series, $(2p_x)-(ms')$, $(2p_y)-(ms'')$, $(2p_u)-(ms''')$, $(2p_z)-(ms')$, and $(2p_z)-(ms'')$. When reduced to international vacuum wave-numbers, the observed values, and those computed by Ritz formulae, agree within one unit on the average. These results are but the framework necessary to an exhaustive analysis of the "ground spectrum" of potassium. They support the Sommerfeld suggestion that there is a relationship between the spark spectrum of an alkali and the arc spectrum of the inert gas with atomic number one less, and should help in resolving the spectrum of argon into series of multiple lines.

For the work of resolving the spectra of argon and the heavier monatomic gases into series,[1] it would be helpful to know as much as possible of the series with fourfold Rydberg constant in the spectra of the alkali metals. When the admirable work of Professor J. C. McLennan[2] on the "ground spectrum" of potassium appeared, it was therein "proposed to make an analysis of the frequencies of the wave-lengths recorded in these tables and of the additional ones given by Schillinger in order to see if . . . . parallelism [with the spectrum of argon] can be established numerically." If, therefore, the series relations given below form part of the results attained by him the priority rests unreservedly with the observer who recorded the supplementary wave-lengths necessary to the series work in question, that is, with Professor J. C. McLennan.

In reducing the wave-lengths λ to vacuum wave-numbers ν of the international system, it has been taken for granted that McLennan's wave-lengths[2] (marked L) refer to the Rowland standards, as is certainly the case with those due to Schillinger[3] (marked S). The correction to international units[4] has, therefore, been applied

[1] See the author's note: "Series of Multiple Lines in the Spectrum of Argon," *Physikalische Zeitschrift*, **21**, 25–28, 1920.

[2] *Proceedings Royal Society London*, A, **100**, 182, 1921.

[3] *Wiener Berichte*, **118** II A, 266, 1909.

[4] See H. Kayser, *Handbuch der Spectroskopie*, **6**, 891, 1912.

to all the wave-lengths used (with exception of those due to Nelthorpe,[1] which are marked *N*). In reducing to vacuum values, the tables of the Bureau of Standards[2] have been used. The small correction from 20° C. (Rowland) to 15° C. (International) has no influence on wave-lengths of less than seven figures.[3]

In Table I is given a "sharp (or second) subordinate series" of multiple lines with fourfold Rydberg constant.

TABLE I

x	$2p_x$		$m=1.5$	$m=2.5$	$m=3.5$
1	75759.2	λ	*L*2049.8 II	*S*4305.17 III	....
		ν	48771.0	23222.22	....
		ms'	124530.2	52537.0	....
2	76418.8	λ	*L*2077.9 X	*S*4186.39 X	*L*2105.45 I
		ν	48111.4	23881.11	47482.03
		ms'	124530.2	52537.7	28936.8
3	77418.40	λ	*L*2122.0 IV	*S*4018.26 I	*L*2062.05 II
		ν	47111.8	24880.32	48481.32
		ms'	124530.2	52538.08	28937.08
4	78722.1	λ	*L*2182.4 I	*S*3818.14 II	....
		ν	45808.1	26184.39	....
		ms'	124530.2	52537.7	....
5	79733.36	λ	*L*2231.67 II	*S*3676.16 I	....
		ν	44796.85	27195.71	....
		ms'	124530.21	52537.65	....
Mean value of ms'	....		124530.2	52537.6	28937.0
Calculated, ms'_1	....		124530.21	52538.05	28937.08
Observed, ms'_1	....		124530.2	52538.08	28937.08
Obs. − calc.	....		0	+0.03	0

W. Ritz's formula[4] has been used for the series $(2p_3)-(ms')$. We find:

$$\nu=77418.40-ms' ; \qquad ms'=\frac{4\times 109737.2}{(m+s'+\sigma'(ms'))^2} ; \qquad s'=0.399985 ;$$

$$\sigma'=-1.809230\times 10^{-7} .$$

The two series $(2p_2)-(ms')$ and $(2p_3)-(ms')$ have been used as stepping-stones to another series relation. They form the three pairs:

48111.4	23881.11	47482.03
999.6	999.21	999.29
47111.8	24880.32	48481.32

[1] Edgar H. Nelthorpe, *Astrophysical Journal*, 41, 16, 1919.

[2] *Scientific Paper* No. 327 (1918) of the Bureau of Standards, Washington.

[3] The correction varies from −0.0018 A (for λ 2000) to +0.0004 A (for λ 10,000).

[4] W. Ritz, *Theorie der Serienspcktren*, Dissertation Göttingen, 1903; *Physikalische Zeitschrift.*, 4, 406–408, 1903; *Annalen der Physik*, 12, 264–310, 1903.

and the lines λ *S*2379.6 I and λ *L*2324.33 III form a pair:

$$\begin{array}{r} 42012.39 \\ 999.10 \\ 43011.49 \end{array}$$

belonging to the column $m=1.5$ of a series system $(2p_y)-(ms'')$ given below in Table II. The linking up of the two systems through parallel columns, $m=1.5$, is shown in Table III.

TABLE II

y	$2p_y$		$m=1.5$	$m=2.5$	$m=3.5$
a..........	72920.03	λ	*L*2149.42 III	*S*4608.49 IV	....
		ν	46510.76	21693.86	....
		ms''	119430.79	51226.17	....
b..........	74007.43	λ	*L*2200.89	*S*4388.30 I	....
		ν	45423.36	22782.37	....
		ms''	119430.79	51225.06	....
c..........	75584.41	λ	*L*2280.05 III	*N*4104.2 VII (FA)	....
		ν	43846.38	24358.46	....
		ms''	119430.79	51225.95	....
d..........	75774.9—	λ	*L*2290.0 (o)	*N*4072.3 II (FA)	....
		ν	43655.9—	24549.3	....
		ms''	119430.79	51225.6—	....
e..........	76250.10	λ	*L*2315.22 IV	*S*3995.23 IV	*L*2089.8— VI
		ν	43180.69	25023.71	47837.50
		ms''	119430.79	51226.39	28412.60
f..........	76419.30	λ	*L*2324.33 III	....	....
		ν	43011.49	....	....
		ms''	119430.79	....	....
g..........	77418.40	λ	*S*2379.6— I	*S*3816.92 I	....
		ν	42012.39	26192.75	....
		ms''	119430.79	51225.65	....
h..........	78473.4—	λ	*L*2440.9— II	*S*3669.06 I	....
		ν	40957.4—	27248.33	....
		ms''	119430.79	51225.1—	....
i..........	80051.0—	λ	*S*2538.7— I	*S*3468.45 I	....
		ν	39379.8—	28824.34	....
		ms''	119430.79	51226.7	....
j..........	81188.26	λ	*S*2614.2 I	*L*3336.7 III	....
		ν	38242.5—	29962.4—	....
		ms''	119430.79	51225.9	....
k..........	82272.64	λ	*S*2690.5— I	*L*3220.0— (o)	....
		ν	37158.2—	31048.2—	....
		ms''	119430.79	51224.4—	....
l..........	83973.82	λ	*L*2819.6 III	*L*3052.85 II	....
		ν	35456.97	32748.02	....
		ms''	119430.79	51225.80	....
Mean value of ms''	....		119430.79	51225.64	28412.60
Calculated, ms''..	....		119430.79	51225.65	28410.62
Observed, ms_e''...	....		119430.79	51226.39	28412.60
Obs.—calc.......	....		0	+0.74	+1.98

Walter Ritz's formula has been used for the series $(2p_e)-(ms'')$ with $2p_e=76250.10$ calculated from $2p_g=2p_3=77418.40$ (and $2p_f=2p_2=76419.-$). We find:

$$\nu=76250.10-(ms'')\ ;\qquad ms''=\frac{4\times 109737.2}{(m+s''+\sigma''(ms''))^2}\ ;\qquad s''=0.434900\ ;$$

$$\sigma''=-1.488891\times 10^{-7}\,.$$

In connection with Table V below a combination with $2p_e$ is given.

TABLE III

z	$2p_z$		$m=1.5$	
A	75230.53	λ ν ms''	*S*2261.8 —I 44200.26 119430.79	*L*2027.8 —I 49299.9 124530.4
B	76419.30	λ ν ms''	*L*2324.33 III 43011.49 119430.79	*L*2077.9—X 48111.4— 124530.7
C	77418.40	λ ν ms''	*S*2379.6 —I 42012.39 119430.79	*L*2122.0—IV 47111.8 124530.2
D	81188.3—	λ ν ms''	*S*2614.2 —I 38242.5 119430.79	*L*2306.58 III 43342.39 124530.7
E	82272.6—	λ ν ms''	*S*2690.5—I 37158.2— 119430.79	*L*2365.8—II 42257.6 124530.2
F	88080.76	λ ν ms''	*L*3189.0 —V 31350.03 119430.79	*L*2742.8—II 36449.7— 124530.4
G	90372.34	λ ν ms''	*S*3440.50 III 29058.45 119430.79	*L*2926.8—I 34158.4— 124530.7—
Mean value of ms''.			119430.79	124530.5

As will be seen, tables I, II, and III have two terms $2p$ in common; e.g., $2p_2=2p_f=2p_B=76419$ and $2p_3=2p_g=2p_C=77418.40$, while tables II and III in addition to this have two other terms $2p$ in common, namely, $2p_j=2p_D=81188.3$ and $2p_k=2p_E=82272.6$. The term $2p_e=76250.10$ is, in the first place, secured by the series relation stated above, in the second place by the combination given below in connection with Table V. The terms, $2p_1$, $2p_4$,

$2p_5$, $2p_a$, $2p_b$, $2p_c$, $2p_d$, $2p_h$, $2p_i$, $2p_l$, have each an alternative value to the one assumed here. To determine the real values requires additional evidence. As a natural point of attack the terms $2p_1$ and $2p_d$ with difference 15.7 have been chosen. They enter as $2p_\beta$ and $2p_\gamma$ into a system of sharp (or second) subordinate series in Table IV.

TABLE IV

u	$2p_u$		$m=1.5$	$m=2.5$	$m=3.5$
a	72074.5	λ	....	S5005.52 II	L2309.0 (0)
		ν	....	19973.12	43296.98
		ms'''	....	52101.4−	28777.5
β	75759.7	λ	L2143.9 VII	S4225.76 IV	....
		ν	46630.48	23658.62	....
		ms'''	122390.2	52101.1	....
γ	75774.9−	λ	S2144.6 I	S4223.15 V	L2127.25 IV
		ν	46615.27	23673.23	46995.57
		ms'''	122390.2	52101.67	28779.33
Mean value, ms'''	....		122390.2	52101.4	28778.4
Calculated, ms'''_γ	....		122390.2	52103.19	28779.33
Observed, ms'''_γ	....		122390.2	52101.67	28779.33
Obs.−calc.	....		0	−1.52	0

In using W. Ritz's formula for the series $(2p_\gamma)-(ms''')$ we find:

$$\nu=75774.9-ms''' ; \quad ms'''=\frac{4\times 109737.2}{(m+s'''+\sigma'''(ms'''))^2} ; \quad s'''=0.408980 ;$$
$$\sigma'''=-1.240454\times 10^{-7} .$$

The terms $2p_1=2p_\beta=75759.7$ and $2p_d=2p_\gamma=75774.9$ are accordingly to be accepted as tested values.

A system of diffuse (or first) subordinate series has been found, and its principal features are indicated in Table V.

As before, W. Ritz's formula was used. For the series $2p_{11}-md$ we find:

$$\nu=62429.08-md ; \quad md=\frac{4\times 109737.2}{(m+d+\delta(md))^2} ; \quad d=0.001614 ;$$
$$\delta=-4.637279\times 10^{-6}.$$

For argon, the author[1] has shown that there is a system of diffuse (or first) subordinate series with $\delta=-1.360133\times 10^{-6}$. The series system in Table V is supported by the combination

[1] *Loc. cit.*

$2p_e - 4d = 76250.10 - 29377.34 = 46872.76$. The line λ *L*2132.8− III has $\nu_{\text{vac. int.}} = 46873.10$.

The series relations shown in tables I–V above are but the framework necessary to an exhaustive analysis of the "ground

TABLE V

v	$2p_v$		$m=3$	$m=4$	$m=5$	$m=6$	$m=7$
I	51635.07	λ	...	...	*L*2985.9 II	*L*2549.6 IV	*L*2348.3 IV
		ν	...	...	33482.33	39211.53	42572.40
		md	...	...	18152.74	12423.54	9062.67
II	62429.08	λ	...	*L*3024.8 III	*S*2257.9	*L*1999.2 II	*L*1873.3 − II
		ν	...	33051.74	44276.59	50005.25	53365.21
		md	...	29377.34	18152.49	12423.83	9063.87
III	64663.53	λ	...	*S*2833.25 I	*L*2149.42 III	...	...
		ν	...	35286.19	46510.76	...	...
		md	...	29377.34	18152.77	...	...
IV	67021.00	λ	...	*L*2655.8 I	*L*2045.7 −	...	...
		ν	...	37643.66	48868.69	...	...
		md	...	29377.34	18152.31	...	...
V	70998.05	λ	...	*L*2402.0 − III	*L*1891.7	...	...
		ν	...	41620.71	52846.30	...	...
		md	...	29377.34	18151.75	...	...
Mean values, *md*	...		...	29377.34	18152.41	12423.68	9063.27
Calculated, md_{II}	...		58977.99	29378.43	18152.50	12423.84	9062.47
Observed, md_{II}	...		?	29377.34	18152.49	12423.83	9063.87
Obs. − calc.	...		?	−1.09	−0.01	−0.01	+1.40

spectrum" of potassium. As they stand, they support the view that there is a relationship between this "ground spectrum" and the spectra of the inert gases with their characteristic "series of multiple lines."

COPENHAGEN
1922

Minor Contributions and Notes

ABSORPTION OF POTASSIUM VAPOR AT HIGH TEMPERATURES, AND ITS BEARING ON THE SELECTION PRINCIPLE OF QUANTUM THEORY

ABSTRACT

In the course of an investigation on the optical properties of non-luminous potassium vapor, it was found that at high temperatures, about 950° C., potassium vapor absorbs λλ 4641, 4642, the first members of the combination series (1s–3d).

In the course of their investigations on the optical properties of non-luminous potassium vapor, the authors found that at about 950° C., the absorption spectrum of potassium vapor consists of a fine dark line at about 4640 A, which phenomenon was embodied in a short note sent to the *Philosophical Magazine*, May 3, 1922, and which will be shortly published.

Subsequent experiments conducted by the authors in this laboratory confirmed that this line was 4641, and 4642, the first members of the combination series (1s–3d). This point, along with the others observed at higher temperatures, was mentioned in a note on the absorption of potassium vapor in the associated series submitted to *Nature* on July 20.

From all these experiments it is beyond doubt that 4641 and 4642 are absorbed at higher temperatures (Fig. 1).

Fig. 1

According to the Bohr-Sommerfield theory of spectral emission, the spectral lines are emitted as the electron changes its habitat from one orbit to another of less energy, the rotational quantum number corresponding to the initial and final orbits changing only by 1, 0, or −1, according to Rubinowicz. In the case of the combination series (1s–3d), the quantum number

changes by two units. Thus the presence of this line in the foregoing spectrum of non-luminous potassium vapor strongly supports the views advanced by Messrs. Foote, Mohler, and Meggers in a short note on the significant exception to the principle of selection, *Philosophical Magazine*, April, 1922.

This absorption experiment, together with the experiments of the foregoing authors, seems to prove conclusively that the method whereby the rotational quantum numbers have been assigned, requires reconsideration.

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DEPARTMENT OF PHYSICS
August 28, 1922

NOTE.—While preparing this short note, *Nature*, July 8, which just reached us, contained a note by S. Datta, recording the same phenomenon. But these experiments and the experiments contained in the original note to the *Phil. Mag.*, May 3, were conducted in this laboratory before Datta's note appeared in *Nature*. We have thought it desirable to send it off for publication.

ERRATA

Vol. 55, June, 1922, "An Estimate of the Distance of the Andromeda Nebula," by E. Oepik:

Page 406, Abstract, line 9, *for* the mass within 150″ of the center *read* the total mass of the nebula.

Page 409, footnote 3, *for* 133 *read* 130.

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THE DURATION OF THE PROCESS OF LIGHT EMISSION IN HYDROGEN

By A. J. DEMPSTER

ABSTRACT

Duration of light emission from hydrogen canal rays.—Observations: Canal rays were allowed to pass through a small hole in the cathode into a second chamber at a very much lower pressure. With certain discharge conditions the rays ceased to be luminous in $1\frac{1}{2}$ to 4 cm; with others a strong continuous bundle masked the dying out phenomenon. By measuring the dependence of the intensity of the bundle on the pressure of the gas in the high vacuum chamber and simultaneously observing the Doppler effect, it is found that the continuous bundle is not due to residual luminosity, but to a second type of long-lived sources carried in from the discharge tube, whose radiation process lasts longer than 5×10^{-7} seconds. A strong electric field is found to have no effect either on the long-lived or short-lived rays. The dependence of intensity upon pressure shows that no rays are present that die out completely in a time much shorter than 10^{-7} seconds. *Interpretation.*—The explanation is proposed of the short-lived particles being neutral hydrogen atoms, and the continuous bundle being due to the dissociation of neutral molecules. *Theories of light emission.*—The classical radiation theory and Bohr's theory are discussed in connection with the above observations, and the observations of the Stark-effect in a variable electric field.

It has been observed by W. Wien[1] and by the author[2] that when a rapidly moving beam of hydrogen canal rays is allowed to pass into a high vacuum, the rays do not cease to give light at once but give a luminous bundle that dies out gradually in 2 to 4 cm. The natural interpretation of the phenomenon is that we observe the gradual decrease of the radiation from the moving sources in the rays and

[1] *Annalen der Physik*, **60**, 597, 1919; *ibid.*, **66**, 230, 1921.

[2] *Physical Review*, **15**, 138, 1920: Abstract of paper read at meeting of the Physical Society, November, 1919.

that after a certain time they are all extinguished. Professor Wien has measured the rate of decrease and finds it in approximate agreement with the value predicted by the classical electron theory for a single radiating atom. Experiments have also been made by W. Wien,[1] G. S. Fulcher,[2] J. Koenigsberg, and J. Kutschewski,[3] H. v. Dechend and W. Hammer[4] on the variation of the intensity of a canal ray bundle with gas pressure. With certain discharge conditions the intensity may be made very low by reducing the pressure of the gas through which the rays move.

The experiments reported in this paper extend the earlier observations and show that none of the rays die out at a rate faster than that observed by Wien and the author, and further that under certain discharge conditions a luminous bundle is formed that dies out at a much slower rate than that previously observed.

APPARATUS

The cathode and essential features of the discharge tube used in the first experiments are shown in Figure 1. The cathode, which had an aluminum face fastened to an iron back, rested on the ground end of a short glass tube sealed as shown, so that the only communication between the upper and lower chambers was a small hole in the center of the cathode 0.25 mm in diameter and 0.25 mm long. The upper chamber was connected to a mercury vapor pump and hydrogen was allowed to stream into the discharge tube. In this way the pressure in the lower chamber could be kept high enough for a discharge to pass while that in the upper was very much less. The canal rays were found to be luminous on first entering the upper chamber and to die out in a distance of 4 to 5 cm. Under the best conditions the upper part of the high vacuum tube was perfectly black and free from any general luminosity. Under certain con-

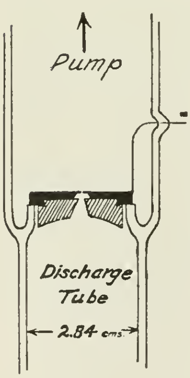

FIG 1

[1] *Annalen der Physik*, 30, 349, 1909. [2] *Astrophysical Journal*, 33, 32, 1911.

[3] *Sitzungsberichte der Heidelberger Akademie*, 1, 4, 7, 1910.

[4] *Ibid.*, 1, 21, 9, 1910.

ditions in addition to the decreasing luminosity, a bundle of uniform intensity could be observed continuing on to the end of the tube 50 cm beyond. This was at first attributed to the presence of sufficient gas in the upper chamber to re-excite a few of the canal rays and to be excited by them. Observation through colored screens and later with a direct vision prism showed that most of the visible light came from Hβ; Hα and Hγ could also be detected and appeared to die out at the same rate as Hβ.

Measurements of a photograph of the luminous bundle made with an ordinary camera and measured with a Hartmann microphotometer are given in Figure 2. The beam was thrown slightly out of focus so as to avoid any apparent decrease of intensity due to the slight divergence of the bundle. In the upper curve the scale readings of the instrument are given corresponding to different distances from the hole. The lower curve gives the background reading, measured to one side of the bundle. It is seen that after 4 cm the intensity has fallen practically to a final steady value. Thus we may conclude that the fastest particles have died out in this distance. The speed of the rays may be obtained from Doppler effect measurements just before the cathode as approximately 4×10^7 cm per second, giving $4\div4\times10^7=10^{-7}$ seconds as an upper limit for the time in which this radiation process is completed.

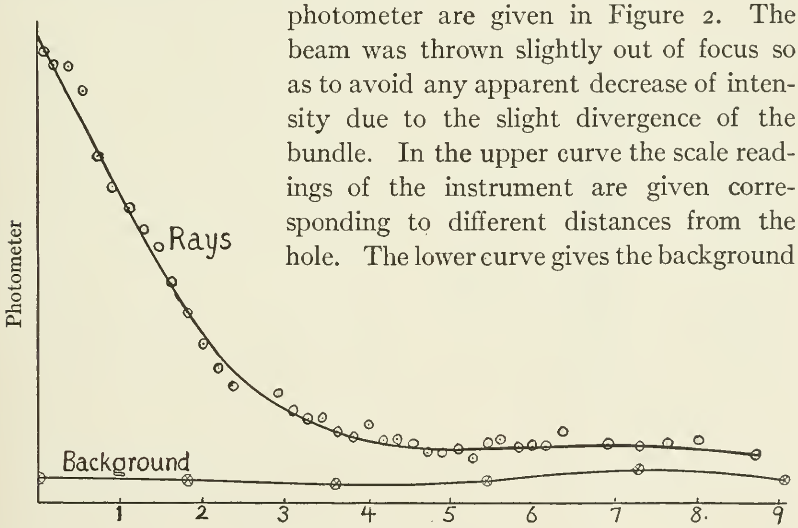

FIG. 2

In the abstract published in the *Physical Review*, 15, 138, 1920, the velocity of the rays was taken as 8×10^7 cm per second from

observations of Wilsar and Stark and W. Steubing[1] on the Doppler effect in a beam behind the cathode. The Doppler effect just before the cathode is considerably smaller than behind, and from the fact that the rays enter the high vacuum almost immediately, it seems that the Doppler effect just before the cathode, viz., 4×10^7 cm per second, should be used.

INTENSITY CHANGES WITH PRESSURE

Further experiments were carried out to find the origin of the continuous bundle and to see if any rays were present with extremely short life. This was done by observing the changes produced in the intensity of the bundle by alterations of the pressure in the high vacuum chamber. It is known from the experiments of Fulcher[2] and Wilsar[3] that the sources of the undisplaced line in the Doppler effect are the gas molecules through which the flying particles move, while the sources of the displaced line are the flying particles that have come from the discharge tube, and are kept excited by the gas through which they move. The effect of reducing the pressure in the high vacuum chamber to zero is thus to suppress the undisplaced light from the gas, and to leave the flying particles free to die out. The luminosity from the moving sources is carried on into the high vacuum chamber and *the initial intensity of the bundle should be reduced only in the ratio of the intensity of the displaced component in the Doppler effect to the total intensity in both the undisplaced and displaced lines.* If the canal ray bundle contained many particles that die out in less than a millimeter, the intensity 1 mm above the hole should be reduced by lowering the pressure in the upper chamber to a much lower value than that given by the above ratio. If the continuous bundle several centimeters above the hole is reduced to a very low value by reducing the pressure, it may be ascribed to residual luminosity, but if it is reduced only in the ratio given above, then it must be due to long-lived luminous sources from the discharge tube.

[1] *Annalen der Physik*, 28, 994, 1909.

[2] *Astrophysical Journal*, 35, 101, 1912. [3] *Annalen der Physik*, 39, 1295, 1912.

EXPERIMENTAL ARRANGEMENT

The first experiments on the alteration of the intensity in the upper chamber with pressure gave very uncertain results, probably due to the fact that the intensity of the Doppler effect, or proportion of moving sources in the rays, varies greatly with the purity of the gas and also with the discharge potentials.

The cathode in Figure 3 was therefore designed in order to observe the Doppler effect at the same time as the changes in intensity were measured. The iron cylinder was 4.4 cm long, had an aluminum face with a hole $1\frac{1}{2} \times 6$ mm, and rested on the polished end of a short glass tube. A window *W* was ground into one side and the Doppler effect in the rays could be observed through it and the prism *P*. A spectroscope with two large Rutherford prisms was used and the displacement at Hβ could be easily followed. Wilsar has shown that the Doppler effect does not alter to any extent with distance behind the cathode, so that the displacements observed may be taken as indicating the velocity of the rays after they have passed into the upper chamber. The small hole in the plate at the top was $\frac{1}{4}$ mm in diameter and 2 mm in length. Other openings were tried, a slit in a thin plate and a hole in a disk, but these gave qualitatively the same results as the hole in the thick plate and did not allow of such great pressure alterations. Two millimeters above the hole a short condenser was arranged to see if the luminous rays were charged or uncharged and to observe whether there was any effect of an electric field in altering the rate of dying out. The pressures were measured on a McLeod gauge, and it was found that a ratio of more than 100 to 1

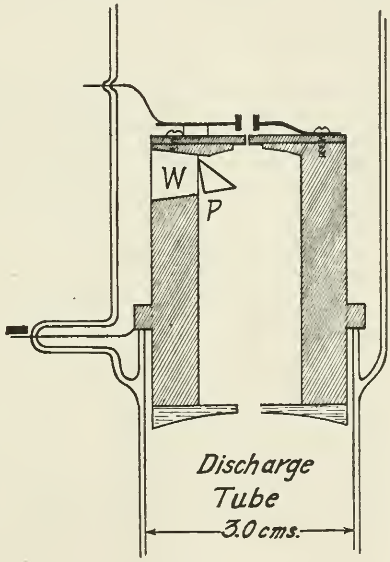

FIG. 3

could be maintained on the two sides of the cathode. The brightness of the bundle at various points was measured by matching its intensity with that of the real image of a small part of a piece of milk glass that was formed inside the tube directly beside the bundle. The image and bundle were viewed through a lens and the color was made as close as possible by colored screens. The intensity of the image was altered by varying the current through a lamp that illuminated the milk glass, and the current readings were later reduced to intensities by means of photometer comparisons.

The discharge tube was run sometimes from an induction coil with a mercury interrupter, but usually from a high voltage alternating current, rectified by a kenotron as used in X-ray equipments. In this way currents up to 10 milliamperes could be sent through the tube at most pressures.

DEPENDENCE OF DYING OUT ON DISCHARGE CONDITIONS

With the cathode in Figure 3 it was found that the dying out was prominent only with high pressures and low voltages in the discharge tube, even when the pressure in the upper chamber was less than 1 per cent of that in the lower. The dying out was observed with pressures in the discharge tube between 0.44 mm and 0.16 mm, voltages 2500–3500; currents up to 9 milliamperes. With the low pressures larger currents were necessary in order to make the dying out effect distinct, as otherwise, with small currents, a continuous bundle was superimposed. Using large currents of 5–10 milliamperes, the Doppler effect appeared suddenly as the pressure was reduced, accompanying an alteration in the discharge conditions before the cathode, and the sudden appearance of the short bundle in the upper chamber, which was previously completely black, was simultaneous with this alteration. The bundle seemed to lengthen from $1\frac{1}{2}$ to about 4 cm as the Doppler displacement increased, with reducing pressure. When the pressure in the discharge tube was reduced to about 0.13 mm, depending on the currents used, a faint continuation of the bundle was observed; and at a slightly lower pressure of 0.12 mm, with a voltage of 5000 and a current of 9.5 milliamperes, the continuation became so

intense that the dying out in the first few centimeters was almost completely masked by the stronger uniform bundle.

As the pressure was reduced still further, for instance to 0.072 mm (voltage 15,000, current 2 milliamperes), the continuous bundle remained very strong in comparison with the faint bundle previously observed and no decrease in intensity in the first few centimeters could be observed. The Doppler effect at this pressure was, however, still very distinct, the displaced line being approximately equal in total intensity to the rest line.

THE EXISTENCE OF LONG-LIVED RADIATING SOURCES

The continuous bundle was at first attributed to the residual gas in the upper chamber which would be excited by the beam of rays passing through it. The decrease in the distinctness of the dying out phenomenon could be associated on this hypothesis with the fact that at high voltages the displaced intensity in the Doppler effect may become very small compared with the intensity in the rest line[1] (Fig. 4) so that we might expect very few luminous particles to enter the upper chamber.

The simultaneous observation of the Doppler effect made this hypothesis untenable and showed that the strong continuous bundle appears before the Doppler effect has become faint. This suggested that the luminosity might be due to moving sources from the discharge tube that have a very much slower rate of dying out than those appearing at higher pressures.

In the earlier experiments with the short cathode the continuous bundle was not so prominent even at low pressures as in the cathode in Figure 3. This is possibly connected with the fact that with the short cathode in Figure 1 the rays make very few collisions before entering the upper chamber, and suggests that the long-lived radiators may be formed as a result of collisions. The great increase in the Doppler effect just behind the cathode over that just in front[2] shows that a few collisions of the moving particles

[1] F. Paschen, *Annalen der Physik*, 23, 260, 1907; B. Strasser, *ibid.*, 31, 916, 1910.

[2] B. Strasser and M. Wien, *Physikalische Zeitschrift*, 7, 746, 1906; W. Wilsar, *Annalen der Physik*, 39, 1291 and 1295, 1912; F. Paschen, *Physikalische Zeitschrift*, 7, 924, 1906; J. Stark, *ibid.*, 7, 747, 1906.

cause a great increase in their luminosity. That the continuous bundle was largely due to moving sources from the discharge tube was proved to be the case by measuring the dependence of the intensity on the pressure in the second chamber while the conditions in the discharge tube were kept unaltered. If the continuous bundle is an excited luminosity in the residual gas, its intensity should be much reduced if the gas pressure in the second chamber is reduced to a low value, while if its intensity is not reduced appreciably the luminosity must be carried in by sources from the discharge tube.

Observations of the intensity of the bundle in the upper chamber about 3 cm above the hole are given in Table I. The first column

TABLE I

Pressure in Discharge Tube	p_1 First Pressure in Upper Tube	p_2 Second Pressure in Upper Tube	Pressure Decrease	Intensity Decrease	Volts on Discharge Tube	Current in Milliamperes	Length of Dark Space	Doppler Effect	Dying Out in First 4 cm
							cm		
.143	.0263	.00126	20.8	>10.0	3000	5.5	1.6	B	Plain
.127	.0145	.0015	9.6	4.0	5000	9.5	2.3	B	Doubtful
.094	.0233	.0015	15.4	1.43	7000	8	2.7	C	Absent
.075	.0158	.00075	21.1	2.1	10,500	5.5	4.0	C	Absent
.0725	.0145	.00067	21.6	3.25	15,200	1.7	4.2	D	Absent
.057	.0123	.0005	24.4	3.4	20,000	......	......	D	Absent
.0747	.0206	.00067	30.8	2.3	11,500	6.2	3.7	C	Absent
.16	.021	.00126	16.7	1.43	3250	8.5	......	B	Plain

gives the pressure in the discharge tube in millimeters of mercury, the second and third the two pressures in the upper tube at which the intensity of the bundle was measured. The pressure change could be made by removing the heater from the mercury vapor pump till the pressure in the upper chamber had increased to the value given by p_1, when a general luminosity began to appear in the gas around the bundle. The heater was then replaced and the intensity observed. It was found that the pump began to act again suddenly, and reduced the pressure in the upper chamber from p_1 to p_2 in a few seconds without altering the pressure in the discharge tube. The fifth column gives the intensity change produced

by the pressure change in the fourth. The eighth column gives the distance in centimeters from the cathode to the end of the Crookes' dark space. The letters under Doppler effect refer to the sketches in Figure 4, which indicate the way in which the appearance of $H\beta$ alters with increasing potentials on the tube.

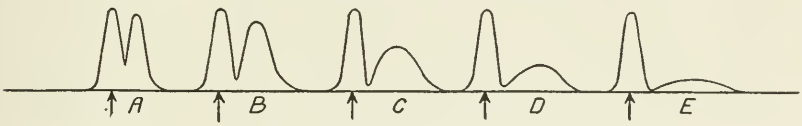

Fig. 4

The first experiment in the table refers to a condition in which the continuous bundle is practically absent, only the faint decreasing bundle appearing. With p_2 the intensity 3 mm above the hole has died down to a very small value, with p_1 there is a continuation of the luminosity. These conditions are similar to those in Fulcher's[1] experiments in which the intensity of the beam at a considerable distance in the high vacuum chamber was found to be proportional to the pressure. In the second experiment, the dying out is almost masked by the continuous bundle; in the third, fourth, and fifth, there is no indication of an initial dying out and the continuous bundle is very strong compared with that in the second. The slight alteration in intensity caused by a pressure change of more than 20 times proves, as discussed above, that the luminosity cannot be excited in the residual gas but must be carried in by luminous sources. In the sixth experiment the Doppler effect has become relatively weaker, and there is in consequence a greater intensity change.

Since no change in the continuous bundle could be observed in 30 cm, we must conclude that under certain conditions particles of a second type are present in the canal rays with a luminous life greater than $30/6\times10^7=5\times10^{-7}$ seconds.

The last two rows in the table refer to measurements of the intensity of the bundle one millimeter above the hole. Here there is also a small intensity decrease produced by a large pressure

[1] *Astrophysical Journal*, **33**, 32, 1911.

decrease, and we must therefore conclude that no considerable proportion of the rays die out in less than one millimeter.[1]

NATURE OF THE PARTICLES WITH LONG AND SHORT RADIATION TIMES

In order to test whether the luminous sources were charged or uncharged a condenser was arranged, as shown in Figure 3, two millimeters above the hole. No effect of an electric field could be observed with either the strong continuous bundle or the bundle that disappeared after 4 cm, even when one plate of the condenser was connected to the anode. We must conclude then that in hydrogen both kinds of sources are in the neutral state. In a recent paper W. Wien[2] has also concluded that the luminosity in the disappearing bundle comes from neutral particles. We may also conclude that a strong electric field, such as used by Stark[3] in observing the electrical separation of spectrum lines, does not cause the extinction of luminosity previously excited in the sources. The observation of the Stark effect in the condenser is planned as a further investigation. On Bohr's theory the initial disturbed orbit would be produced in the lower chamber and correspond to zero field, while the final orbit is reached in a strong field. The observations may be expected to throw light on the various theories of the atomic processes discussed below.

No definite conclusion can be drawn from these experiments alone as to the nature of the two kinds of radiating particles. We might think of neutral atoms and neutral molecules, for both occur in the canal rays,[4] but since the radiation is always almost entirely

[1] In a recent article in the *Annalen der Physik* for January, 1923, Professor Wien develops a very complete theory of the dying-out phenomenon in connection with the changes of the charge on the flying sources. His experiments show that there is very little further change in the dying-out phenomenon when the pressure in the high-vacuum chamber is reduced below 0.005 mm. The pressures p_2 in Table I are below the values at which there can be any effect from the residual gas as calculated by Professor Wien from the various free paths occurring in the rays. In his notation we may say that the experiments in this paper show that there are two distinct values of the constant l, which indicates the distance covered by the rays before the radiation is emitted. In Professor Wien's observations of the dying-out phenomenon the potentials on the discharge tube are given as less than 5,000 volts, so that under his discharge conditions the long-lived sources were probably unimportant.

[2] *Annalen der Physik*, **69**, 325, 1922. [3] *Ibid.*, **43**, 968, 1914.

[4] J. J. Thomson, *Rays of Positive Electricity*, p. 67.

the hydrogen series lines we should endeavor to ascribe the light in both cases to atoms. We may provisionally adopt the explanation that the comparatively faint short bundle observed at high pressures and low voltages is due to atoms formed in the discharge tube that have become neutralized before entering the upper chamber and emit light in the neutral state. The time for the duration of the radiation previously observed by Wien and the author then applies to these neutral atoms. The sources that give rise to the strong continuous bundle may be supposed to be atoms formed by the relatively slow dissociation of neutral molecules that have come from the discharge tube. Experiments by G. P. Thomson[1] show that charged hydrogen atoms are more numerous at high pressures and charged molecules at low, and we may assume that the same applies to the neutral constituents of the rays. On this view the neutral hydrogen molecule either does not dissociate, or is not formed in the discharge tube with potentials below 3000 volts. With higher potentials, its collisions while still in the lower chamber cause some instability, resulting in dissociation in the upper chamber with a consequent emission of the hydrogen series lines. The lower limit for the time required for this radiation process, 5×10^{-7} secs., as deduced above, applies on this view to the rate of dissociation of instable hydrogen molecules.

INTERPRETATION IN TERMS OF ATOMIC PROCESSES

The observed duration of the radiation from a moving beam of canal rays may be interpreted in terms of the atomic process in several ways. In Wien's earlier papers a mean value for the velocity is used, and the exponential intensity decrease observed is supposed to be the sum of exactly similar intensity decreases for the radiation from individual flying atoms. This interpretation is in accord with that required by the classical theory of continuous radiation from an electronic oscillator and the observed value for the damping coefficient also agrees well with the numerical value predicted on this theory.[2] Objections to this apparent con-

[1] *Philosophical Magazine*, 40, 242, 1920.

[2] H. A. Lorentz, *Theory of Electrons*, p. 260.

firmation of the simple radiation picture were raised by Seeliger,[1] who pointed out that the particles are excited at various distances in front of the hole. I have also found that if we assume a distribution of velocities, similar to that shown by Doppler effect curves, instead of a single velocity as assumed in Wien's calculations, the observed intensity decrease may even be obtained from particles that emit at a uniform rate and cease suddenly. Such a type of radiation process was discussed in a paper by Epstein.[2] Recently Mie[3] has developed a theoretical curve for the decrease, from consideration of the phenomena to be expected with Bohr's atom model.

A second interpretation may be called, on Bohr's picture of the radiation process, the theory of delayed jumps—the displaced electron may remain for some time in an outer non-radiating orbit and the duration of the actual light emission may be comparatively brief. Thus the pencil of decreasing luminosity observed in the second chamber may be made up of individual flashes or short streaks of light emitted when the electron does return to the inner orbit. The curve in Figure 2 is thus to be regarded as a decay curve for the semi-stable excited atoms formed in the discharge tube, analogous to the exponential decay curves of radioactive substances. This "quantum" interpretation has been discussed by O. Stern and M. Vollmer[4] in relation to a paper by Einstein.[5]

A third interpretation is the dissociation theory suggested above to explain the continuous bundle. Here we think of the emission process as following a dissociation process.

STARK EFFECT OBSERVATIONS AND DURATION OF THE EMISSION

A discussion of Stark's experiments on the effect of an electric field on spectrum lines, and their bearing on the duration of the emission process has recently been published by Foersterling.[6]

[1] *Jahrbuch der Radioaktivität*, **16**, 420, 1920.

[2] *Münchener Berichte*, p. 73, 1919.

[3] *Annalen der Physik*, **66**, 237, 1921.

[4] *Physikalische Zeitschrift*, **20**, 183, 1919.

[5] *Ibid.*, **18**, 123, 1917.

[6] *Zeitschrift für Physik*, **10**, 357, 1922.

In Stark's[1] discharge tube the flying luminous particles move through holes in the cathode into a strong electric field. If the radiation from an atom lasts 10^{-7} seconds as deduced above, the sources will move 1–4 cm during the process, and should carry their light into the electric field.

Stark states that under the conditions in his discharge tube most of the light must have come from the moving particles and adopts precautions to avoid the Doppler effect from their motion. In Stark's experiments the dark space was 5 to 10 centimeters in length and it is probable that the gas pressure was less than 0.05 mm of mercury, giving a mean free path greater than 1.5 cm. This according to Wien's experiments is also approximately the free path for changes of sign in the canal rays, so that the radiating particles should move undisturbed for about this distance. We have shown above that their luminosity is not affected in intensity by a strong electric field. His observations, however, show no unaffected lines in the field with $H\alpha$, $H\gamma$, and $H\delta$ (p-components). Foersterling concludes that the radiation from an atom must be capable of being altered during the process of emission if the atom moves into an electric field. He proposes the idea of some continuous connection between the atom and the energy to be radiated that would allow of such continuous alterations in frequency as are observed, and would abandon Sommerfeld's idea of the energy radiation process in itself being distinct from the releasing of the energy in the atom.

If, however, we are to keep the fundamental idea of Bohr's theory that the frequency radiated is determined by the energy difference between two orbits, the following considerations lead us to suppose that the elementary radiation process is completed in a very brief time. In his 1915 paper Stark observes that the field between the cathode and the secondary electrode has a very strong gradient over about 0.2 cm near the cathode. The lines are split up in this varying field so that the displacements follow sharply the variations in the field strength. These observations are predicted with remarkable detail by the theory of Epstein on the basis of a calculation of the modifications introduced by an electrical

[1] *Annalen der Physik*, **43**, 968, 1914; **48**, 194, 1915.

field in the Bohr non-radiating orbits. The significant point is that the final as well as the initial modified orbit determines the frequency of the light emitted. If in the varying part of the field a frequency characteristic of one field intensity is emitted, the electron must have passed from the initial to the final orbit while the flying atom has covered less than 0.2 mm. We are also bound to say that the energy difference must have been completely radiated in the same short interval. The only logical alternative would be for the atom to act in some way as a receiver of energy and to emit it later in such a manner that the frequency is altered to suit the new field strengths existing at each instant. This alternative suggested by Foersterling makes the frequency radiated independent of any orbital considerations whatsoever, and so removes the basis of Epstein's theory and the Bohr theory of light emission. The successes of these theories, however, should, at least, be considered as confirming Bohr's idea of connecting radiation frequencies with differences in orbital energies.

We are thus led to the conclusion that the whole emission process on the Bohr picture, including the transition between two stationary orbits, and the radiation of the energy difference, must occur while a flying atom with a speed of approximately 6×10^7 cm per second covers 0.2 mm; that is, in less than 3×10^{-10} seconds.

The success of Holtsmark's[1] theory of the broadening of spectral lines also gives indirect evidence on this point. The broadening is supposed to be due to the Stark effects produced in the radiating atoms by the superposition of the electric fields from other flying atoms. This resultant field varies very rapidly and the radiation emitted has a frequency corresponding to the field that happens to exist at the particular instant of the light emission.

It is of interest to consider whether the three interpretations of the canal ray experiments, discussed above, are compatible with the very brief duration of the elementary atomic radiation process, as deduced from Stark's experiments. The first classical interpretation gives too long a time as pointed out by Foersterling. The second interpretation of the duration of the luminosity as due to "delayed jumps" might agree as far as the brevity of the actual

[1] *Annalen der Physik*, 58, 577, 1919.

emission is concerned, and may apply to the faint short-lived bundle, but it is impossible to have "delayed jumps" in Stark's experiments, since it is necessary to have the initial and final orbits both characteristic of some definite electric field. The third explanation proposed above to explain the long-lived continuous bundle, in which the emission from an atom is regarded as following the dissociation of a molecule, may be reconciled with Stark's observations, since it seems probable that most of the moving intensity in Stark's experiments was of this kind, inasmuch as the pressure in his tube was such as to give a dark space 5 to 10 cm in length. Here the initial orbit, radiation emitted, and final orbit may all be characteristic of the point in the field at which the dissociation of the molecule leads to light emission.

Although we may thus obtain agreement between Bohr's theory and canal ray experiments, by supposing a duration of the light emission process of less than 3×10^{-10} seconds, we are led thereby to disagreement with the observed homogeneity of spectrum lines. The value for the half width for $H\alpha$, .047 A, as observed by Michelson[1], agrees with the value computed from the Doppler effect caused by the kinetic motions of hydrogen atoms if the temperature is assumed to be 135° C. The finite wave train of 9 cm caused by a limitation of the time for the emission to 3×10^{-10} seconds would cause a half width of at least .01 A.[2] This would leave only .037° A as due to Doppler effect and this is smaller than that demanded by the kinetic theory even if the tube were at room temperature. With helium a very great discrepancy may be deduced.

SUMMARY

The experimental results may be summarized as follows:

1. The previously reported experiments on the dying out of a beam of moving luminous sources in hydrogen canal rays in 2–4 cm have been repeated, and the conditions investigated under which the dying out is most prominent. The upper limit for the time of duration of this radiation process is 10^{-7} seconds.

2. No luminous rays are present that die out in a much shorter distance.

[1] *Philosophical Magazine* (5) **34**, 280, 1892. [2] *Astrophysical Journal*, **2**, 252, 1895.

3. With different discharge conditions a second type of moving luminous particles appear that show no dying out. Their radiation process lasts longer than 5×10^{-7} seconds. With low pressures they are much more intense than the short-lived particles.

4. Both types of luminous particles are neutral, and their luminosity is not altered in intensity by a strong electric field.

The discussion of the experiments in connection with other phenomena bearing on light emission shows that the first type of particles may be neutral hydrogen atoms. The second type of luminosity may be a secondary phenomenon following the dissociation of a neutral molecule. Stark effect observations in a variable electric field lead us to assume that the duration of the elementary atomic radiation process requires less than 3×10^{-10} seconds. This short time may be reconciled with one interpretation of the experiments in this paper, but leads to disagreement with the duration of the atomic radiation set by the classical interpretation of the homogeneity of spectrum lines.

The author's thanks are due to Professor Frost and Professor Parkhurst, of the Yerkes Observatory, for the use of their Hartmann microphotometer, and to Dr. F. Kannenstine for many mechanical suggestions.

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A STUDY OF THE GREEN AURORAL LINE BY THE INTERFERENCE METHOD[1]

By HAROLD D. BABCOCK

ABSTRACT

Green auroral line; observations with Fabry-Perot etalon.—It is found that with a camera having a speed of $f/3$ or more the persistent green radiation in the background of the night sky can be observed through a Fabry-Perot interferometer on almost any night between third and first quarters of the moon. No dispersion apparatus is required. Moderate cloudiness does not hinder the observations. Orders of interference amounting to 3700, 8450, 15,500, 31,000, and 85,000 waves were successively used in photographing the rings and from the sharpness it is concluded that the *line-width* is not more than 0.035 A. No appearance of doubling or other complexity has been noted. The surface *brightness* of the diffuse green auroral light is found to be of the order of 10^{-8} times that of the green radiation from a mercury arc in vacuum. Considerable variations in the intensity have been noted, probably amounting to at least three- or fourfold. A consistent series of eleven interferometer measurements of the *wave-length* is given. Two pairs of planes, three different separations, and three different pairs of metal films, both gold and silver, were used. The green mercury line and the strongest yellow neon line were used as reference standards. The wave-length obtained is $5577.350 \pm .005$ I.A.

INTRODUCTION

An extensive list of papers gives evidence of the interest which investigators have shown in the spectrum of the polar light. While the beautiful researches of Störmer, Birkeland, Vegard, and others have contributed greatly to our knowledge of the nature and causes of the aurora, the origin of the prominent green line characteristic of this light is still unexplained.

It has long been recognized that on nights when no auroral display is visible there still may be observed in the background of the sky a faint yellow-green spectral line which has been identified with the strong line in the spectrum of the aurora itself. The importance of further study of this light is emphasized by several recent investigations. Lord Rayleigh[2] has shown that the green line can be photographed in England on two nights out of three, even when the sky is partially clouded. He also found that it was more intense at Terling near London than some three degrees of latitude farther north, near Newcastle. Slipher[3] has published the results of

[1] *Contributions from the Mount Wilson Observatory*, No. 259.

[2] *Proceedings of the Royal Society*, A, **100**, 367, 1922.

[3] *Astrophysical Journal*, **49**, 266, 1919.

photographic determinations of the wave-length of the green line made with higher dispersion than had been employed previously. He found the wave-length to be 5578.05 A from three plates made with exposures of about 100 hours each. The investigation of Vegard[1] leaves no question as to the identity of the persistent green line with that present in the spectrum of visible streamers and discusses the difficulties of accounting for it, as well as the possible manner of its excitation.

The extreme faintness of this persistent light, on the one hand, and the evanescent character of most other auroral phenomena, on the other, have hitherto restricted observers to the use of low-dispersion prismatic instruments for such investigations.

It is the purpose of the present paper to describe work done in Pasadena and on Mount Wilson during recent months without the customary dispersing apparatus, and to present the results of new measurements of the wave-length of the green line made by the interference method, together with some observations on its width and intensity.

Inasmuch as this line is the only emission line found within a long range of the spectrum of the night sky, it should, if sufficiently homogeneous, give interference rings when a Fabry-Perot etalon is placed in front of a camera focused for parallel light and pointed toward the sky. Superposed light, having either a continuous or a band spectrum, would spread a uniform illumination upon the ring system, and the visibility of the latter would depend not only upon the sharpness of the spectral line but also upon its brightness relative to the background. The available data upon the brightness made it seem probable that under good observing conditions the background illumination would not be a very serious obstacle, and this has proved to be the case.

LINE WIDTH AND INTENSITY

An etalon having glass planes 1 mm apart was mounted in front of a hand magnifier lens of about 50 mm equivalent focal length and 25 mm clear aperture. The planes were covered with very thin

[1] *Geofysiske Publikationer*, 2, No. 5, "Utgit av den Geofysiski Kommission"; *Philosophical Magazine*, 42, 47, 1921.

films of gold cathodically deposited, this metal being chosen because of its high coefficient of transmission for green light. The first plate was exposed for ten hours in Pasadena on the night of February 25–26, 1922, and upon development the clear impression of a system of circles was found. No attempt was made to maintain the temperature constant during the exposure and no means were employed for reducing the background illumination. The instrument, packed in a wooden box, was merely directed toward a point low in the northern sky, away from the glare of the city lights,and left throughout the night with an alarm clock set to close the shutter before the first light of dawn. A photographic emulsion prepared by Mr. C. E. K. Mees of the Eastman Kodak Company was used. This is called an "Astronomical Plate" and has a uniformly high sensitivity throughout the green region of the spectrum. Several other emulsions tried later were found less rapid.

Instrumental improvements were at once effected, and it was soon found that the first exposure was not exceptional. On the contrary, almost every plate exposed under fair conditions has shown at least some trace of the interference ring system, and even on partially cloudy nights good photographs have been obtained.

Without relating in detail all of the trials made, it will prove sufficient to describe here the instrument as used in the later stages of the work. The camera is supplied with a Dallmeyer cinematograph anastigmat lens of aperture ratio $f/1.9$, having an equivalent focal length of 76.8 mm. It has a spiral focusing mount and is rigidly fastened in a light metal box, provided with a brass plateholder. The interferometer is supported upon the same aluminum base plate, close in front of the camera lens. It consists of a pair of planes of either glass or fused quartz, held parallel to one another by an invar separator, the system being contained in a steel tube carrying the adjusting springs at one end. A cover of sheet aluminum incloses both etalon and camera, and the combination may be quite freely handled without disturbing the adjustment. The clear aperture of the interferometer plates when in position is 25 mm, thus reducing the effective aperture of the camera to about $f/3$. The instrument is mounted in a wooden case which is provided with an extension 1.5 meters long in front. A piece of plate glass

closes the opening, and a simple sliding shutter is actuated by a cord wound on the spindle of the ringing mechanism of an ordinary alarm clock. The interior of the compartment containing the apparatus is provided with a thermostat of bimetallic strip and a distributed heating resistance. The entire case is packed in wool in a much larger wooden box. These precautions make it possible to reduce the variations of temperature to less than $0^\circ 1$ C. during an exposure.

It was found that the 1 mm etalon gave sharp fringes. At this point, the following question might be raised: Is it certain that the fringes photographed are produced by the green line? An examination of the negatives under a measuring machine shows that the scale of the ring system itself provides an affirmative answer, for a rough value of the wave-length is given by the expression,

$$\lambda = \frac{2eS}{8F^2}$$

where e = thickness of etalon; F = equivalent focal length of camera; S = difference in squares of successive ring diameters. A measurement of S and its probable error upon a plate taken with the 1-mm etalon, together with the known values of e and F, gives for the wave-length the value 5525 ± 70 A. The difference between this approximation and the known value of the wave-length is thus well within the probable error. Observations to be described later remove all uncertainty in this regard.

The etalon plates were next separated more widely. The second distance chosen was 2.4 mm and no difficulty was experienced in securing sharp fringes with this order of interference, namely, about 8450 waves. The next higher order employed was 15,500 waves, which was obtained by means of a plane-parallel glass plate of suitable thickness and index of refraction, lightly gilded on both sides. Sharp fringes were photographed with this instrument.

The fused quartz plates referred to above were then separated about 8.7 mm by pieces of invar and subsequent photographs, at about 31,000 waves retardation, still showed sharp rings. A thicker plate of plane-parallel glass was then used, giving an order of interference for the green line of approximately 85,000 waves. With

this high order of interference the rings are very small, but they are still well defined and a conservative estimate of their width places this quantity at not over one-half of the spectral range of the etalon, which is 0.066 A. It would appear safe to say that the width of the persistent green auroral line does not exceed 0.035 A. This is approximately the width of the finer arc lines of iron when excited in a vacuum by a moderate electrical current.

It may be mentioned here that the spectral range, or the extent of spectrum contained between any two points in the interference pattern whose orders of interference differ by unity, is given by the expression

$$\text{Spectral range} = \frac{\lambda}{p} = \frac{\lambda^2}{2en}$$

where λ = wave-length; p = order of interference; e = thickness of etalon; n = index of refraction of the medium between the parallel planes.

None of the photographs obtained show any signs of doubling or dissymmetry in the line; the number of different orders of interference employed, together with the fortuitous values of their ratios, would indicate that the line is either single or a doublet having a separation less than 0.035 A, or, if it consists of a primary and satellites, the latter must be of low intensity.

With these high orders of interference and the short-focus camera, the ring systems are on so small a scale that further large increase of resolving power is impracticable. To increase the scale by the use of a longer-focus lens requires an etalon of proportionately greater diameter, if one is to maintain sufficient photographic speed to secure an observation in a single night. A larger etalon is now under construction in the instrument shop of the observatory with which still higher resolving power may be brought to bear upon the green line while still permitting the use of very rapid camera lenses.

Most of the photographs discussed here were made when the moon was below the horizon throughout all or nearly all of the exposure time. Under this condition it is found that, even in Pasadena, fair contrast may be secured between the rings and the back-

ground without the use of a color screen, except when clouds or fogs diffuse the light from street lamps. On Mount Wilson no trouble arises from artificial illumination if the northern sky is used, and, even when the moon is at the quarter phase, good plates may be obtained if a filter is used to cut off light of wave-length less than about 5000 A.

The best screen is found to be one designed for isolating the green line of mercury from the other lines in the spectrum of that element. The intense absorption bands of neodymium in the yellow and the extinction of the entire spectrum below λ 5200 leave only a small portion of the continuous spectrum effective, since the plates used are comparatively insensitive to red light. The amount of background fog is thus appreciably reduced.

When all observing conditions are as uniform as possible there still appear large differences of intensity among the photographs secured. It would seem that variations in brightness of perhaps three- or fourfold are of frequent occurrence. At times, the intensity was found exceptionally high, the best instances of this being the observations of July 11–12 and July 12–13. There appeared concurrently at the sun's limb a remarkable prominence which subsequently extended over one-fourth of the solar circumference, and this event was probably associated with the increase of intensity noted. Since the present year is near a minimum in the cycle of sun-spot activity, it would not be expected to offer the best opportunities for a study of this kind, although, if systematic observations had been made, some further correlation between intensity of persistent auroral light and solar phenomena might have been revealed.

A consideration of the data regarding exposures made upon the aurora and the green mercury line leads to an estimate of the order of magnitude of the ratio of the surface brightnesses. The mercury lamp was inclosed in a box having an opening covered by the filter which transmitted only the green radiation. This light fell upon a white screen having a good diffusing surface, which acted as a reflector and furnished light to the etalon and camera. The area of the luminous mercury vapor used was about 3 square centimeters, and it was distant approximately 60 centimeters from the diffusing

screen. The reduction of surface brightness is then found by comparing the area of a hemisphere of radius 60 cm with the area of the source. This ratio of areas is about 7000:1. The average exposure time for the aurora was about nine hours and for the mercury light about five seconds, but, on account of slower plates being used for the mercury, it is fair to take the corrected ratio of exposure times as about equal to 10^4. Since the auroral plates were generally weaker than the comparison photographs, it follows that in round numbers the ratio of the surface brightness of the green radiation from a mercury arc to the surface brightness of the diffuse auroral light is about 10^8. A promising field is offered here for more extended quantitative work upon the brightness of the green auroral light and its variations.

It is worthy of note that, for studies of the intensity of monochromatic sources such as the green auroral line, instruments like the one described offer certain advantages. This is especially true when the etalon consists of a plane-parallel glass plate with gilded planes, for such a plate is always in adjustment if reasonably protected from temperature variations, and its gold films are fairly permanent. If need be, the metal films can be protected by glass covers, one of which may be a color screen, cemented on with Canada balsam. Results comparable with those of other observers may be obtained if the camera speeds are specified and the absorption factor is measured. Since a moderate order of interference is in general best suited to intensity observations, there is required a plane-parallel glass plate of sufficient size which is not too thick. But extreme accuracy in the surfaces is not essential, except when wave-length measurements are to be made upon the rings, and no great difficulty is found in preparing suitable plates.

It may be remarked that a plane-parallel glass plate affords a distinct advantage over an air etalon giving the same order of interference when used with short-focus lenses in the study of widths of spectral lines. It may easily be shown that, for the same value of focal length, and the same integral and fractional orders of interference, the diameter of a ring is directly proportional to the index of refraction of the medium between the reflecting planes. Thus, the use of a glass plate is equivalent to an increase of focal length

in the ratio 3:2 without loss of speed, spectral range, or resolving power. This increase of scale is very valuable when the limiting order of interference is approached.

The exposure times were dependent upon the hours of darkness; in each case, all the available time of a single night was used. Near summer solstice this amounts to only six hours at Mount Wilson, since exposures were not made when the sun was less than 18° below the horizon. No observations were made in June, but in July good plates were secured with about seven hours' exposures.

WAVE-LENGTH MEASUREMENTS

Turning now to the question of the wave-length of the green line, it may be recalled that the recent measurements of Slipher[1] and of Vegard[2] are of outstanding weight in comparison to those made previously. An uncertainty of 1 A in the values obtained by these observers is improbable. On this account it was deemed sufficient to begin measurements upon interference plates taken with an etalon having a spectral range of about 1.5 A, corresponding to a distance of 1 mm between the planes. The practice was adopted of photographing the green line of mercury through the instrument before and after each exposure to the aurora, always diffusing the light from a large surface in order to make the illumination of the etalon similar in the two cases. At times, the bright yellow line of neon λ 5852.488 was used in addition to the green line of mercury. This was conveniently supplied by a 5-watt Osglim lamp with a specially made filter to reduce the intensities of the other lines in the spectrum. The spectra of mercury, iron, and neon were used for determining the integral orders of interference.

As indicated above, there is no ambiguity attached to the wave-length derived from the 1-mm etalon if the uncertainty of the preliminary value is appreciably less than the spectral range. This condition is more than fulfilled in the present instance by the agreement between the measurements of Slipher and Vegard. But the use of two other widely different orders of interference furnishes an independent check. Since none of the orders employed bears a simple multiple relation to the others, the consistent values of the

[1] *Loc. cit.* [2] *Loc. cit.*

wave-length yielded by the three etalons can only arise from a unique system of corresponding whole orders, unless a preliminary wave-length, differing by more than 12 A from the known value, is adopted. This may be seen from an inspection of the spectral ranges of the various etalons, which are very approximately 1.535, 0.652, and 0.178 A, respectively, at wave-length 5577. The lowest integers having the same ratios as these numbers are 8, 19, and 69, and $8\times1.535=12.3$ A. Such an error in the observations of previous investigators is inadmissible.

In Table I are presented the results derived from all the plates thus far obtained which are suitable for wave-length determinations.

TABLE I

WAVE-LENGTH OBSERVATIONS ON GREEN AURORAL LINE

Date	Station	Reference Line	Etalon	Observed λ	Weight
			mm	I.A.	
1922, Sept. 20–21...	Mt. Wilson	5460.746	8.7	5577.348	2
Sept. 21–22...	Mt. Wilson	5460.746	8.7	.356	1
Sept. 27–28...	Mt. Wilson	5460.746	8.7	.353	1
Oct. 18–19...	Mt. Wilson	{5460.746 5852.488}	8.7	.344	5
Oct. 25–26...	Mt. Wilson	{5460.746 5852.488}	2.4	.346	3
Nov. 16–17...	Pasadena	5460.746	1.0	.372	1
Nov. 17–18...	Pasadena	5460.746	1.0	.355	1
Nov. 20–21...	Pasadena	5460.746	2.4	.353	1
Nov. 22–23...	Pasadena	5460.746	2.4	.350	1
Nov. 24–25...	Pasadena	5460.746	8.7	.352	2
Nov. 25–26...	Pasadena	5460.746	8.7	.352	2
Weighted mean.	....	....	....	5577.350	....

The weights given in the last column are based upon the difficulty of measurement and the degree of constancy of the etalon between comparison plates. Under the best observing conditions the change in thickness of the etalon during the night was found to be so small that if either comparison plate had been used without the other the error introduced would not have exceeded 0.002 A.

The values of the wave-lengths of the reference lines are those given by Fabry and Buisson[1] and the Bureau of Standards[2] for the

[1] *Journal de Physique* (5th series), 9, 189, 1919.

[2] *Scientific Papers of the Bureau of Standards*, No. 329, 1918.

mercury and the neon lines, respectively, in terms of the primary standard of wave-length. The numbers in the column headed "Observed λ" are uncorrected either for change of phase in the interferometer or for dispersion of the atmosphere, due to the observations not being made under standard conditions, since an examination of the magnitudes of these corrections showed them to be small compared to the variations found. Two different pairs of gold films were employed on the fused quartz plates, one of these pairs of films having a small admixture of silver in the gold, and the observation of November 22–23 was made with pure silver films on glass plates. It seems probable that errors due to the imperfections of the interferometer and its adjustment are nearly eliminated when the separate values are combined, and it is thought that the weighted mean derived from the observations is reliable to approximately one part in one million. This mean is

$$\lambda = 5577.350 \pm 0.001 \text{ I.A.}$$

the probable error being appended to show the accordance of the data rather than to indicate the limit to the uncertainty in the wave-length.

This value of the wave-length is 0.48 A less than that of Slipher, and 0.63 A less than the measurement of Vegard, assuming that these observers refer their results to the Rowland scale, which seems to be implied in their papers. From an extensive study of the differences between the Rowland and the International scales made at this observatory, it is found that at λ 5577 the latter is 0.220 A below the former; and this correction has accordingly been applied before making the foregoing comparisons.

It has been pointed out[1] that for interferometer plates the scale in angstroms per millimeter at any point in the ring system is given by

$$\frac{d\lambda}{dr} = \frac{\lambda}{F^2} r$$

where F is the equivalent focal length of the lens which forms the image of the rings, and r is the radius to the point. For the present series of photographs, F has the value 76.8 mm, so that the coefficient of r is approximately 0.95. From this it is found that the

[1] *Mt. Wilson Contr.*, No. 137; *Astrophysical Journal*, 46, 236, 1917.

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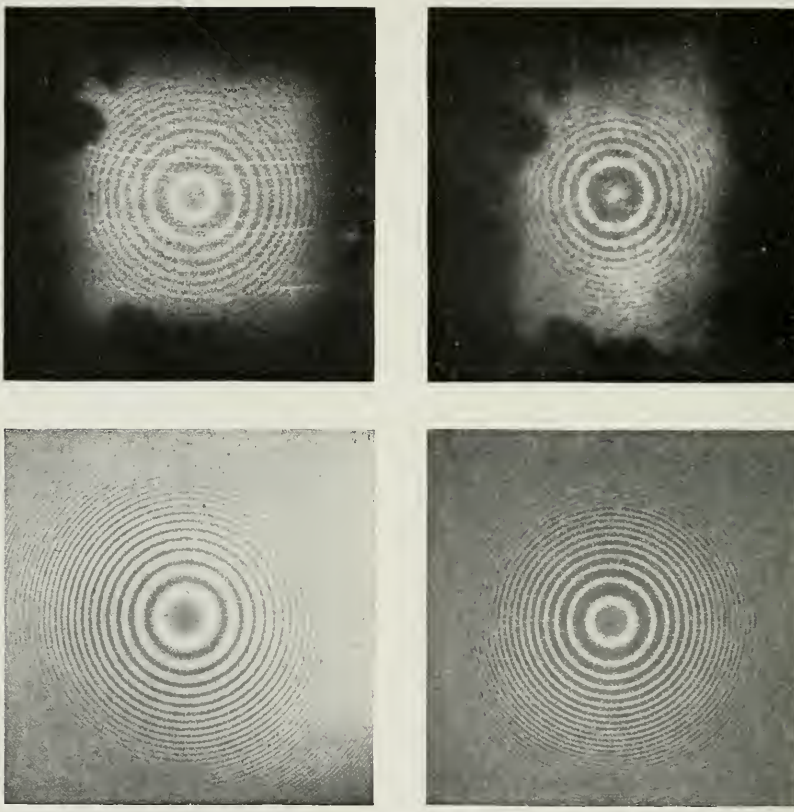
PLATE XVII



a b c d

Interference Rings from Green Auroral Line and from Artificial Sources

(*a*) Aurora. July 26–27, 1922, glass plate etalon, order of interference 15500. Exposure $6^h\ 50^m$, $f/1.9$.

(*b*) Aurora. October 25–26, 1922, air etalon, order of interference 8455. Exposure 10^h, $f/3$.

(*c*) λ 5461 of mercury, glass plate etalon, order of interference 15800.

(*d*) λ 5852 of neon, air etalon, order of interference 8058.

actual dispersion employed in these observations on the green auroral line varied from 8.5 A per mm for the largest ring used with the 1-mm etalon to 0.57 A per mm for the smallest one measured with the 8.7-mm etalon. The linear diameters of the interference rings were measured with a precision screw under a microscope fitted with a binocular eyepiece, at a magnification of about tenfold. On the best plates of the aurora as many as ten rings were measured, while on others only five were used. For the comparison plates, measurements were ordinarily made on from six to ten rings. The constant, $K\lambda$,[1] was determined from the comparison plates on account of the greater ease with which these plates could be measured, and the value of K to be used for the auroral rings was then derived from the ratio of the wave-lengths. The advantage of measuring several rings is especially great in the case of such photographs as those under discussion, for it largely eliminates the errors due to accidental distribution of silver grains in the emulsion and to slight imperfections in the film. Each ring furnishes an independent value of the fractional order, and thus of the wave-length, and, when reduced by the method indicated, all the results obtained from a given photograph are of practically the same reliability, out to a considerable distance from the center, depending upon the actual scale of the plates. No other measurements are required than those on the diameters themselves.

In Plate XVII are shown reproductions of the auroral photographs made on July 26–27, *a*, and on October 25–26, *b*, together with comparison photographs from mercury, *c*, and neon lines, *d*. The photographs of July 26–27 were made with a plane-parallel glass plate giving orders of interference of about 15,500 for the auroral line, and 15,800 for the green mercury line. The exposure time for the aurora was 6 hours, 50 minutes with the camera working at $f/1.9$. The plates of October 25–26 were made with an air etalon with orders of interference 8455 for the auroral line and 8058 for the yellow neon line (*d*). The exposure was 10 hours at $f/3$ for the aurora. The sky was partly cloudy during the last third of the exposure. A uniform enlargement of 4 diameters has been used in preparing these illustrations.

[1] *Mt. Wilson Contr.*, No. 202; *Astrophysical Journal*, 53, 44, 1921.

Plate XVIII shows at *a* the auroral exposure made on October 18–19 with an air etalon giving an order of interference of 31,000. At *b* and *c* are shown neon and mercury comparison plates used for determining the wave-length. At *d* is shown the auroral plate of November 17–18, made in Pasadena with a 1-mm etalon. The observation made on the aurora with the highest order of interference is not suited to reproduction, on account of the very small scale of the original.

The three primary data required concerning a spectral line are its wave-length, intensity, and width or character, whether single or complex, sharp or diffuse. The adaptability of the Fabry-Perot etalon to studies of all three kinds is well illustrated here. No other instrument offers such advantages in the problem at hand for precise measurements of wave-length and for examining the line width, while its usefulness for intensity observations is hardly secondary to that of a prism. It was long ago pointed out by Fabry and Perot[1] that the intensity of the maxima of the interference rings would be equal to that of the incident light if there were no absorption in the metallic films. If high resolving power is required, the reflecting power must be considerable, and this entails increased absorption, but for intensity observations the films may be made exceedingly thin, so that the loss by absorption is greatly reduced. The use of gold in place of silver for work in the green is a further help. Although measurements upon gold are not available it seems probable that an etalon employing this metal may be used for intensity observations with an efficiency of at least 60 per cent. The efficiency of a glass prism with its collimating lens cannot greatly exceed 75 per cent. This difference in favor of the prism is to a considerable extent offset by the fact that for weak images it is much easier to judge the blackening upon the interference pattern of the etalon than upon a single, narrow, and generally short image of the slit which is given by the prism.

The value of the wave-length obtained here is not sufficiently different from that of Slipher to provide any obvious new possibilities for identifying the line, although it may prove an important aid in the solution of the difficulty. In this connection, it is interesting to

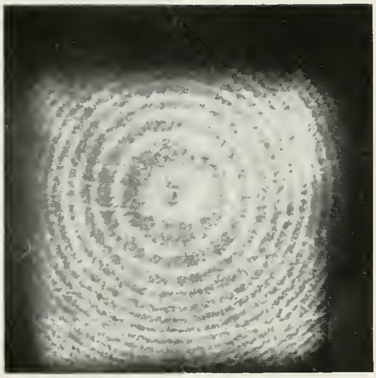
[1] *Annales de Chimie et de Physique* (7th series), 12, 462, 1897.

PLATE XVIII

a

b

c



d

INTERFERENCE RINGS FROM GREEN AURORAL LINE AND FROM ARTIFICIAL SOURCES

(*a*) Aurora. October 18–19, 1922, air etalon, order of interference 31000. Exposure $9^h\ 30^m$, $f/3$.

(*b*) Plate for wave-length comparison; neon λ 5852, order of interference 29500.

(*c*) Plate for wave-length comparison; mercury λ 5461, order of interference 31600.

(*d*) Aurora. November 17–18, 1922, 1 mm air etalon, order of interference 3600. Exposure, made in Pasadena, 10^h, $f/3$.

recall the work of Fabry and Buisson[1] upon the relation between width of spectral line, atomic weight, and absolute temperature for a radiating gas. In their interesting paper they give results of laboratory tests of the theory that under suitable conditions of excitation the width is dependent solely upon the Doppler effect due to motion of the radiating centers in accordance with the kinetic theory of gases. The equation stated by them is

$$\Delta = 0.82 \times 10^{-6} \lambda \sqrt{\frac{T}{M}}$$

where Δ = width of line of wave-length λ; T = absolute temperature; M = atomic weight. Measurements are given for hydrogen, helium, neon, and krypton. In the case of hydrogen, there may be some question regarding the conclusion drawn, as Saha[2] has pointed out, but for the other three elements the observations appear to furnish satisfactory confirmation of the theory.

An application of this theory to the aurora, though not conclusive, may be of interest. If the absolute temperature of the earth's atmosphere at the height of the aurora is taken at 218 °C., as indicated by Humphreys,[3] and the width of the green auroral line is put equal to 0.035 A, the equation above gives for M the value 3.8, while if the width is subsequently found to be less than 0.035 A, the value of M will be increased accordingly. It is evident, then, that if hydrogen is postulated as the origin of the auroral line, the absolute temperature cannot much exceed one-fourth that of the isothermal layer. Helium, on the other hand, would correspond well with the probable temperature; but, if a heavier element is involved, the line-width must be expected to be less than the upper limit observed, since the absolute temperature can certainly not be much higher than 218°. It is possible that further studies of the line-width and an examination of the spectra, under extreme conditions of excitation, of the gases which are likely to be responsible may result in the discovery of a line corresponding to the new position found for the auroral line.

Mount Wilson Observatory
December 1922

[1] *Op. cit.*, 2, 442, 1912.

[2] *Philosophical Magazine*, 40, 159, 1920.

[3] *Physics of the Air*, p. 70

POLE-EFFECTS, PRESSURE-SHIFTS, AND MEASUREMENTS OF WAVE-LENGTHS IN THE SPECTRUM OF MANGANESE

By GEORGE SPENCER MONK

ABSTRACT

Interference measurements in the arc spectrum of manganese between λ 4470 *and* λ 6021.—Pole-effects, pressure-shifts, and wave-lengths for the stronger lines of manganese between λ 4470 and λ 6021 have been measured. A carbon arc with the lower, positive, pole bored out and filled with manganese dioxide was used as a source. (1) *Pressure-shift:* The interference method used does not seem to be suitable for observations on pressure-shift in manganese, on account of the intrinsic wideness of many lines, and their widening under pressure. The evidence obtained indicates a correspondence between unsymmetrical broadening of the lines under pressure, and pole-effect. (2) *Pole-effect:* Little or no pole-effect is observed with a carbon arc whose lower pole is filled with manganese chloride. There are objections, however, to its use. A moderate effect is observed at the positive pole of a carbon arc filled with manganese dioxide, and a larger effect with an arc between metallic poles. (3) *Wave-lengths:* Only about 1 millimeter of the center of a 12 mm to 15 mm, 5-ampere arc on a 220-volt circuit was used, and it is believed that pole-effect is entirely absent from the measurements. The wave-lengths of about forty lines have been measured with an accuracy estimated at 0.002 A. Lines of neon and lines of iron of group *a* were used as standards, the strongest component of Hg 5461 serving as an auxiliary standard.

On account of the prevalence of manganese as an impurity in iron, and the consequent appearance in the spectrum of iron of the stronger manganese lines, the spectrum of manganese is of interest as a possible source of standards of wave-length. Also, a knowledge of the correct wave-lengths of many manganese lines is necessary for comparison with the corresponding wave-lengths in the sun. This paper contains the results of interferometer measures of the wave-lengths of a number of lines in the region λ 4470 to λ 6021, together with an examination of the stronger lines in that region for pole-effect, and their behavior under moderate differences of pressure.

Wave-lengths of manganese lines have been published by a number of observers. Chief among these are those due to Exner and Haschek,[1] Kilby,[2] and Fuchs,[3] while some of the stronger lines

[1] *Die Spektren der Elemente*, Leipzig, 1912.

[2] *Astrophysical Journal*, **30**, 243, 1909.

[3] *Zeit. für Wiss. Phot.*, **14**, 239, 1915.

have been measured as impurities in the arc spectrum of nickel by Hamm,[1] and of iron by Burns,[2] who used the interference method.

THEORY AND APPARATUS

From elementary theory, it is easily deduced that the fractional order of interference at the center of the ring system formed by the passage of light through a parallel-plate interferometer is expressed by

$$E+(n-1)=\frac{D_{n-1}^2}{D_n^2-D_{n-1}^2}(1+D_n^2/8F^2m^2)\,, \tag{1}$$

where E is the fractional part of the whole order of interference at the center of the ring system, D is the diameter of a ring, F is the focal length of the lens or mirror projecting the fringes on the slit of a spectrograph, m is the magnification of the spectrograph, and n is an integer referring to the number of the ring counting from the center outward. This equation holds only when the magnification of the spectrograph is constant for all wave-lengths. For the apparatus used in this investigation, F is 500 mm, the maximum D is 25 mm, and m is 1.07. Hence, neglecting the second term in the bracketed factor on the right-hand side of equation (1) introduces a negligible error of one part in about thirty-five hundred for the largest fringe measured. Also, it can easily be shown that the difference between the squares of the diameters of contiguous fringes for a given wave-length is a constant. Hence, equation (1) can be put into the simpler form

$$E+(\text{integer})=\frac{D^2}{D_k^2-D_{k-1}^2} \tag{2}$$

in which the denominator is the average of all the differences of diameters squared for a given line. Thus, by the use of equation (2), as many determinations of E, the fractional order of interference at the center of the ring system for a given line, may be made as the number of fringe diameters measured. In practice, this number was limited to eight or nine. Equation (2) is due to St. John and Babcock, and its use has been explained by them.

[1] *Zeit. für Wiss. Phot.*, 13, 105, 1913. [2] *Ibid.*, 12, 207, 1913.

The conditions in the foregoing theory are present in the apparatus used. Light from a source L (Fig. 1) is focused on the aperture of a parallel-plate interferometer by a concave mirror C_1, of 17 cm diameter and 60 cm focal length. By means of a plane mirror M, the light is then reflected to concave mirror C_2, which is set at a distance from the slit S equal to its focal length. Thus the fringes produced at the interferometer are in focus on the slit. For most of the plates, C_2 is of 50 cm focal length, although some were taken with mirrors of 60 cm and 25 cm focal length. Auxiliary

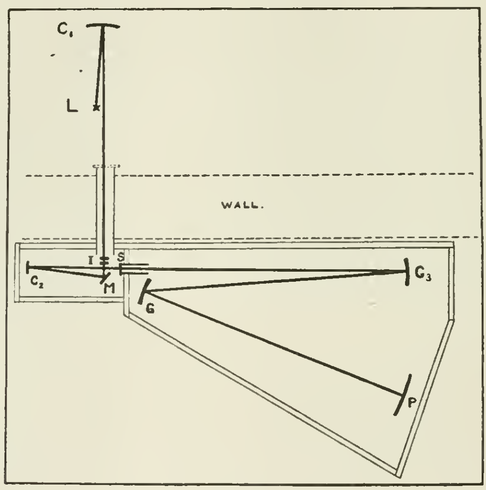


FIG. 1

dispersion of the fringes for different wave-lengths is obtained by the use of a concave grating G, on which the incident rays are made parallel by the use of the concave mirror C_3, of approximately 6 meters focal length. This form of the apparatus is due to Pfund. The grating, ruled by Professor Michelson, is of good quality and has a ruled surface 2 inches by 5.5 inches, with 15,000 lines to the inch. Its focal length is 21 feet. A negligible amount of astigmatism exists in the spectrograph owing to the small angle, about two degrees, between the incident and reflected beams at C_3. The grating and plate-holder are mounted at the ends of a rigid steel bar, which is movable about a vertical axis passing through the

center of the grating. The photographic plate is 10 inches long by 2 inches wide, bent so that the fringes are in focus over its entire length. The plate-holder is so mounted in guides that it can be moved a small distance horizontally in a direction at right-angles to the beam of light by means of a push-screw, thus permitting exposures, with the source under different conditions, to be photographed so that the spectrum lines are side by side on the same plate. The housing of the auxiliary spectrograph is large enough to permit exposures in the yellow of the second order.

The interferometer is equipped with a number of etalons ranging in thickness from 2.5 mm to 40 mm. The etalons are brass rings with three invar studs exactly equal in length to within a fraction of a light-wave. During this investigation, a pair of quartz mirrors was used, accurately flat to a fraction of a fringe, and sputtered to an appropriate thickness of silver by cathodic deposition. The aperture of the interferometer, upon which the light of the source is focused, is placed between the source and the silvered surfaces, and as near the latter as possible.

The entire instrument, inclosed in a tight, double-walled box, is located in a constant-temperature room in the basement of Ryerson Laboratory. The source of light and the concave mirror C_1 are in an adjoining room, the light being projected on the interferometer through a hole in the wall, over one end of which is a plate-glass cover. With these precautions, the interior of the interferometer box and spectrograph is free from temperature effects sufficient to cause displacements of the fringes. In order to obtain a check on instrumental changes of any sort during exposures, a short exposure to a Cooper-Hewitt lamp was made on each plate before and after each of the principal exposures. In addition, the order in which the exposures to manganese arc and standards were made was varied, to prevent systematic errors due to possible changes. Some few plates were taken with simultaneous exposures of manganese and standard source, but it was found that successive exposures with the Hg spectrum as a check gave satisfactory results. It might be mentioned in this connection that the character of the lines examined made the highest degree of accuracy of measurement impossible. Where the character of the inter-

ference fringes and the nature of the source permit the greatest accuracy, simultaneous exposures are to be recommended.

In all, about fifty plates of the manganese spectrum were taken and measured. The source was a vertical carbon arc, mounted so that quick and accurate adjustments could be made for focus and setting of the image of the source on the interferometer aperture. In the lower, positive, electrode, about 12 mm in diameter, was drilled a 3 mm hole, packed with MnO_2. The upper, negative, electrode was a solid carbon, 9 mm in diameter. Attempts were made to use a metallic manganese arc, in the hope that the extremely steady conditions of the Pfund iron arc could be obtained, but metallic manganese spluttered so badly that it proved unmanageable. Some exposures were made with a carbon arc as described, the lower pole being charged with manganese chloride instead of manganese dioxide. This arc required too frequent charging, and the location of the center of the arc was an uncertain matter. The manganese dioxide formed a bead almost equal in steadiness and longevity to the bead of the Pfund iron arc. The current was maintained at 5 amperes on a 220-volt circuit, and the distance between the electrodes was from 12 to 15 mm.

For the red region, Wratten Panchromatic plates were used. Cramer old-style X-ray plates, having a pronounced maximum of sensitiveness to the green, were used for wave-lengths below λ 5600. Exposure times ranged from about four minutes for the brightest component of Hg λ 5461 to an hour for some of the faint lines of manganese.

The etalons used were 5 mm and 7.5 mm. The choice of these distances rather than distances with a greater difference between them was necessitated by the character of the lines measured. Most of them were intrinsically very wide. The principal effect of the small range is an increase of the uncertainty of the phase-change correction. A comparison of the measures with those of other observers indicates that no systematic error has been thus introduced.

The image of the source in focus on the interferometer was about 5.5 times the size of the source. This gave an image of the arc between 70 and 80 mm in length. The aperture of the interferometer was rectangular in shape, and 6 mm high by 13 mm wide.

Thus, the central portion of the arc selected was well within the dimensions prescribed by the Committee on Standard Wave-Lengths. The interferometer was so placed that an image of its aperture coincided with the ruled area of the grating and was in sharp focus thereon. It is believed that on account of the large magnification of the arc, and the careful selection of its center, the wave-lengths determined are entirely free from pole-effect.

MEASUREMENT AND COMPUTATION

The plates were measured on a Gaertner comparator, readings being made to hundredths of a millimeter. Two or three settings were made and averaged for each fringe. In some exceptional cases, such as in preliminary determinations of the phase-change correction where the etalon used was so small that a mirror of comparatively short focal length was used to project the fringes on the slit, the readings were made to thousandths of a millimeter. From the measured diameters of the fringes, the fractional orders of interference were computed to the third decimal place according to formula (2). The integral part of the order of interference was computed on a calculating machine by a quick and easy method similar to that described by Meggers. The complete order of interference thus obtained, multiplied by the wave-length of the standard line, gave the double distance between the interferometer plates. This value, divided by the orders of interference for the other lines on the plate, gave the wave-lengths of those lines to the fourth decimal place.

As standards, in the red, lines of neon were used, in particular λ 5944.834; in the green, the Fe line λ 5497.522 was used as a standard, although many of the plates were measured in terms of Hg λ 5461, this line being measured in terms of the Fe and neon lines for both etalons. The adopted values of the wave-length of the bright component of Hg λ 5461 were: for the 5 mm etalon, λ 5460.7443; for the 7.5 mm etalon, λ 5460.7434. The difference is due to a greater nearness of a fainter component for one of the etalons. Both of these values are slightly larger than the value λ 5460.7424 obtained by Fabry and Buisson with a 5 mm etalon. A number of plates were taken with neon, mercury, and iron, giving a check on the phase-change corrections used. A curve

plotted for the corrections thus derived agrees well with that obtained from the manganese measures. In compiling the final table of wave-lengths, the formula for the phase-change correction derived by Meggers was used. Table I shows the application of the formula to three lines and a comparison with the values obtained by calculating the phase-change correction for the region on the basis of the neon lines, assuming the Bureau of Standards values as correct.

TABLE I

REDUCTION FOR PHASE-CHANGE

$$\Delta_1=\frac{e_2c}{e_2-e_1}; \quad \Delta_2=\frac{e_1c}{e_2-e_1}; \quad e_1=5.0\text{ mm}; \quad e_2=7.5\text{ mm}$$

c = difference between uncorrected wave-lengths of a line for two etalons

LINE	UNCORRECTED		c	CORRECTED	BY NEON LINES	ADOPTED
	7.5 mm	5.0 mm				
6013........	.4964	.4978	−0.0014	.4936	.4946	.494
6016........	.6459	.6474	−0.0015	.6432	.6440	.644
6021........	.8015	.8028	−0.0013	.7987	.7995	.799

PRESSURE-SHIFTS

Table II gives the results of measures on a number of lines photographed with the source subject to pressures ranging from 5 cm to 160 cm of mercury. A plate taken with a metal electrode showed fringes so widened for moderate pressures that measures were impossible. It was not considered advisable to make a more comprehensive study of pressure-shifts in manganese at this time in view of the small extent of the region investigated.

TABLE II

Line	Displacement per Atmosphere High-Low	Line	Displacement per Atmosphere High-Low	Line	Displacement per Atmosphere High-Low
4709..........	0.004	4783.........	0.004	5470.........	0.007
4727..........	0.004	4823.........	0.006	5481.........	0.008
4739..........	0.002	5341.........	0.004	5516.........	0.003
4754..........	0.006	5377.........	0.008	5537.........	0.008
4761 } *........	0.014	5394.........	0.006	6013.........	0.009
4762 }	0.006	5399.........	0.009	6016.........	0.011
4765 } *........	0.009	5420.........	0.010	6021.........	0.013
4766 }	0.006	5432.........	0.003		

* Close pairs, measures doubtful.

It is important to note a fact which does not appear from Table II. The displacements for λ 4754, λ 4783, and λ 4823 are from settings on what appears to be the center of the most dense portion of the fringes. Actually, in pressures higher than atmospheric, these fringes were drawn out so far toward the center of the ring system for each of the lines that it was doubtful where the center of the line really lay. In other words, these three lines were unsymmetrically widened toward the red, a condition frequently noted in certain groups of lines photographed under high pressures, and strikingly illustrated in the plates of Gale and Adams[1], taken with the source under a pressure of 8 atmospheres. The displacements indicated in Table II for λ 6013, λ 6016, and λ 6021 are based on observations at atmospheric pressure or less, no measures being made on plates taken at higher pressures.

POLE-EFFECT

The pole-effect for manganese has been investigated by Brendel-Wirminghaus[2] for wave-lengths shorter than λ 5600. This investigator used a carbon arc, one pole drilled out and filled with manganese chloride. She concluded from her results that the spectrum of manganese showed no pole-effect. The present writer made a number of exposures with this form of arc, and observed little or no pole-effect at the negative, carbon, pole. At the positive pole, which was drilled out and filled with manganese chloride, the results were inconclusive, a few plates showing an appreciable effect, and many, none at all. An objection to the use of manganese chloride is in the uncertain manner in which the arc burns. Most of the time, since the chloride is used up quickly, the arc issues from the positive pole at a point between 2 and 5 mm below the end of the electrode, rendering observations for pole-effect useless.

With a metallic arc, the values shown in Table III have been obtained, using the third order of the 10-inch Michelson grating in the 30-foot Littrow spectrograph. A comparison with the results of St. John and Babcock is given in Table III. Several exposures

[1] *Mt. Wilson Contr.*, No. 58; *Astrophysical Journal*, **35**, 10, 1912.

[2] *Zeit. für Wiss. Phot.*, **20**, 229, 1921.

using the interference method with a carbon arc, in which the lower, positive, pole was drilled out and packed with manganese dioxide, gave the results in Table IV.

TABLE III

Line	Monk	St. John and Babcock
	Neg. Pole-Center	Neg. Pole-Center
6013	0.018	0.019
6016	0.017	0.020
6021	0.019	0.018

TABLE IV

Line	No. Plates	Positive Pole-Center	Line	No. Plates	Positive Pole-Center
4472	1	.003	5255	2	.003
4490	1	.003	5341	2	.003
4498	1	.003	5377	2	.008
4502	1	.004	5394	3	.003
4709	3	.002	5420	3	.005
4727	3	.004	5470	3	.003
4739	3	.004	5481	3	.006
4754	2	.008	5505	2	.005
4783	2	.009	5516	4	.005
4823	2	.007	6013	4	.007
5150	2	.005	6016	4	.008
5196	2	.005	6021	4	.012

λ 4754, λ 4783, λ 4823, which were mentioned in connection with the discussion of pressure-shifts, as having an abnormal widening toward the red under pressure, are here shown to have

(*a*) (*b*) (*c*)

FIG. 2

larger pole-effects than other lines in the same region. It is also of interest, in connection with pole-effect, to notice the difference in appearance of the three types of arc used. These are illustrated

in Figure 2: (*a*) is almost uniform in intensity from pole to pole, the arc at the positive pole seeming to spring from the crater in which the manganese chloride has sunk; (*b*) has a marked condensation of the discharge at the point on the manganese oxide bead from which the arc springs, and is brighter there; (*c*) has this same feature at the positive pole, and an excessively great intensity at the negative pole.

WAVE-LENGTHS

The adopted wave-lengths for the lines measured are shown in Table V, compared with the results of other observers. A difference of 0.1 A in the case of λ 4739 is undoubtedly due to an error or misprint in the lists of other observers, since no manipulation of the whole number part of the order of interference will give an approximation to their results. The value for λ 5377 is in some doubt owing to the presence of a faint line close on the violet side. While the probable errors in column 8, based on the collected measures from each plate, are as low as 0.001 A, this does not represent the consistency of the measures of the individual fringes for a single exposure. These were not so good, especially for the very wide lines. Consequently, it is believed that claim cannot be made to greater accuracy than 0.002 A.

The usefulness of manganese lines in the region studied as possible secondary standards is in general impaired by two considerations. Many of the lines are eliminated on account of their intrinsic width. Most of those which are designated by Janicki as sharp, simple lines are as a rule most affected by pressure-shift and pole-effect. Among these latter are λλ 4754, 4783, 4823, 6013, 6016, 6021, which on account of their intensity, appear most frequently in the arc spectrum of iron. With exposures carefully made on the center of the arc, they should be useful as standards of measurement.

No lines of any prominence were measured in the region between λ 5537–λ 6013. While there are several lines of moderate intensity in the neighborhood of λ 5800, none of the plates taken showed them with sufficient density for measurement. Three other lines, λ 4709, λ 4727, and λ 4739, all sharp, and having the advantage of comparative isolation in the spectrum, gave promise of usefulness

as standards. The values obtained for these lines are however so different from the values obtained by other observers that

TABLE V

Line	Rowland	I. A. Units					Prob. Error	No. Plates
	Exner & Haschek	Kilby	Fuchs	Burns	Hamm	Monk		
4470	.33	.138	.142	.141	.147	.140	.0006	7
4472	.98	.800	.793	......	......	.796	.0004	7
4490	.27	.071	.078	......	......	.081	.0002	6
4498	.09	.898	.897	.903	......	.900	.0004	6
4502	.40	.218	.223	.219	.228	.222	.0003	7
4605	.52	.378	.367	......	......	.365	.0005	6
4626	.69	.552	.546	......	......	.542	.0004	6
4671	.89	.694	.688	......	......	.684	.0008	2
4701	.31	.150	.158	......	......	.154	.0008	3
4709	.89	.708	.704	......	......	.713	.0002	11
4727	.70	.476	.462	.464	......	.479	.0002	11
4739	.30	.004	.001	......	......	.106	.0002	11
4754	.24	.046	.048	......	......	.041	.0004	6
4761	.73	.521	.527	.524	.528	.526	.0002	6
4762	.60	.375	.375	.373	.374	.375	......	2
4765	.08	.852	.856	.863	.861	.861	.0003	6
4766	.63	.414	.426	.422	.421	.429	.0004	5
4783	.62	.454	.432	.434	......	.428	.0006	6
4823	.71	.521	.522	......	......	.516	.0006	6
5117	.10	.944	.935	......	......	.940	......	1
5150	.10	.937	......	......	......	.930	......	2
5255	.48	.330	.330	......	......	.320	.0004	4
5341	.25	.068	.070	......	......	.066	.0012	7
5377	.86	.623	.634	......	......	.619	.0007	11
5394	.89	.679	.677	......	......	.676	.0006	15
5399	.70	.494	.506	......	......	.487	.0006	9
5407	.67	.429	.432	......	......	.428	.0009	4
5413	.90	.690	.696	......	......	.683	.0010	5
5420	.61	.371	.368	......	......	.361	.0007	6
5432	.75	.553	.555	......	......	.545	.0006	11
5457	.64	......	.468	......	......	.463	......	1
5470	.88	.644	.640	......	......	.642	.0005	8
5481	.61	.401	.395	......	......	.387	.0006	5
5505	.10	.874	.877	......	......	.874	.0006	9
5516	.00	.774	.773	......	......	.773	.0005	11
5537	.99	.753	.749	......	......	.759	.0007	9
5551	.20	.991	.987	......	......	.984	......	1
5567	.02	......	.764	......	......	.761	......	1
6013	.74	.480	.484	......	......	.494	.0006	12
6016	.90	.631	.636	......	......	.644	.0005	13
6021	.05	.794	.787	......	......	.799	.0008	14

further measurement is necessary. It is planned to continue this investigation in the arc spectrum of manganese, using a cadmium

tube as a standard source, for such lines as these, for fainter lines, and for other parts of the spectrum.

In conclusion, the writer wishes to acknowledge the help and guidance of Dr. H. G. Gale, under whose direction this work was begun and carried on, and whose liberality in time and equipment made the results possible. Thanks are also given to Messrs. St. John and Babcock, who furnished details of their own apparatus, to Dr. Elias Klein for taking and measuring several plates, and to Mrs. Monk for aid in measuring and computing.

Ryerson Laboratory
April, 1923

A NEW METHOD FOR THE MEASUREMENT OF THE DEPTH OF THE CHROMOSPHERE[1]

By PHILIP FOX

ABSTRACT

Depth of the chromosphere.—(1) *Prism micrometer method of measurement* involves reflecting light from each of two diametrically opposite edges of the solar image on to the slit of a spectroscope by means of prisms whose distance apart can be varied by screw adjustments. The apparatus is shielded from the sun's heat by means of a plate equal in size to the solar image. The double thickness of the reversing layer is determined from the difference between the minimum and maximum separations of the prisms for which light from each shows the chromosphere lines. (2) *Preliminary observations* at Yerkes Observatory, under unfavorable conditions of seeing, indicate that for a clear sky the depth would be over 10″ for Hα and over 8″ for D_3.

In 1909 Hale and Adams published a paper on the "Photography of the Flash Spectrum without an Eclipse."[2] In this paper they described the apparatus and method of operation for holding the limb of the sun exactly tangential to the slit of the spectrograph. The success of the observations depends on holding the bright lines of the chromosphere continually in view during the exposures. This is a matter of considerable delicacy, for, if the photosphere falls on the slit, its brilliant spectrum will blot out the fainter chromospheric lines, and if the slit is too far from the limb the lines of the lower-lying vapors do not appear.

The success of their observations suggested that if such an apparatus could be adapted for simultaneous visual observations of the "flash" spectrum from points on the limb diametrically opposite we would have a very delicate method of measuring the depth of the reversing layer and possibly also the diameter of the sun.

As the only observational evidence on the thickness of strata of vapors of the reversing layer heretofore available comes from the length of cusps shown by objective-prism spectra of the chromosphere taken at times of solar eclipses,[3] it was thought advisable to test this new suggestion with a view to supplementing these results.[4]

[1] This method was first suggested by the writer at the Washington meeting of the American Section of the International Astronomical Union on April 4, 1922.

[2] *Astrophysical Journal*, **33**, 222–230, 1909.

[3] Frost, *Astrophysical Journal*, **12**, 321, 1900; S. A. Mitchell, *ibid.*, **15**, 119, 1902.

[4] Since writing this paragraph my attention has been directed to St. John's observations of the depth of the reversing layer in which he used photographs of spectra made with radial slit. *Astrophysical Journal*, **32**, 64, 1910.

Various observers in studying spectroscopically the problem of the rotation of the sun have used a device[1] which can be easily adapted to the present needs. Certain features of the apparatus used by the investigators of the flash spectrum and of the solar rotation were combined in a device, which may be called a prism micrometer, shown in Figure 1. This was made up by Mr. Küng in the shops of the Physical Laboratory of Northwestern University. The set of prisms was loaned to Dearborn Observatory through the courtesy of Professor H. D. Curtis, of Allegheny Observatory.

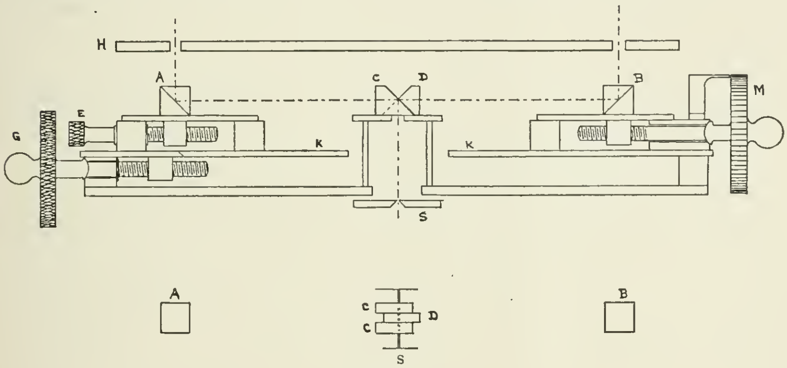


FIG. 1

Prisms *A* and *B*, Figure 1, reflect light from the limb of the sun respectively to prisms *C* and *D*, which are mounted in fixed position over the slit of a spectroscope *S*. Prisms *A* and *B* are set at approximately equal distances from the slit and are separated by a distance about equal to the diameter of the solar image; the adjustments are made by means of screw *E* and the micrometer screw *M*. After the approximate adjustment is made, the screw *E* is not again touched. The carriage *K*, on which the two prisms *A* and *B* are mounted, can be moved to follow the image by means of the screw *G*. Any considerable movement to this screw will lengthen the light path for one limb and shorten that from the other. If the driving clock is well rated, this maladjustment of focus should not be serious. The plate *H* is to shield the apparatus from the sun's heat.

[1] W. S. Adams, Carnegie Institution of Washington, *Publication No. 138*, 3, 1911; Schlesinger, *Allegheny Observatory Publications*, 3, 103, 1914.

In determining the thickness of the strata of vapors of the reversing layer, the process of measurement is to bring the sun central on the shield *H;* by means of the screw *G* the carriage is moved until the chromospheric spectrum from *A* is visible in the spectroscope; holding this with the screw *G*, prism *B* is moved by the micrometer screw *M* until the same spectrum from *B* is also seen. The setting of *M* is read for the smallest distance between *A* and *B* where these spectra from both limbs are visible. Prisms *A* and *B* are then separated by *M*, always adjusting with *G*, until the maximum diameter for simultaneous visibility of the spectrum from both limbs is reached. At this point *M* is again read. The difference of these two readings of *M*, with perhaps some correction dependent on slit width, will give the double depth of the reversing layer for the particular line under observation.

Professor Frost offered the facilities of the Yerkes Observatory for carrying out this investigation. Inasmuch as the advantages of the large solar image given by the 40-inch refractor are obvious, his offer was accepted and the observing has been carried on there. The prism micrometer was mounted before the slit of the auto-collimating stellar spectroscope of that Observatory. This spectroscope has a lens of 30 inches focal length. The dispersion piece is a Michelson grating, which was used in the first order. The apparatus was assembled in the early days of September. The adjustments offered some difficulty and even when these were completed it was found that the operation of the instrument was quite inconvenient. The observations were made during two visits to the Yerkes Observatory, in the middle and end of September. On neither occasion was the sky favorable; observations were made in intervals between clouds or through thin clouds or haze. It was found possible, however, to measure the depth of the reversing layer for the hydrogen line $H\alpha$ and for the helium line D_3. The results are collected in Table I.

All of the measures were made in the north and south direction. The slit of the spectrograph was set at widths between 0.05 mm and 0.08 mm. The aperture of the telescope was reduced to about 20 inches.

It is seen that the results have a fairly large rånge. The smaller values, in general, were obtained when the sky was thick and the

sky spectrum, therefore, very bright. Under such circumstances, the bright line of the reversing layer is lost at no very great distance from the limb, on account of the diminished contrast. Under uniform conditions, the results seem to be fairly accordant. It is a safe estimate that under good atmospheric conditions, high transparency and steadiness, the Hα line could be observed consistently at depths of 10″ and D_3 fully 8″. The chromospheric

TABLE I

Measured Thickness of the Reversing Layer

1922	Line	Double Depth	Single Depth	Remarks
		R		
Sept. 15	Hα	4.1	8″.5	Through breaks in clouds. With Dr. Lee
		4.5	9.3	Through breaks in clouds
		4.6	9.5	Through breaks in clouds. With Dr. Lee
		4.2	8.7	Through breaks in clouds
		4.7	9.7	Through breaks in clouds
		4.5	9.3	Through breaks in clouds
Sept. 16		5.2	10.8	
Sept. 17		5.2	10.8	
		4.7	9.7	Clouds at end
Sept. 30		4.6	9.5	Hazy
		4.5	9.3	Hazy
		4.6	9.5	Hazy
Oct. 1		4.4	9.1	Very hazy
		4.7	9.7	With Professor Barnard
Sept. 15	D_3	2.8	5.8	Thin clouds
		2.5	5.2	Thin clouds
Sept. 16		4.1	8.5	Fairly transparent. Cloud intervals
		3.9	8.1	Fairly transparent. Cloud intervals
Sept. 30		2.5	5.2	Very thick, high cirrus clouds
Oct. 1		3.0	6.3	Thick
		3.2	6.6	Thick

reversals for certain other lines of the spectrum were also seen, but under the unfavorable conditions of the sky it seemed useless to try to use them for measurement. The results which are tabulated above must be considered as of preliminary character. It is hoped to extend the series of observations at a later date.

I wish to acknowledge with thanks the assistance given in this investigation by Professors Curtis, Frost, and Crew.

Dearborn Observatory
October 9, 1922

DISPERSION OF LIGHT BY AN ELECTRON GAS

By LEIGH PAGE

ABSTRACT

Electro-magnetic theory of the dispersion of an electron gas is deduced. The phase velocity in such a gas is found to be a function of the wave-length slightly greater than the velocity of light in a vacuum. From Shapley's estimate of a difference between the velocities of blue and yellow light not greater than 0.1 meter per sec., an upper limit to the *number of electrons per cc in interplanetary space* can be computed. This number is $3(10)^{11}$ for electrons or $5(10)^{14}$ for protons. The expressions obtained for the electric and magnetic vectors are checked both by the law of conservation of momentum and that of energy.

From the agreement in phase of the blue and yellow light curves of distant stars Shapley[1] has drawn the conclusion that the velocity of blue light through space cannot differ from that of yellow light by as much as one-tenth of a meter per second. While it is not likely that any considerable amount of matter in the atomic or molecular state is present in interstellar space, it may well happen that certain regions are traversed by swarms of electrons or protons. Hence it seems of interest to investigate whether the laws of electromagnetism lead to dispersion of light passing through an electron gas. For the sake of simplicity the electron gas will be supposed to be of uniform density, and the light will be assumed to be plane polarized. In connection with the exponentials employed, the real part alone will be understood to be taken. The electromagnetic quantities are expressed in Heaviside-Lorentz rational units.

Consider a train of plane waves traveling in the X direction. Let the electric vector have the Y direction. Then, at any one point,

$$E_y = E_0 e^{-i\omega t} .$$

Suppose an electron to be present at this point. If v denotes the velocity of the electron in the Y direction due to the alternating electric field of the wave, the equation of motion of the electron is

$$N\frac{d^2v}{dt^2} - M\frac{dv}{dt} = -E_0 e^{-i\omega t} ,$$

[1] *Bulletin Harvard Coll. Observatory*, No. 763, Jan. 1922.

where

$$M \equiv \frac{m}{e}, \qquad N \equiv \frac{e}{6\pi c^3},$$

m and e designating the mass and charge respectively.

Hence, as $N\omega$ is small compared to M,

$$v = \frac{iE_0}{M\omega} e^{-i(\omega t + \delta)},$$

where

$$\delta \equiv \tan^{-1} \frac{N\omega}{M}.$$

If n denotes the number of electrons per unit volume, the density of charge ρ is equal to ne, and

$$\rho v = \frac{ineE_0}{M\omega} e^{-i(\omega t + \delta)}$$

$$= -\frac{ne}{M\omega^2} e^{-i\delta} \dot{E}. \tag{1}$$

Now, the equations of the electromagnetic field are

$$\nabla \cdot \boldsymbol{E} = \rho, \tag{2}$$

$$\nabla \cdot \boldsymbol{H} = 0, \tag{4}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c} \dot{\boldsymbol{H}}, \tag{3}$$

$$\nabla \times \boldsymbol{H} = \frac{1}{c} (\dot{\boldsymbol{E}} + \rho \boldsymbol{v}). \tag{5}$$

Substituting in (5) the value of $\rho \boldsymbol{v}$ given by (1), this equation becomes

$$\nabla \times \boldsymbol{H} = \frac{1}{c} \left(1 - \frac{ne}{M\omega^2} e^{-i\delta} \right) \dot{\boldsymbol{E}}. \tag{5'}$$

Eliminating $\boldsymbol{H}$ by the usual method, taking note of the fact that ρ is constant and hence

$$\nabla \nabla \cdot \boldsymbol{E} = \nabla \rho = 0,$$

it is found that

$$\nabla \cdot \nabla \boldsymbol{E} - \frac{1}{c^2} \left(1 - \frac{ne}{M\omega^2} e^{-i\delta} \right) \ddot{\boldsymbol{E}} = 0,$$

and an exactly similar equation in $\boldsymbol{H}$ is obtained by eliminating $\boldsymbol{E}$. These are the equations of a wave traveling with wave slowness

$$S=\frac{1}{c}\left(1-\frac{ne}{M\omega^2}e^{-i\delta}\right)^{\frac{1}{2}}.$$

Put the complex wave slowness equal to $S'+iS''$, where S' and S'' are real. Then, provided $\frac{ne}{M\omega^2}$ is small compared to unity,

$$\begin{aligned} q&=\frac{1}{S'} \\ &=c\left(1+\frac{ne^2}{2m\omega^2}\right) \end{aligned} \tag{6}$$

is the velocity of propagation, and

$$\omega S''=\frac{neN}{cM^2} \tag{7}$$

is the coefficient of absorption.

Equation (6) shows that the phase velocity is greater than the velocity of light *in vacuo* in much the same manner as in the case of a dielectric in the region of anomalous dispersion. Written in terms of wave-length this equation has the form

$$q=c\left(1+\frac{ne^2}{8\pi^2mc^2}\lambda^2\right).$$

If now, q_B represents the velocity of blue light, and q_Y that of yellow light,

$$q_Y-q_B=\frac{ne^2}{8\pi^2mc}(\lambda_Y^2-\lambda_B^2).$$

Taking 0.1 meter/sec. as the greatest allowable value of the difference of the two velocities, an upper limit to the number n of electrons per cubic centimeter may be calculated. This number is $3(10)^{11}$ per cubic centimeter for electrons, and $5(10)^{14}$ for protons, the latter having a smaller dispersive effect on account of their greater mass.

The electric and magnetic field strengths in the wave are

$$E_y = Ae^{-\omega S'' x} e^{i\omega(S'x-t)} ,$$

and

$$H_z = Be^{-\omega S'' x} e^{i[\omega(S'x-t)+\epsilon]} ,$$

where

$$B \equiv Ac\sqrt{S'^2+S''^2} ,$$

$$\epsilon \equiv \tan^{-1} \frac{S''}{S'} .$$

These expressions may be checked by both the law of conservation of momentum and that of energy. Consider a layer of the electron gas of unit cross section at right angles to the direction of propagation of the light and of thickness δx. The loss in momentum of the radiation passing through this layer in a unit time is

$$\tfrac{1}{2}AB - \tfrac{1}{2}AB(1-2\omega S''\delta x)$$

$$\doteq \tfrac{1}{2}A^2 \frac{\rho}{c} \frac{N}{M^2} \delta x .$$

Dividing this by the number $n\delta x$ of electrons in the layer, the loss in momentum per unit time per electron is

$$\tfrac{1}{2}A^2 \frac{e}{c} \frac{N}{M^2} .$$

But this is just the radiation pressure on an electron.[1]

The energy lost by the waves which pass through this layer in a unit time is

$$\frac{c}{2} AB - \frac{c}{2} AB(1-2\omega S''\delta x)$$

$$\doteq \tfrac{1}{2}A^2 \rho \frac{N}{M^2} \delta x$$

or, dividing by the number of electrons, the energy lost to each electron is

$$\tfrac{1}{2}A^2 e \frac{N}{M^2} .$$

[1] Leigh Page, *Astrophysical Journal*, 52, 67, 1920.

Now, the energy radiated away by each electron in the course of its oscillations under the influence of the electric field in the wave is

$$\frac{e^2}{6\pi c^3}\left(\frac{dv}{dt}\right)^2$$

$$=\tfrac{1}{2}A^2e\frac{N}{M^2}$$

per unit time.

The energy transferred to the electrons through the agency of light pressure is of a smaller order of magnitude and does not appear on account of the approximations made.

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February 1923

ON THE SPECTRUM OF NEUTRAL HELIUM

By C. V. RAMAN and A. S. GANESAN

ABSTRACT

Silberstein's formula for the lines of neutral helium.—As objections to the validity of the formula $\nu=4N\left(\frac{1}{n_1^2}-\frac{1}{m_1^2}+\frac{1}{n_2^2}-\frac{1}{m_2^2}\right)$ it is pointed out that the ionizing potential computed by means of this formula does not agree with that given by experiments; that no definite principle of selection has been given to indicate observable lines; that there is no arrangement of lines as regards series relationship, division into singlet and doublet systems, or of intensities. The difference between the actual and the calculated frequencies is shown to be the same as that to be expected on the assumption that the coincidences are purely accidental; and the same frequencies can be obtained with different sets of numbers.

In his paper on this subject contributed to the September number of the *Astrophysical Journal* (56, 119, 1922), Dr. Silberstein has suggested a formula for the explanation of the spectrum of neutral helium. This formula may be written as follows:

$$\nu=4N\left\{\left(\frac{1}{n_1^2}-\frac{1}{m_1^2}\right)+\left(\frac{1}{n_2^2}-\frac{1}{m_2^2}\right)\right\},$$

where n_1, n_2, m_1, m_2 are four independent integers such that $m_1>n_1$ and $m_2>n_2$. "This formula amounts, obviously, to putting $\nu=\nu_1+\nu_2$ where ν_1 and ν_2 are frequencies of any two lines of ionized helium, observed or only theoretical. This would then be a new kind of combination principle, the sum of the frequencies belonging to one atomic system (ionized helium) giving the frequency for another system (neutral helium)." With this formula Dr. Silberstein has succeeded in explaining 84 of the total 111 observed lines of the helium spectrum, to a very close approximation.

Now, the question is whether the formula has any physical basis or significance. Silberstein derives it on the assumption that "the mutual perturbation of the two electrons is practically nil or negligible," and in support of this says that "as a matter of fact we have no good evidence that the electrons, especially as trabants of the nucleus within the atom, do act upon each other at all, and

some bold modern physicist, encouraged by the recognized prohibition to radiate, might deny the electrons the right to interact while they are busy obeying the orders of the central body driving them around on stationary orbits."

But there are some very weighty objections against the formula and the results derived therefrom.

The ionization potential as calculated with this formula is quite different from that which is actually observed. The ionization potential is the work per unit charge required in moving an electron out of the influence of the atom. On the assumption that the two electrons do not act on each other, the ionization potential would obviously be

$$V=\frac{4\times 109723.2}{8102}=54.2 \text{ volts.}$$

But actually it is known that to doubly ionize the helium atom about 80 volts are necessary.

The formula, as it stands, represents a very large number of lines and there is absolutely no principle of selection followed which prima facie indicates the lines to be chosen and those to be rejected. As a matter of fact, the selection actually made appears to be more or less haphazard, the only aim being to choose such numbers as happen to give values coinciding most closely with the observed values. The want of a definite principle of selection is a strong objection against accepting the proposed formula as having any physical significance.

Again, as is well known, the observed lines of the helium spectrum fall into certain definite series which Silberstein dismisses as empirical, but which nevertheless have a definite physical meaning. Thus in each series as we pass from the less to the more refrangible part of the spectrum the lines become more and more crowded, grow fainter and fainter, and tend to converge to a definite limit. But Silberstein's formula shows no arrangement either with reference to series relationships or with reference to singlet and doublet systems of lines.

Nor does it suggest any arrangement with regard to the intensity of the lines. Bohr's theory, which has successfully explained the

spectra of hydrogen and ionized helium, suggests that the probability of electrons remaining in distant orbits is very small and that more electrons are to be found in orbits closer to the nucleus. And from the analogy with the spectra of hydrogen and ionized helium, we should expect that more intense lines are obtained by using comparatively smaller quantum numbers than for the less intense ones. In Silberstein's table of values some of the first lines of great intensities are not to be seen at all (probably no combination within the limits prescribed for m and n gives them), and of the rest, more frequently than not, the more intense lines are obtained by larger quantum numbers than the less intense ones. Table I gives some examples.

TABLE I

Observed ν	Intensity	Quantum Numbers, and Calculated ν
9231.86, 9230.83	200 (P.d.)	Not included
25708.63	10 "	Not included
31361.12	8 "	$\left(\frac{16}{4} \cdot \frac{19}{8}\right)$ 31358.
33944.75	6 "	$\left(\frac{15}{4} \cdot \frac{30}{7}\right)$ 33950.
37537.5	1 "	$\left(\frac{4}{3} \cdot \frac{18}{5}\right)$ 37536.
37798.22	"	$\left(\frac{4}{3} \cdot \frac{20}{5}\right)$ 37794.
41150	5 (S.d.)	Not included
21211.35	3 "	$\left(\frac{6}{5} \cdot \frac{16}{5}\right)$ 21205.5
27374.48	1 "	$\left(\frac{6}{4} \cdot \frac{9}{5}\right)$ 27377.
4857.34	20 (S.s.)	$\left(\frac{10}{7} \cdot \frac{15}{4}\right)$ 4856.7
22528.65	1 "	$\left(\frac{7}{5} \cdot \frac{11}{5}\right)$ 22527.4

Many of the lines can be obtained by numbers other than those chosen by Silberstein (confined, of course, between his limits

$m \ngtr 32$ and $n \ngtr 9$), and in some cases these agree more closely with the observed values than those given by Silberstein. Thus to give a few examples:

TABLE II

Observed	Silberstein's Values		Other Values	
24830...........	$\left(\frac{13}{4} \cdot \frac{n}{n}\right)$	24834.	$\left(\frac{10}{11} \cdot \frac{11}{9}\right)$	24833.
26938...........	$\left(\frac{30}{4} \cdot \frac{n}{n}\right)$	26943.	$\left(\frac{16}{5} \cdot \frac{20}{6}\right)$	26935.4
28574...........	$\left(\frac{4}{3} \cdot \frac{16}{7}\right)$	28577.	$\left(\frac{11}{4} \cdot \frac{26}{9}\right)$	28572.8
36208...........	$\left(\frac{5}{4} \cdot \frac{20}{4}\right)$	36208.7	$\left(\frac{10}{4} \cdot \frac{10}{5}\right)$	36208.7
36592...........	$\left(\frac{5}{4} \cdot \frac{25}{4}\right)$	36603.	$\left(\frac{17}{4} \cdot \frac{17}{6}\right)$	36585.
			$\left(\frac{13}{4} \cdot \frac{32}{6}\right)$	36596.7
19824...........	$\left(\frac{5}{4} \cdot \frac{14}{6}\right)$	19827.	$\left(\frac{6}{\text{[illegible]}} \cdot \frac{23}{9}\right)$	19828.
			$\left(\frac{14}{5} \cdot \frac{22}{9}\right)$	19828.
			$\left(\frac{28}{6} \cdot \frac{24}{7}\right)$	19827.
20358...........	$\left(\frac{5}{4} \cdot \frac{16}{6}\right)$	20352.	$\left(\frac{17}{5} \cdot \frac{20}{9}\right)$	20358.3
			$\left(\frac{11}{5} \cdot \frac{32}{8}\right)$	20357.6
27508...........	$\left(\frac{9}{4} \cdot \frac{18}{8}\right)$	27516.	$\left(\frac{11}{4} \cdot \frac{16}{9}\right)$	27507.6

The average difference between the actual frequencies of the helium lines and the values nearest to them given by Silberstein's formula is exactly what we should expect from the simple theory of probability on assuming the coincidences to be fortuitous. Thus for instance, if $m \ngtr 32$ and $n \ngtr 9$, the total number of lines obtained with Silberstein's formula, lying between $\nu = 28000$ and $\nu = 29000$,

is found to be 161. If these lines were evenly distributed between these limits, the interval between successive lines would be $\frac{1000}{161} = 6.1$ and the maximum difference of any value of ν between 28000 and 29000, chosen at random, and the nearest number in the list would be 3.0. The minimum error being 0, the average error would be 1.5. Taking the 19 observed lines of helium between these limits and finding for each the nearest number from the list, the average actual error is found to be 2.6; that is, actually greater than the number indicated on the assumption that the coincidences are purely fortuitous. This is what we should expect; for, owing to the non-even distribution of the numbers, there are certain gaps with differences of about 20 to 25, and some 3 of the 19 lines falling in these gaps eventually increase the average error. From Silberstein's table the average error for the 15 lines which he has given between these limits is about 3.5. This is due to the fact that there are combinations other than Silberstein's that are nearer the observed values.

These facts clearly show that coincidences between the actual frequencies and those given by Silberstein's formula must all be regarded as purely fortuitous and having no physical significance whatever.

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ON THE SPECTRUM OF HELIUM, A REJOINDER TO C. V. RAMAN AND A. S. GANESAN

By LUDWIK SILBERSTEIN

ABSTRACT

Consideration of objections to the Silberstein formula for the spectrum of neutral helium. (1) The author disclaims ignoring the fact that the observed lines fall into certain *definite series*, and points out that the formula covers twenty-three of the thirty so-called combination lines. (2) A discussion of the *ionization potential* was purposely omitted from his original paper since the mutual relation of the electrons in the normal state of the atom is still an open question. He has preferred to deal with it in a separate paper. (3) In order to show that *the coincidences of lines are not fortuitous* he computes that for $3 \leqq n \leqq 8$ and $4 \leqq m \leqq 20$ the theoretical number of lines in the interval $19{,}800 \leqq \nu \leqq 37{,}800$ is 631. Of these, 44 coincide with observed lines, of which there are 96 in this interval. The probability that these coincidences are accidental is of the order 10^{-13}.

The substance of the objections raised against the spectrum formula

$$\nu = 4N\left(\frac{1}{n_1^2}+\frac{1}{n_2^2}-\frac{1}{m_1^2}-\frac{1}{m_2^2}\right) \equiv 4N\left(\frac{m_1}{n_1}\cdot\frac{m_2}{n_2}\right) \tag{1}$$

by Raman and Ganesan in this issue of the *Astrophysical Journal* has already been given in Professor Raman's letter published in *Nature* of November 25 (**110**, 700, 1922), to which I replied at some length in *Nature* of January 13 (**111**, 46, 1923). The full paper, moreover, upon which the last said letter was based, was read at the Boston meeting of the American Physical Society and is now in the course of publication in the *Philosophical Magazine.* It contains all numerical details relevant to the question. Under these circumstances it will be enough to make here but a few concise remarks, the more so, as my reply in *Nature* could not possibly have reached Calcutta before Raman and Ganesan wrote their note which arrived at Chicago in the middle of January.

In the first place then I have never "dismissed as empirical" the fact that the observed lines "fall into certain definite series." On the contrary, I have emphatically expressed the regret for seeing the series *lacerated* through the proposed representation. At the same time, however, I drew attention to some regularities.

I should like to add that with the thirty so-called "combination lines," of which my formula covers twenty-three, no such claims or objections can be associated. Many of these lines can scarcely be said to be represented at all by the empirical "combinations," eight of these showing deviations of 2 to 3 and one even of 7.2 A (cf. Fowler's *Report*, p. 94), whereas they are covered by my formula with particular ease, as for instance the three *C.d.* lines, all represented by $\left(\frac{5 \cdot m_2}{4 \cdot 4}\right)$.

With regard to the ionization potential of the (normal) helium atom, I have purposely abstained from dealing with it, since there was no occasion of descending to one-quantic orbits, the majority of my cases concerning $n \geqq 4$. Essentially relevant for the ionization work is the behavior of the intra-atomic electrons within the region of the few first quantum numbers, so that the actual applications made of my formula do not by any means exclude a treatment of the ionization problem based on appropriate electronic interaction. The configuration of the electrons in the normal helium atom, even on Coulomb lines, is still an open question (and a very hard one), and may here be left alone. I deal with it in another paper which, however, has nothing in common with the present discussion. In other words, a formula may be successful in the region $\nu = 17{,}000$ to 38,000, without pretending to cover the extreme ultra-violet Lyman series, of which λ 504 or 506 A (the equivalent of the ionization potential of 24.5 volts) is the limit.

The most crushing contention of the opponents of my formula, however, and the easiest to repel, seems to be, that the coincidences given by it "must all be regarded as purely fortuitous" and therefore as "having no physical significance whatever." The latter verdict of Raman and Ganesan had better be left alone, as it would scarcely be feasible to discuss here the subtle concept of "physical significance" and the still harder question, whether a given set of conceptual results does or does not possess it. It is enough to repel the imputation of fortuitousness, and since this has been most particularly dealt with in the aforesaid letter and paper, it will suffice to point out the reason for the discrepancy of opinion held in this respect by the writer and by his two opponents. Raman

and Ganesan obtain with my formula such large numbers of theoretical lines (as 161 between $\nu=28{,}000$ and 29,000) and thence also such a high estimate of the probability of chance coincidences, because they insist, very unjustly, upon pushing m as high as 32 (and n up to 9), although such high numbers were purposely avoided, the table in my first paper containing "32" only once, and "31" not at all. The significant fact, and one that speaks very strongly in favor of the proposed formula, is that if we limit the quantum numbers to the intervals

$$3 \leqq n \leqq 8 \qquad \text{and} \qquad 4 \leqq m \leqq 20, \tag{2}$$

the number of theoretical lines is cut down very considerably and yet a large number of good coincidences still remains. One such example is mentioned, and estimated for its probability, in *Nature* of January 13. Another, still more convincing example (sacrificing only one out of 45 lines) is this:

The number of distinct theoretical lines satisfying the conditions (2) and falling within the interval

$$\nu = 19{,}800 \text{ to } 37{,}800 \tag{3}$$

is 631 only. Of these as many as 44 coincide with observed lines, whose total in the interval (3) is 96. The mean deviation is $|\delta\nu| = 2.57$. Now, a straightforward computation will show that the probability of such an event considered as a fortuitous set of coincidences is, in round figures,

$$P_{44} = 1 - \Theta(7), \tag{4}$$

where Θ is the error-function. This formula speaks for itself. In fact, for $x=4.80$, the largest argument one has ever encountered in error-function tables, $1-\Theta(x)$ is just of the order 10^{-11}. For $x=5.30$ it is about 20 times smaller, and for $x=7$, as in our case, it is well below 10^{-13}. In fine, the "fortuitous" hitting of these 44 lines under the said circumstances is as improbable as making forty "heads" in succession in tossing a coin.

Rochester, N. Y.
March 31, 1923

SPECTROSCOPIC OBSERVATIONS OF BOTH COMPONENTS OF THE VARIABLE DOUBLE STAR X OPHIUCHI[1]

By PAUL W. MERRILL

ABSTRACT

Spectra and magnitudes of both components of X Ophiuchi.—This double star of separation 0″.22 consists of a typical long-period variable of class M6e of apparent mag. 6.8 to about 12, and a star of class K0 of mag. 8.9. The former can be observed spectroscopically at maximum, the latter at minimum. Twenty spectrograms have been secured of which six show the K-type spectrum. The absolute magnitude of the K-type star as determined from these spectrograms by Mr. Adams is 2.4; from this it follows that the absolute magnitude of the variable at maximum is 0.3.

Radial velocity of both components.—The radial velocities derived from the dark lines of the M- and K-type components are −70.6 and −70.8 km, respectively, while the bright lines of the M star yield a mean value of −83.4 km. It is probable that the displacements of the dark lines of the variable and of Me stars in general give essentially the star's radial velocity. Near maximum the wave-lengths of the bright lines are shortened by about 0.2 A, by motions within the star or by other causes, and for several weeks after maximum are algebraically smaller than at other times. This effect is probably typical of variables of class Me.

X Ophiuchi presents the only known instance in which a long-period variable is one component of a visual double star. The duplicity of this star was discovered by Hussey[2] in 1900. His observations of position angle and distance were 195°.2 and 0″.22, respectively. Remeasurement by Van Biesbroeck[3] twenty years later showed the position angle to have decreased by 24°, the separation being the same. He decided that the northern component is the variable, and Gingrich[4] has confirmed this by his measurements of parallax photographs.

It will obviously be many years before the orbital elements are accurately known, but as a first approximation we may assume a circular orbit whose plane is perpendicular to the line of sight. The radius of the relative orbit would then be 0″.22, and the period 300 years. Using the spectroscopic parallax,[5] 0″.005, determined by Mr. Adams (see p. 252), the distance between the stars is calculated to be forty-four astronomical units, and the combined mass 0.95 that

[1] *Contributions from the Mount Wilson Observatory*, No. 261.

[2] *Astronomical Journal*, 21, 35, 1900.

[3] *Popular Astronomy*, 29, 278, 1921.

[4] *Mt. Wilson Contr.*, No. 238; *Astrophysical Journal*, 56, 132, 1922.

[5] Measurement of the trigonometric parallax by Van Maanen yielded the value 0″.000. *Publications Astronomical Society of the Pacific*, 33, 319, 1921.

of the sun. If each of the stars has one-half of the combined mass, the orbital speed of each with respect to the center of mass is 2.2 km. These values are, of course, subject to large corrections.

The period of the light variations is 337 days, which is about the average for long-period variables. The mean range in brightness is, however, only 2.1 mag., which is considerably less than usual for variables of this type; but this refers to the combined magnitude of the two components. The range of the variable component is considerably greater, probably about 5 magnitudes. The spectrum at maximum is class M6e, but at minimum this spectrum is overpowered by that of the southern component, which is of class Ko. Six spectrograms (see Table II), taken near minimum, show the Ko spectrum fairly free from interference by that of the variable star. From these Mr. Adams has had the kindness to determine the absolute magnitude, finding it to be +2.4. The apparent magnitude is 8.9, and the spectroscopic parallax 0."005. The available photometric data may be combined to yield the results indicated in Table I.

TABLE I

MAGNITUDES OF THE COMPONENTS OF X OPHIUCHI

COMPONENT	MAXIMUM			MINIMUM		
	Vis.	Ph.	Abs. Vis.	Vis.	Ph.	Abs. Vis.
Combined..........	6.7	8.3	0.2	8.8	9.9	2.3
Northern...........	6.8	8.6	0.3	12	14	6
Southern...........	8.9	9.9	2.4	8.9	9.9	2.4

Thus it happens that at maximum the K-type component contributes only one-tenth of a magnitude to the combined light, while at minimum it preponderates so strongly that this relationship is reversed and the *variable* contributes only about the same proportion. When the combined magnitude is brighter than 7.5, the M-type spectrum is recorded in the $H\gamma$ region with very little interference from the other component. On the other hand, when the combined light is fainter than 8.0 mag., the K-type spectrum appears quite pure in this region, although near $H\beta$ the bands of the M-type star show plainly until magnitude 8.6 or 8.8 is reached. Assuming color indices of 1.0 and 1.8 mag.[1] for the K- and M-type stars, respec-

[1] A mean value used by the Harvard College Observatory for Me variables. It may vary during the light-cycle.

tively, as in Table I, we find that at combined magnitude 7.5 the M-type star is 1.1 mag. brighter visually, and 0.3 mag. photographically than the K-type component; when the combined magnitude is 8.0, the M-star is 0.3 brighter visually and 0.5 fainter photographically than the other. Thus it appears that the spectrum of either star may be obtained practically pure, if it is a half-magnitude brighter photographically than the other.

This star offers a unique opportunity to derive a value of the absolute magnitude of a long-period variable by a direct method, through comparison with the K-type component whose absolute magnitude can be determined spectroscopically in the usual way. The distances of these variables are, in general, so great that trigonometric parallaxes can scarcely be used except in a statistical manner. The high absolute magnitude, 0.3 at maximum, found for the variable component of X Ophiuchi, is close to the mean value for giant M stars; and since the variable component appears to be a typical long-period variable, the presumption is that long-period variables in general are giant stars. This is borne out by the fact that the mean absolute magnitude of these stars derived by several statistical investigations is about the same as that found for X Ophiuchi. Discussions of the causes of variability and of the evolutionary relationships of these stars must take into account their high luminosity at maximum.

Observations at the Harvard College Observatory,[1] at Ann Arbor,[2] and at Mount Wilson have shown the spectrum to be of class M at maximum. In the spectral classes M0–M10, which are to replace the subdivisions Ma–Mc,[3] the class of X Ophiuchi would be M6e. Photographs of the spectrum have been secured here at times fairly well distributed over the light-curve. The spectrum shows striking changes, aside from the effect of the K-type component, which correspond to those regularly exhibited by long-period variables.[4] Hence the northern component of this double star may be regarded as a typical long-period variable.

[1] Henry Draper Catalogue, *Harvard Annals*, **97**, 258, 1922.

[2] *Publications of the Observatory, University of Michigan*, **2**, 58, 1916.

[3] *Transactions of the International Astronomical Union*, **1**, 97, 1922.

[4] *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 185, 1921.

The Mount Wilson spectrograms are listed in Table II. The first three columns give, respectively, the date, visual magnitude, and the number of days before (−), or after (+) maximum. I am indebted to Mr. Leon Campbell, of the Harvard College Observatory, for the photometric data. The fourth and fifth columns give the velocities derived from the absorption lines of the M- and K-type components, respectively, with the number of lines on which each determination is based. The last column gives the velocity derived from the emission lines, and the lines used on each plate.

TABLE II

RADIAL VELOCITIES OF X OPHIUCHI

Date	Mag.	Phase	Velocity			
			M	K	E	
1919 June 9*	6.6	−11	....	....	−87.7 γδ	
June 10*	6.6	−10	....	....	−84.7 γδ	
1920 Apr. 8	7.3	−53	....	....	−73.9 γδ	
May 1	6.9	−30	−69.5 12	....	−76.9 γδ	
May 4*	6.8	−27	−68.1 12	....	−78.6 γδ	
May 31	6.6	0	−69.9 34	....	−83.2 γδ	4202
July 5*	7.0	+35	....	....	−86.9 γδ	4202
July 6	7.0	+36	−70.0 7	....	−83.1 γδ	4202
Sept. 2	8.3	+94	....	−69.3 10	−78.6 γδ	4202, 4308, 4571
Sept. 27	8.8	+119	....	....	−75.2	4202, 4308, 4571
1921 Apr. 26	7.0	−20	−72.7 10	....	−83.5 γδ	
Apr. 26†	7.0	−20	....	....	−84.6 γδ	
May 24*	6.6	+8	−70.4 19	....	−89.3 γδ	
June 22*	7.3	+37	−73.3 24	....	−88.0 γδ	4202
Aug. 11	8.2	+87	....	−72.9 17	−89.6 γδ	4202, 4308, 4571
Sept. 21	8.2	+123	....	−73.2 18	....	....
Oct. 11	8.4	+143	....	−73.7 18	....	....
1922 June 14*	7.6	+60	....	....	−86.0 γδ	4202, 4571
Aug. 9	8.6	+116	....	−64.6 14	(−65.)	4202, 4308, 4571
Sept. 8	8.7	+146	....	−71.2 13	(−82.)	4308
Mean	......	......	−70.6	−70.8	−83.4	

* Sixty-inch telescope. † Three-prism spectrograph.

With a single exception the plates were made with one-prism spectrographs attached to the 60-inch or the 100-inch telescope; the dispersion at Hγ is 36 A per mm. The second plate, of April 26, 1921, was secured with a three-prism spectrograph belonging to the Lowell Observatory. This was being used for a special investigation in connection with the Hooker telescope by Dr. V. M. Slipher, and it was through his courtesy that the exposure on X Ophiuchi was made. The dispersion at Hγ with this instrument is 12.5 A per mm. On this plate the bright Hγ and Hδ lines are narrow and fairly well defined, but they are not quite so sharp as

the comparison lines of the iron arc. The measured velocity agrees with that derived from the one-prism spectrograms, and gives a desirable check on their accuracy. Measurements of the emission lines on the last two plates were rendered uncertain by the presence of the strong K-type spectrum. The values derived from them were not used in forming the mean.

The discrepancy of 12.8 km between the apparent velocities from the bright and dark lines of the M-type star corresponds in sign and magnitude to similar displacements observed in the spectrum of *o* Ceti and of many other Me variables. It is of great interest to find that the velocity from the K-type star is practically identical with that from the absorption lines of the M-type star. Since the orbital speed is probably about 2.2 km and the inclination small, the relative radial velocity of the two components must be very nearly zero. Hence we are led to the conclusion that the result derived from the dark lines of the M-type spectrum represents essentially the radial velocity of the star, and that the bright lines are displaced toward the violet by some unknown cause. It is probably safe to generalize this statement and make it apply to all variable stars of class Me in which the usual relative displacement of bright and dark lines exists. Support is lent to this assumption by the fact that, in a solution for solar motion based on the radial velocities derived from the bright lines of 83 variables, the K-term was found to be -12 km.[1]

The apparent velocities from the emission lines of X Ophiuchi are plotted against the light phase in Figure 1. The observations extend in phase from -53 days to $+119$ days, covering about one-half of the period. The velocities are not constant, but appear to have low values for a month or two after maximum light. Of the Ann Arbor spectrograms, one taken in 1914 at a phase of $+24$ days gave a velocity of -91 km, and three others taken in 1915 gave a normal place of -84.6 km at -69 days. Thus they exhibit the same effect as the Mount Wilson spectrograms, although to a lesser degree. A close quantitative agreement is perhaps not to be expected, especially as the record shows that the Ann Arbor velocities have rather low weight. In view of the lack of regularity in the light-curve, it would not be surprising if different cycles

[1] *Popular Astronomy*, 29, 637, 1921.

yielded somewhat different velocity effects. Three other stars, namely, R Leonis, χ Cygni, and T Cephei, seem to exhibit the same effect, and fragmentary observations indicate about the same behavior by a half-dozen other variables. It thus seems to be typical of Me variables that the bright lines have slightly shorter effective

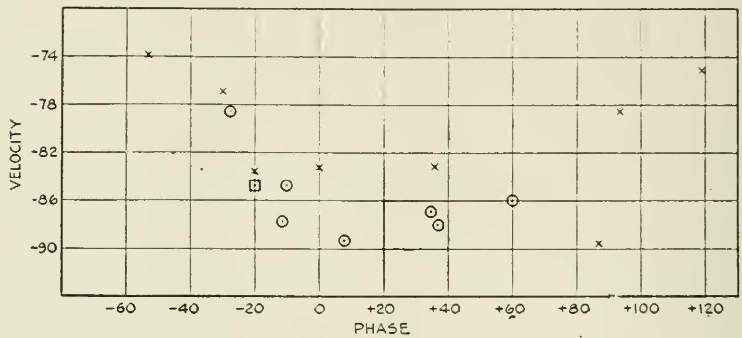

FIG. 1.—Phase in days counted from maximum light, and apparent velocities (km per sec.) derived from emission lines. ×, single-prism spectrograph attached to the 100-inch telescope; ⊙, single-prism spectrograph attached to the 60-inch telescope; □, three-prism spectrograph attached to the 100-inch telescope.

wave-lengths during several weeks after maximum than at other times.[1] This is about the time of the maximum relative intensity of the hydrogen lines.[2]

If the displacements of the bright hydrogen lines are due to an outflow of incandescent hydrogen, it follows that on first coming into view after minimum the gas has a low velocity, and is subject to acceleration while under observation, or is replaced by other hydrogen having a higher outward velocity. This would indicate the existence of agencies acting during an appreciable fraction of the light-period, rather than a sudden outburst followed by a gradual resumption of normal conditions.

MOUNT WILSON OBSERVATORY
March 1923

[1] Errors of measurement and the inclusion, during the declining phase, of other bright lines than those due to hydrogen do not seem adequate to explain the observed effects. These questions will, however, be considered in another contribution.

[2] *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 185, 1921.

THE

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THE CROSSED-ORBIT MODEL OF HELIUM, ITS IONIZATION POTENTIAL, AND THE LYMAN SERIES

By LUDWIK SILBERSTEIN

ABSTRACT

The energy formula for crossed-orbit model of the normal helium atom.—From simple dynamical considerations it is shown that the energy of the model, with the two orbits assumed as rigorously circular, is given by the formula, $E = -7N_{\infty} ch \left[1 - \frac{1}{4\pi} F\left(\frac{i}{2}\right) \right]$, where i is the mutual inclination of the planes of the two orbits and F is the complete elliptic integral of the first kind. From this general formula it is found that for Bohr's model, in which $i = 120°$, the *ionizing potential* is 24.35 volts, as compared with 24.5 volts recently obtained by Lyman. By means of the *formula for the wave-lengths* of the emitted radiation, $\nu = N\left[3 - \frac{7}{4\pi} F\left(\frac{i}{2}\right) \right]$, it is found that the simple rational values, $-\cos i = \frac{5}{9}, \frac{3}{5}, \frac{2}{3}, \frac{4}{5}$, correspond to the lines, $\lambda_1 = 584.4$, $\lambda_2 = 537.1$, $\lambda_3 = 522.3$, and $\lambda_4 = 515.7$, of *the Lyman series*. The author disclaims any responsibility for the dynamical legitimacy of the model. In an appended note two more lines are shown to be covered by $-\cos i = \frac{5}{6}$ and $\frac{7}{13}$, and a regularity of the whole array of fractions is pointed out.

The model of the normal helium atom proposed by Bohr in his Fysisk Forening lecture[1] consists of two electrons describing around the nucleus two equal one-quantic orbits (1_1) which are quasi-circular and whose planes are mutually inclined at $i = 120°$, and are themselves spinning "slowly" around the permanent axis of angular momentum of the whole system. Bohr states (p. 33) that the

[1] Translated in *Zeitschrift für Physik*, **9**, 1–67, 1922.

analysis of the said configuration, conducted with the aid of Kramers, requires a large amount of computation which had thus far led to no conclusive results, although it seems promising with regard to a correct ionization potential. More than a year has now elapsed without any further announcements to that effect being published by Bohr or Kramers. In the meantime, J. H. van Vleck has announced in a note to his paper[1] that "calculation has given an ionization potential of 20.7 volts," for the aforesaid model, that is. Yet, the details of computation leading to this disappointing potential value not being very transparent, one did not feel ultimately convinced as to the fate of this latest model, a successor to three or four luckless ones. Under these circumstances it has seemed worth while to attempt an independent evaluation by a straightforward method which recently suggested itself. Its publication in the present paper seems the more justified, as the result arrived at was surprisingly close to Lyman's latest estimate, which, as I am informed by Professor Lyman, is one volt lower than that usually quoted, and amounts ultimately to 24.5 volts.

It will be expressly understood, however, that the dynamical possibility of the model, as satisfying permanently and with sufficient accuracy the equations of motion, will be entirely left to the judgment of others. In fine, the legitimacy of the model itself, with its sufficiently "quasi-circular" orbits, being taken for granted, I propose merely to compute its energy (with the usual Coulomb law of interaction between all the three bodies), and thence the required ionization potential.

By a well-known theorem, due to Burgers, the average kinetic energy of the system is equal to minus one-half its average potential energy, and since the total energy, E, remains constant, we have

$$\dot{E}=\bar{E}=\tfrac{1}{2}\bar{E}_{pot},$$

and, if a be the radius of either orbit, and $\rho=1/r$ the reciprocal of the mutual distance of the electrons at any instant,

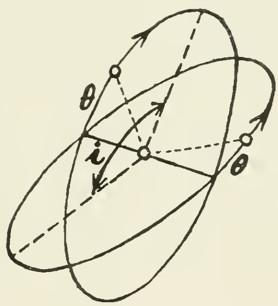
$$E=-\frac{e^2}{a}[2-\tfrac{1}{2}a\bar{\rho}].$$

[1] *Philosophical Magazine*, 44, 869, 1922.

The configuration being symmetrical by assumption, the two trabants will pass the nodes simultaneously, being then on the opposite ends of a diameter. Thus, if the "slow" spinning of the orbits themselves (as announced by Bohr, *loc. cit.*) is slow enough compared with the orbital angular velocity of the electrons, the approximate value of the radius, a, can be determined by the same reasoning as for Bohr's older ring electron (which gave a much too high potential), which, in usual symbols, gives $e^2/a=\frac{7}{2}chN_\infty$. Consequently,

$$E=-7N_\infty ch(1-\tfrac{1}{4}a\bar{\rho})\,. \tag{1}$$

It remains to find $\bar{\rho}$. Now if the mutual inclination, i, of the orbit planes be defined as explained in the figure, and if the azimuth angles, $\theta=\tilde{\omega}t$, be counted from the nodes, we have, by elementary geometry, at any instant t,



$$r^2/a^2=4\cos^2\theta+2(1+\cos i)\sin^2\theta\,,$$

whence, the required average,

$$\bar{\rho}=\frac{1}{4\pi a}\int_0^{2\pi}\frac{d\theta}{\sqrt{1-\frac{1}{2}(1-\cos i)\sin^2\theta}}\,,$$

or $\pi a\bar{\rho}=F(k)$, where F is the complete elliptic integral of the first kind, modulo

$$k=\sqrt{\frac{1-\cos i}{2}}\,.$$

If we put, as usual, $k=\sin\alpha$, then $\alpha=\frac{1}{2}i$, the semi-inclination. Thus, writing now α instead of the modulus itself, as is customary in elliptic tables, we have for the average reciprocal distance of the two trabants,

$$\bar{\rho}=\frac{1}{\pi a}F\left(\frac{i}{2}\right), \tag{2}$$

a handy kinematical formula which may be interesting on its own account.

Substituting this into (1), and leaving, for the present, i unspecified, we have for the negative energy of the system, or of what will hereafter be referred to as the i-model,

$$-E=7N_{\infty}ch\left[1-\frac{1}{4\pi}F\left(\frac{i}{2}\right)\right]. \tag{3}$$

Next, drag away one electron to "infinity," allowing the other to settle itself in a one-quantic orbit. Since the negative energy of this residue, the ionized atom, He^{+}, is simply $4Nch$, where $N=N_{\infty}:(1+m/M)$, and since the small difference $N_{\infty}-N$ is irrelevant for the purpose in hand, we have for the ionization work,

$$W=Nch\left[3-\frac{7}{4\pi}F\left(\frac{i}{2}\right)\right], \qquad N \doteqdot 1.0973\cdot 10^{5},$$

or, for the equivalent wave-number (of the flash emitted at the return of the vagabond),

$$\nu=N\left[3-\frac{7}{4\pi}F\left(\frac{i}{2}\right)\right]. \tag{4}$$

Now, for Bohr's model, $i=120°$, and, by a four-figure table, $F(60°)=2.1565$. Thus $\nu=1.7987N$, and since N is equivalent to 13.54 volts, the corresponding ionization potential amounts to

$$V=24.35 \text{ volts}, \tag{5}$$

which is remarkably close to 24.5, the latest observed value. The corresponding wave-length, or the limit of Lyman's (and Fricke's) extremely ultra-violet series, would be $\lambda_{\infty}=506.6$ A.

Whether this is a mere "chance" coincidence, or has some "deeper significance" (whatever that means), I am unable to say. Still less, whether this model of normal helium is a dynamically legitimate one. This, as stated before, will be left entirely to those who have proposed it.

Thus far the particular case of the Bohr model. Now, by way of mere curiosity, suppose, for the moment, that there are dynamically possible (and stable) states of the i-model also for some inclinations other than 120°. Then its energy will be as in (3),

and the wave-number of radiation emitted at the passage from He^+ to this i-model will be given by (4). Bohr's model corresponds to $\cos i = -\frac{1}{2}$. He supports this choice only by a terse appeal (p. 32) to the quantizing principle of angular momentum.

At any rate, it has seemed interesting to apply formula (4), even regardless of its significance or deduction, to some other simple rational values of $\cos i$, especially with a view of covering, perhaps, some of the four observed members of Lyman's series, $oS - mP$, which are $\lambda_1 = 584.4$, $\lambda_2 = 537.1$, $\lambda_3 = 522.3$, $\lambda_4 = 515.7$, with the aforesaid λ_∞ as limit.

The region beyond 500 A being thus far barren or unexplored, values of $\cos i > -\frac{1}{2}$ are without interest, and thus the next simple ones worth considering are $-\frac{2}{3}$, $-\frac{3}{4}$, and so on. The results obtained on this somewhat adventurous quest were as follows:

$$\cos i = -\tfrac{2}{3}, \quad \frac{i}{2} = 65^\circ\!.905, \quad F = 2.340_4, \quad \text{gave} \quad \lambda = 537.2,$$

remarkably close to the observed λ_2; the next tried, $\cos i = -\frac{3}{4}$, gave $\lambda = 561.9$, which is without interest; but the very next trial,

$$\cos i = -\tfrac{4}{5}, \quad \frac{i}{2} = 71^\circ\!.565, \quad F = 2.578_1, \quad \text{gave} \quad \lambda = 582.7,$$

which roughly corresponds to Lyman's first line, and

$$\cos i = -\tfrac{3}{5}, \quad \frac{i}{2} = 63^\circ\!.435, \quad F = 2.257_3, \quad \text{yielded} \quad \lambda = 522\ 9,$$

which is close enough to the observed λ_3. But one more member, 515.7, observed by Lyman, remained uncovered. Working back from this, the required F will be found, by (4), to be 2.2131, and the corresponding semi-inclination, $61^\circ\!.97$, whence $-\cos i = 0.558$, whereas the nearest simple fraction, $\frac{5}{9}$, is 0.555..... But whether 5 and 9 are still "simple" integers must be left to everyone's own judgment. Taking $\frac{5}{9}$, we have $i/2 = 61^\circ\!.870$, $F = 2.2094$, and $\lambda = 515.1$. In fine, formula (4) gives the correct ionization potential (5) for $-\cos i = \frac{1}{2}$, and, as a curious addition, the lines

$$\lambda_4, \quad \lambda_3, \quad \lambda_2, \quad \lambda_1,$$

respectively, for $-\cos i = \frac{5}{9}, \frac{3}{5}, \frac{2}{3}, \frac{4}{5}$, with the deviations -1.7, $+0.6$, $+0.1$, and $+0.6$ angstroms, the experimental error limits being ± 0.2 A.

Whether or not these additional states of the model are dynamically and otherwise legitimate could be decided only by a thorough analysis which the writer is not in the position to offer.

Rochester, N.Y.
March 3, 1923

NOTE ADDED MAY 12, 1923

Since the above was printed, the long awaited investigation of Dr. Kramers (*Zeitschr. für Physik*, **13**, 312–341, 1923) has reached America. The net result of this investigation, in which the computation of the mutual perturbation of the electrons is pushed to the second approximation, is that the Bohr model gives a much too low ionization potential, namely 20.7 volts (identical with van Vleck's previous result), and that the contemplated quantized motion is not even stable in the mechanical sense of the word. Kramers, however, who speaks also in the name of Bohr (*loc. cit.*, pp. 339, 340), does not at all seem to be discouraged by this unexpected "negative result" of his laborious computations. On the contrary, after a consultation with Dr. Bohr, he sees in this failure only a proof that "already in this simple case [of two trabants only] ordinary mechanics loses its validity," and does not swerve from his belief in "the correctness" of the model itself. In fine, Kramers and Bohr are unanimous in condemning, not the model, but "ordinary" mechanics. As yet, however, they do not see their way for proposing any definite modification or cutting of classical mechanics to make it applicable to atomic systems.

Under these circumstances, the alternative method of computation given above seems the more interesting. Originally dictated by the writer's ignorance of details, this method now appears brutally simple and "classically" wrong. For it amounts to adding, in (1) or (3), to the quantized central energy the averaged perturbation function—a theorem known to be valid only when the latter is a small fraction of the former. But it is precisely such a manifest infringement of the classical laws yielding fairly correct results, which may serve as a hint how to modify these mechanical laws for intra-atomic purposes. This is the reason why the foregoing treatment is here left unchanged.

In the second place, it may be well to add the following peculiarities of the spectrum formula (4) as such, noticed in the meantime.

If the simple rational values of $-\cos i$ are written out orderly, descending in magnitude,

$$\frac{4}{5}\left(\frac{3}{4}\right)\frac{2}{3}\left(\frac{5}{8}\right)\frac{3}{5}\left(\frac{4}{7}\right)\frac{5}{9},$$

every second (bracketed) covers no observed line, while the remaining ones represent orderly the first four members of the Lyman series $\mathrm{o}S-mP$. Now, extrapolating this intermittence and continuing the regular sequence of the last three fractions by

$$\left(\frac{6}{11}\right) \frac{7}{13},$$

one would expect the former to cover no line, and the latter to cover the line $\mathrm{o}S-5P$ which, though not yet observed, can confidently be expected. Now, the wave-length of this line, with Lyman's $\mathrm{o}S$ and the usual $5P$, would be $\lambda_5=512.1$, while our formula (4) gives, for $\cos i=-7/13$,

$$\lambda=512.3.$$

Turning to the left hand of the above sequence, the next fraction $\frac{5}{6}$ has naturally seemed worth trying. To this value of $-\cos i$ corresponds $i/2=73^\circ.221$, $F=2.6642$ and, by (4),

$$\lambda=601.2$$

which is very close to the "single line at 600.5 ± 0.3" repeatedly observed by Lyman. Moreover, the combination line

$$\mathrm{o}S-1S=198{,}300-32{,}033$$

would lie at $\lambda=601.3$, which is still closer to our result.

Gathering the scattered results we have the following correlation (in which bracketed fractions cover no lines):

$$\begin{array}{c|cccccccccccc}
\frac{5}{6} & \frac{4}{5} & \left[\frac{3}{4}\right] & \frac{2}{3} & \left[\frac{5}{8}\right] & \frac{3}{5} & \left[\frac{4}{7}\right] & \frac{5}{9} & \left[\frac{6}{11}\right] & \frac{7}{13} & \cdots & \frac{1}{2} \\
\mathrm{o}S-1S & \lambda_1 & & \lambda_2 & & \lambda_3 & & \lambda_4 & & \lambda_5 & & \lambda_\infty
\end{array}$$

The regular intermittency, as far as $\mathrm{o}S-mP$ is concerned, is manifest. The position of $\mathrm{o}S-1S$, the "queer" line (Compton), is correspondingly queer. Yet even this fits into the further regularity of the whole sequence of fractions, pointed out to me by my friend, Professor A. S. Eve, of Montreal, to wit, that the differences between the successive fractions are all of the form $\frac{1}{np}$, thus $5\cdot5-4\cdot6=1$, $4\cdot4-3\cdot5=1$, and so on.

The substance of this note was given in a paper read at the Washington meeting of the Physical Society, April 21, and the whole subject was expounded in a lecture delivered two days later at the Bureau of Standards, followed by a discussion.

INVESTIGATIONS ON PROPER MOTION

TENTH PAPER: INTERNAL MOTION IN THE SPIRAL NEBULA MESSIER 33, N.G.C. 598[1]

By ADRIAAN VAN MAANEN

ABSTRACT

Measures of internal motion in the spiral nebula M 33 (N.G.C. 598).—Comparison of two photographs taken in 1910 and 1922 by Ritchey and Humason, respectively, gives, with respect to twenty-four comparison stars, the annual proper motion of the nebula, $\mu_\alpha = +0''.003$, $\mu_\delta = -0''.004$, and the motions of 399 nebular points freed from this motion. The internal motions are shown on Plate XIX. They can be interpreted as a rotation or as a motion outward along the arms of the spiral, preferably the latter. Taken as a rotation, the motions indicate periods from 60,000 to 240,000 years.

Reality of measured displacements in spirals.—A summary of the results for seven spirals, M 33, 51, 63, 81, 94, 101, and N.G.C. 2403, shows that the displacements found cannot have been caused by (*a*) the telescope, (*b*) the quality of the plates, (*c*) the measuring instrument, (*d*) the measurer. Apparently they must be accepted as representing actual internal motion. As such they are in agreement with the theory of cosmogony lately proposed by Jeans.

Parallaxes of the larger spiral nebulae.—These seem to lie between a few ten-thousandths and a few thousandths of a second of arc. The corresponding diameters range from several light-years to several hundred light-years. The larger spirals are therefore enormous as compared with our solar system, but small in comparison with the system of the Milky Way.

In 1921 a preliminary note on the internal motion in this nebula was published in the *Proceedings of the National Academy of Sciences;*[2] it included measures of thirty nebulous points on two plates taken at the 25-foot focus of the 60-inch reflector and of twenty-two points on the photographs taken at the 80-foot focus of the same instrument. The measures were all made with the monocular arrangement of the Zeiss stereocomparator, while those planned for several hundred nebular points on the first pair of plates were postponed until the new stereocomparator, then under construction in the instrument shop of the observatory, should be completed. Before this was finished, the plate taken by Mr. Duncan in 1920 was accidentally broken by him and it was necessary to wait until a new exposure could be made. This was secured by Mr. Humason on September

[1] *Contributions from the Mount Wilson Observatory*, No. 260.

[2] *Mt. Wilson Comm.*, No. 71.

23 and 24, 1922. As the older plate taken by Mr. Ritchey dates from August 5, 6, and 7, 1910, the interval is 12.133 years.

Twenty-four comparison stars and 400 points, presumably belonging to the nebula, were selected for measurement. The measures and reductions were carried out in the same way as for other spirals studied.

The measures were made in four positions, east, west, north, and south, respectively, in the direction of increasing readings of the micrometer screw. The measures in right ascension were combined into one set, and those in declination into another; then the measured quantities were multiplied by 0.688 to reduce the values expressed in parts of the micrometer screw to annual motions in thousandths of a second of arc. These quantities, m_α and m_δ, respectively, were used as the first members in equations of condition of the form:

$$\left.\begin{aligned} m_\alpha &= a+bx+cy+dx^2+exy+fy^2+\mu_\alpha \\ m_\delta &= a'+b'x+c'y+d'x^2+e'xy+f'y^2+\mu_\delta \end{aligned}\right\} \qquad (1)$$

in which $a \ldots . f$, $a' \ldots . f'$, are the plate constants, x and y the co-ordinates in right ascension and declination, and μ_α and μ_δ the annual proper motions. By a least-squares solution the plate constants were determined from two sets of equations of the form (1), yielded by the twenty-four comparison stars. These constants were substituted into equations of the form (1) for all objects measured, thus giving for both stars and nebular points μ_α and μ_δ, the components in right ascension and declination of the motions with respect to the mean of the comparison stars.

For the comparison stars these quantities are given in the fourth and fifth columns of Table I; the second and third columns give the positions with respect to the center of the nebula, accurate to a tenth of a minute of arc.

In order to derive the internal motions of the nebula, the values μ_α and μ_δ of the nebular points must be freed from the motion of the nebula as a whole. One point, No. 367, shows so large a motion ($\mu = 0\overset{''}{.}136$) that it cannot be a part of the nebula, and is accordingly omitted in the following discussion.

a) The mean motion of the 399 remaining points is

$$\mu_\alpha = +0''.002; \qquad \mu_\delta = -0''.003^5.$$

b) Combining the mean motion in quadrants I and III, we have

$$\mu_\alpha = +0''.004^5; \qquad \mu_\delta = -0''.003^5$$

while for quadrants II and IV

$$\mu_\alpha = +0''.003^5; \qquad \mu_\delta = -0''.005^5.$$

TABLE I

CO-ORDINATES AND ANNUAL MOTIONS OF THE COMPARISON STARS

No.	x	y	μ_α	μ_δ
a	− 1'.6	− 4'.5	−0''.003	0''.000
b	− 2.9	− 2.7	− 3	− 2
c	− 2.7	−11.0	− 11	+ 8
d	− 8.0	−11.4	+ 5	− 7
e	− 7.0	− 5.7	− 2	− 8
f	−13.2	− 1.4	+ 2	+ 8
g	−11.9	+ 2.4	+ 6	− 7
h	− 8.6	+ 2.4	− 4	+ 3
i	− 7.6	+ 3.7	− 7	+ 3
j	− 1.4	+ 1.7	+ 7	0
k	− 3.3	+ 6.4	− 2	+ 6
l	− 2.1	+11.4	− 2	+ 1
m	+ 0.5	+10.3	+ 4	− 2
n	+ 5.2	+ 8.9	+ 3	− 5
o	+11.6	+ 4.0	− 5	+ 4
p	+ 4.1	+ 4.0	0	+ 6
q	+ 7.0	+ 2.1	0	+ 8
r	+ 1.3	+ 3.2	− 4	− 15
s	+ 8.0	− 1.1	+ 4	− 10
t	+10.7	− 6.8	− 4	− 4
u	+ 6.4	− 9.2	+ 1	+ 1
v	+ 1.7	−10.3	0	+ 4
w	+ 3.3	− 4.0	+ 13	+ 1
x	+ 0.5	− 2.5	+0.002	+0.008

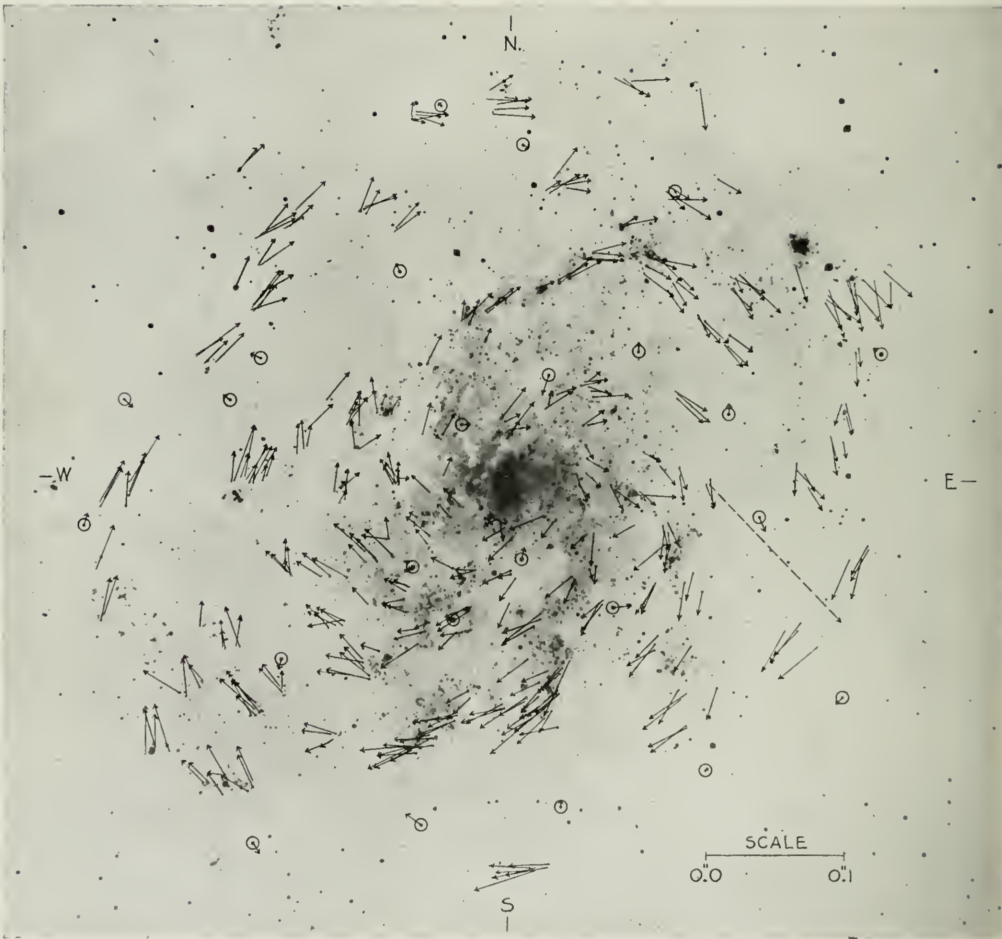
All four quadrants combined give

$$\mu_\alpha = +0''.004; \qquad \mu_\delta = -0''.004^5.$$

c) Using only the 293 points, within 10' from the center of the nebula, which have a more symmetrical distribution, we find for quadrants I and III

$$\mu_\alpha = -0''.004; \qquad \mu_\delta = -0''.003^5$$

PLATE XIX



INTERNAL MOTIONS IN MESSIER 33

The arrows indicate the directions and magnitudes of the annual motions. Their scale (0."1) is indicated on the illustration. The scale of the nebula is 1 mm = 12."8. The comparison stars are inclosed in circles.

while for quadrants II and IV

$$\mu_\alpha = +0''.003; \qquad \mu_\delta = -0''.004^5.$$

For all quadrants combined, we find

$$\mu_\alpha = +0''.003^5; \qquad \mu_\delta = -0''.004.$$

For the motion of the nebula as a whole, the mean result from the three methods is

$$\mu_\alpha = +0''.003, \qquad \mu_\delta = -0''.004.$$

Subtracting these values from the annual motions, μ_α and μ_δ, of the individual points, we derive the internal motions, which are given in the fourth and fifth columns of Table II. These motions are plotted in Plate XIX; for the comparison stars, which are surrounded by circles, the motions of Table I are used; the motion of point 367 is indicated by a broken line. The scale of the motions is indicated in the lower right-hand corner. The length of the arrows represents the motions during an interval of about 2,500 years.

The second and third columns of Table II give the co-ordinates of the points measured, with respect to the center of the nebula, to tenths of a minute of arc. In a few cases, points close together thus appear with identical co-ordinates. This is of no consequence, because, if necessary, these cases can be distinguished by their motions.

The internal motions are resolved into: (*a*) rotational and radial components; (*b*) components along and at right angles to the spiral arms (stream and transverse components).

The plane of the nebula is probably inclined a little to the tangential plane of the celestial sphere, but, as the axes are in the ratio of about 1 to 1.25, the components will in no case be affected by more than 25 per cent, or in the mean by about 10 per cent.

The results are given in the sixth, seventh, eighth, and ninth columns of Table II, where the positive sign is used for motions in the direction N-E-S-W, and outward.

TABLE II

Co-ordinates and Annual Internal Motion

No.	x	y	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
1	−12′.7	− 4′.5	+0″.007	+0″.031	+0″.027	−0″.017	+0″.026	−0″.019
2	−12.7	− 4.3	+ 9	+ 28	+ 23	− 18	+ 21	− 20
3	−12.9	− 2.9	+ 13	+ 32	+ 28	− 20	+ 29	− 19
4	−12.7	− 1.0	+ 15	+ 27	+ 26	− 17	+ 28	− 12
5	−12.7	− 0.8	+ 16	+ 27	+ 26	− 17	+ 28	− 13
6	−12.6	− 0.6	+ 13	+ 26	+ 25	− 14	+ 27	− 10
7	−11.9	− 1.0	+ 2	+ 25	+ 25	− 3	+ 25	− 1
8	−11.8	− 0.5	+ 11	+ 25	+ 25	− 10	+ 26	− 6
9	−11.8	− 0.3	+ 17	+ 27	+ 29	− 13	+ 31	− 7
10	−11.8	− 0.2	+ 22	+ 33	+ 33	− 22	+ 38	− 11
11	−11.4	− 8.7	0	+ 35	+ 28	− 21	+ 23	− 27
12	−11.1	− 8.4	− 7	+ 25	+ 24	− 9	+ 23	− 12
13	−11.1	− 8.6	0	+ 24	+ 19	− 14	+ 18	− 17
14	−10.7	− 8.6	− 11	+ 33	+ 32	− 13	+ 30	− 18
15	− 9.5	− 9.5	− 15	+ 16	+ 22	0	+ 21	− 4
16	− 9.4	− 9.5	− 14	+ 14	+ 20	0	+ 19	− 5
17	− 8.9	− 9.4	− 10	+ 1	+ 8	+ 6	+ 10	+ 3
18	− 8.7	− 9.4	− 14	+ 24	+ 26	− 9	+ 22	− 16
19	− 8.9	−10.0	− 11	+ 15	+ 18	− 5	+ 16	− 10
20	− 8.1	− 9.9	− 20	+ 11	+ 22	+ 4	+ 23	− 3
21	− 8.0	− 9.7	− 11	+ 23	+ 23	− 12	+ 18	− 18
22	− 8.0	− 9.5	− 11	+ 24	+ 23	− 12	+ 20	− 17
23	−10.2	− 6.7	− 27	+ 15	+ 28	+ 13	+ 30	+ 6
24	−10.0	− 6.8	− 2	+ 28	+ 26	− 13	+ 20	− 19
25	− 9.7	− 6.5	− 11	+ 23	+ 25	− 5	+ 24	− 10
26	− 9.5	− 6.4	− 9	+ 11	+ 14	+ 2	+ 14	− 1
27	− 9.5	− 4.6	+ 2	+ 18	+ 15	− 10	+ 13	− 12
28	− 8.9	− 5.2	− 1	+ 19	+ 17	− 9	+ 12	− 14
29	− 8.7	− 5.4	− 2	+ 14	+ 13	− 5	+ 11	− 9
30	− 8.4	− 5.2	+ 3	+ 22	+ 16	− 15	+ 10	− 19
31	− 8.3	− 5.2	− 11	+ 29	+ 30	− 6	+ 27	− 14
32	− 8.1	− 6.8	− 14	+ 17	+ 22	− 2	+ 20	− 10
33	− 8.1	− 7.0	− 15	+ 21	+ 26	− 3	+ 24	− 11
34	− 8.0	− 7.3	− 24	+ 22	+ 32	+ 3	+ 32	− 5
35	− 7.8	− 7.0	− 17	+ 16	+ 23	+ 3	+ 22	− 6
36	− 8.1	− 7.5	− 12	+ 26	+ 28	− 7	+ 24	− 16
37	− 7.8	− 7.5	− 14	+ 14	+ 19	+ 1	+ 19	− 5
38	− 7.8	− 7.3	− 16	+ 22	+ 27	− 4	+ 25	− 11
39	− 7.0	− 6.7	− 16	+ 17	+ 23	0	+ 23	− 7
40	− 7.0	− 6.7	0	+ 13	+ 9	− 9	+ 6	− 11
41	− 7.2	− 6.5	− 12	+ 15	+ 19	− 2	+ 18	− 7
42	− 6.7	− 3.0	− 6	+ 12	+ 13	+ 1	+ 13	− 2
43	− 6.8	− 3.0	− 2	+ 22	+ 21	− 8	+ 19	− 11
44	− 6.8	− 3.0	− 13	+ 15	+ 19	+ 6	+ 19	+ 3
45	− 6.8	− 2.9	− 15	+ 14	+ 19	+ 8	+ 20	+ 5
46	− 7.0	− 2.9	+ 1	+ 24	+ 21	− 10	+ 21	− 11
47	− 8.6	− 0.2	+ 2	+ 22	+ 22	− 2	+ 22	+ 2
48	− 8.6	0.0	+ 15	+ 40	+ 40	− 15	+ 41	− 12
49	− 8.3	0.0	+ 6	+ 17	+ 17	− 6	+ 17	− 3
50	− 8.1	0.0	+ 7	+ 37	+ 37	− 7	+ 38	0
51	− 8.1	0.0	+ 16	+ 24	+ 25	− 15	+ 27	− 10
52	− 8.1	− 0.2	+ 13	+ 28	+ 27	− 13	+ 28	− 9
53	− 8.0	− 0.2	+ 2	+ 13	+ 13	− 2	+ 13	− 1
54	− 7.6	0.0	+ 4	+ 16	+ 16	− 4	+ 16	− 3
55	− 7.5	+ 0.2	+ 5	+ 15	+ 15	− 6	+ 16	− 3
56	− 7.6	+ 0.2	+ 11	+ 19	+ 19	− 11	+ 20	− 9
57	− 8.0	0.0	+ 12	+ 23	+ 22	− 13	+ 23	− 10
58	− 9.1	+ 3.7	+ 20	+ 20	+ 26	− 12	+ 27	− 9
59	− 9.4	+ 3.5	+ 17	+ 16	+ 21	− 11	+ 22	− 7
60	− 9.7	+ 3.8	+ 37	+ 24	+ 38	− 24	+ 41	− 15
61	− 9.7	+ 3.8	+ 21	+ 12	+ 19	− 15	+ 21	− 11
62	− 7.6	+ 5.2	+ 11	+ 18	+ 21	0	+ 21	+ 3
63	− 7.8	+ 5.2	+ 17	+ 20	+ 25	− 4	+ 25	− 5
64	− 8.0	+ 5.2	+ 26	+ 8	+ 20	− 18	+ 20	− 18
65	− 8.0	+ 5.2	+ 20	+ 26	+ 33	− 2	+ 33	− 2
66	− 7.8	+ 5.4	+ 21	+ 24	+ 32	− 4	+ 32	− 5
67	− 8.4	+ 5.9	+ 11	+ 23	+ 24	+ 5	+ 24	+ 6
68	− 7.6	+ 5.7	+ 21	+ 11	+ 21	− 10	+ 21	− 11
69	− 7.8	+ 6.7	+ 12	+ 19	+ 22	+ 2	+ 22	+ 3
70	− 7.6	+ 6.5	+ 24	+ 15	+ 27	− 7	+ 27	− 9

TABLE II—*Continued*

No.	x	y	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
71	−7′.8	+7′.5	+0″.029	+0″.014	+0″.031	−0″.011	+0″.031	−0″.010
72	−7.6	+7.5	+26	+15	+29	−7	+29	−6
73	−7.5	+7.6	+16	+19	+25	+3	+25	+4
74	−7.0	+7.8	+15	+9	+18	−4	+18	−4
75	−6.8	+7.8	+18	+15	+23	+1	+23	0
76	−6.7	+8.1	+22	+21	+30	+3	+30	+3
77	−8.3	+9.4	+15	+17	+22	+4	+22	+2
78	−8.3	+9.5	+17	+17	+24	+2	+24	−1
79	−4.6	+8.3	+10	+25	+21	+18	+23	+13
80	−4.5	+8.1	+9	+17	+16	+11	+17	+7
81	−4.5	+8.3	+24	+10	+26	−1	+25	−6
82	−4.5	+8.4	+23	+6	+23	−6	+22	−10
83	−5.6	+2.4	+16	+20	+24	−8	+24	−8
84	−6.2	+1.6	+17	+17	+20	−13	+20	−13
85	−6.2	+1.0	+2	+15	+15	−1	+15	−2
86	−6.4	+1.0	−1	+21	+20	+5	+21	+3
87	−6.5	+0.8	+1	+22	+22	+2	+22	0
88	−4.9	+2.1	+8	+16	+18	0	+18	−1
89	−4.8	+2.1	+19	+17	+23	−11	+22	−12
90	−4.9	+1.7	+8	+16	+18	−2	+18	−2
91	−4.9	+1.7	+12	+17	+20	−6	+20	−6
92	−4.6	+1.6	0	+22	+21	+7	+21	+7
93	−4.8	+1.0	−2	+21	+20	+6	+21	+3
94	−4.6	+1.0	+15	+9	+12	−13	+9	−15
95	−5.2	−0.3	+4	+4	+3	−5	+4	−4
96	−5.4	−0.3	+3	+21	+21	−4	+20	−6
97	−5.1	−0.3	+13	+12	+11	−15	+10	−15
98	−5.1	−0.6	+2	+22	+21	−5	+21	−4
99	−5.2	−0.8	+5	+9	+8	−7	+8	−7
100	−5.9	−3.0	−18	+11	+18	+11	+20	+8
101	−5.6	−3.0	−20	+17	+25	+9	+26	+4
102	−4.9	−3.0	−25	+14	+25	+13	+27	+8
103	−5.6	−4.6	−15	+15	+21	+3	+21	−3
104	−5.6	−4.6	−9	+12	+15	0	+14	−4
105	−5.4	−4.6	−20	+9	+20	+9	+22	+3
106	−5.1	−4.6	−16	+11	+18	+6	+19	−1
107	−5.1	−4.5	−20	+8	+19	+9	+21	+3
108	−5.2	−4.5	−24	−4	+11	+21	+17	+17
109	−4.3	−4.0	−9	+12	+15	0	+15	−3
110	−4.0	−5.4	−13	+4	+13	+4	+13	−4
111	−4.5	−5.6	−15	+18	+23	−5	+18	−15
112	−4.5	−5.7	−27	+5	+24	+12	+27	0
113	−4.5	−6.0	−14	+16	+21	−5	+17	−14
114	−4.3	−6.2	−21	+17	+27	−3	+21	−17
115	−4.3	−6.4	−31	+7	+30	+12	+31	−7
116	−4.9	−6.2	−22	+2	+18	+13	+22	+1
117	−5.6	−8.1	−21	+2	+19	+10	+21	0
118	−5.4	−8.0	−22	+7	+22	+6	+23	−5
119	−5.6	−8.3	−18	−7	+11	+16	+18	+7
120	−5.4	−8.3	−14	−5	+6	+13	+11	+10
121	−3.2	−8.4	−32	−2	+29	+14	+32	0
122	−3.0	−8.4	−24	−7	+19	+16	+25	+3
123	−3.0	−8.4	−29	−14	+22	+23	+31	+9
124	−3.0	−8.6	−27	−13	+22	+20	+29	+7
125	−2.9	−8.6	−28	−2	+27	+7	+28	−2
126	−2.7	−8.4	−27	+1	+26	+5	+26	−7
127	−2.2	−8.4	−20	−3	+19	+8	+20	−4
128	−2.4	−8.3	−27	−15	+21	+22	+30	+8
129	−2.2	−8.1	−28	−1	+26	+9	+27	−8
130	−2.4	−8.1	−21	−5	+19	+11	+21	0
131	−1.9	−7.8	−22	−11	+20	+15	+24	+1
132	−1.6	−7.6	−27	−10	+25	+15	+29	0
133	−1.7	−7.5	−28	−3	+27	+8	+26	−11
134	−1.7	−7.5	−27	−6	+25	+11	+27	−8
135	−1.6	−7.3	−21	−11	+18	+15	+24	+1
136	−1.6	−7.5	−15	−3	+14	+7	+15	−3
137	−1.1	−6.8	−13	−12	+11	+14	+17	+5
138	−1.3	−6.7	−15	−8	+15	+8	+17	0
139	−2.7	−5.1	−27	−18	+16	+28	+29	+14
140	−2.5	−4.8	−22	−6	+17	+15	+23	0
141	−2.4	−4.5	−11	−5	+7	+9	+12	+2
142	−2.9	−4.6	−14	−2	+11	+9	+14	0

TABLE II—*Continued*

No.	x	y	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
143	−3′.2	−4′.1	−0″.016	+0″.005	+0″.016	+0″.005	+0″.017	−0″.003
144	−3.3	−4.0	−10	0	+8	+6	+10	+2
145	−3.2	−3.8	−16	−3	+10	+12	+15	+6
146	−3.2	−3.5	−15	+9	+17	+3	+18	0
147	−3.2	−3.3	−13	+3	+11	+7	+13	+3
148	−3.0	−3.3	−12	+5	+13	+3	+13	−1
149	−4.5	−2.5	−12	+13	+17	+5	+18	+1
150	−4.5	−2.2	−22	+20	+28	+11	+29	+5
151	−4.1	−2.5	−17	+18	+24	+7	+24	+3
152	−4.0	−2.5	−15	+16	+21	+5	+22	+2
153	−3.5	−2.4	−20	+19	+27	+5	+27	+4
154	−3.8	−1.9	−18	+3	+15	+10	+15	+10
155	−3.7	−1.9	−2	+16	+15	−6	+15	−6
156	−2.5	−2.1	−8	+10	+13	−1	+13	−2
157	−2.4	−1.7	−17	+13	+21	+4	+21	+4
158	−2.9	−1.1	−4	+3	+4	+3	+5	+2
159	−3.3	−0.2	−1	+12	+12	−1	+12	−3
160	−3.5	−0.2	−5	+23	+23	+4	+23	0
161	−3.5	−0.2	−6	+13	+14	+6	+14	+3
162	−3.5	−0.2	−10	+16	+17	+9	+18	+6
163	−3.3	0.0	−8	+11	+12	+8	+13	+6
164	−3.3	0.0	+7	0	0	−7	−2	−7
165	−3.3	−0.2	+1	+15	+15	−1	+14	−4
166	−4.0	+2.1	−4	+26	+15	+21	+23	+12
167	−3.8	+2.2	+3	+8	+8	+1	+8	0
168	−3.7	+2.2	+6	+10	+12	0	+10	−6
169	−3.5	+7.6	+19	+14	+23	+5	+24	+1
170	−3.5	+7.8	+17	+18	+22	+11	+24	+7
171	−3.0	+11.1	+1	+10	+3	+10	+6	+8
172	−3.0	+11.1	+9	−2	+8	−5	+7	−6
173	−2.9	+11.1	+23	+5	+24	0	+23	−6
174	−2.7	+11.1	+19	−7	+17	−10	+15	−13
175	−2.9	+11.1	+19	−1	+18	−6	+16	−9
176	−2.9	+11.3	+23	−2	+21	−8	+20	−12
177	−0.6	+11.8	+30	−1	+30	−3	+29	−9
178	−0.6	+11.9	+16	+11	+16	+10	+18	+6
179	−0.2	+11.9	0	−1	0	−1	0	−1
180	−0.2	+11.8	+9	−1	+9	−2	+9	−4
181	−0.3	+11.6	+25	+4	+25	+3	+25	−3
182	−0.5	+11.4	+25	+1	+25	+1	+24	−5
183	−0.5	+11.3	+30	−1	+30	−3	+28	−9
184	+1.4	+9.2	+16	+22	+13	+24	+21	+17
185	+1.7	+9.1	+19	+3	+18	+7	+19	−1
186	+1.7	+9.1	+17	−2	+17	0	+15	−8
187	+1.4	+9.1	+26	+9	+25	+12	+27	+1
188	+1.3	+8.9	+22	+14	+20	+17	+25	+5
189	+1.1	+8.9	+11	+7	+10	+8	+13	+3
190	−0.6	+5.2	+18	+15	+20	+12	+23	−4
191	−0.6	+5.1	+24	+19	+26	+16	+30	−4
192	−0.6	+5.2	+16	+10	+17	+8	+18	−4
193	−1.3	+5.1	+18	+18	+23	+11	+25	−3
194	−1.3	+4.9	+9	+9	+12	+6	+13	0
195	−1.4	+4.9	+2	+15	+14	+7	+13	+8
196	−1.1	+4.8	+8	+18	+12	+16	+18	+7
197	−0.6	+4.3	+3	+10	+6	+9	+10	+3
198	−2.5	+3.5	+10	+21	+21	+11	+23	0
199	−2.7	+1.4	+6	+22	+23	+3	+22	−6
200	−2.2	+1.4	+4	−3	0	−5	−3	−4
201	−1.9	+1.4	+9	+14	+16	+1	+14	−8
202	−1.9	+0.2	+4	+13	+12	−5	+9	−10
203	−2.4	−0.5	−13	+12	+14	+10	+16	+8
204	−1.4	−0.8	−11	+12	+16	+3	+16	+1
205	−1.7	−1.1	−9	+10	+14	+1	+14	+1
206	−1.6	−2.9	−16	+8	+18	0	+17	−6
207	−1.1	−2.9	−12	0	+11	+4	+12	0
208	−1.0	−2.9	−18	−6	+15	+12	+18	+4
209	−1.3	−4.0	−22	−14	+18	+19	+23	+12
210	−1.1	−4.0	−19	−7	+16	+13	+19	+5
211	−1.1	−4.1	−23	−15	+19	+20	+24	+12
212	−1.4	−4.8	−14	−14	+9	+17	+17	+10
213	−1.3	−4.8	−16	−2	+15	+5	+16	−2
214	−0.5	−1.4	−5	+5	+6	−3	+7	−2

TABLE II—*Continued*

No.	x	y	μ_{α}	μ_{δ}	Rotational	Radial	Stream	Transverse
215	−0′.3	−1′.3	−0″.010	−0″.005	+0″.009	+0″.007	+0″.008	+0″.008
216	−0.2	−1.3	+2	−14	−4	+14	−8	+11
217	0.0	−1.4	−12	+5	+12	−7	+13	0
218	0.0	−1.4	−16	−9	+16	+10	+13	+13
219	+0.2	−1.7	−17	−16	+18	+14	+14	+18
220	+0.2	−2.9	−21	−10	+22	+8	+23	0
221	+0.2	−3.0	−10	−2	+10	+2	+10	−1
222	+0.3	−3.0	−11	−16	+12	+14	+17	+9
223	+0.2	−3.8	−8	−18	+9	+17	+15	+13
224	+1.1	−4.1	−27	−21	+32	+13	+34	−1
225	+1.1	−4.5	−28	−11	+30	+2	+27	−13
226	+1.0	−4.6	−23	−13	+25	+8	+26	−4
227	+0.6	−5.6	−22	−17	+25	+13	+27	−7
228	−0.2	−7.2	−26	−14	+26	+14	+29	−4
229	−0.2	−7.3	−29	−10	+28	+11	+29	−10
230	0.0	−7.2	−25	−4	+25	+4	+23	−11
231	+0.6	−7.0	−18	−10	+19	+8	+20	−4
232	+0.6	−6.8	−34	−23	+36	+19	+41	−5
233	+1.1	−6.4	−21	−10	+23	+4	+22	−7
234	+1.3	−6.0	−19	−8	+20	+3	+18	−9
235	+1.7	−5.9	−27	−31	+34	+23	+41	−1
236	+2.1	−5.7	−12	−26	+20	+21	+28	+7
237	+1.7	−5.9	−18	−18	+22	+12	+25	+1
238	+1.7	−6.0	−15	−12	+17	+8	+19	−3
239	+1.6	−6.5	−11	−21	+15	+18	+23	+7
240	+1.6	−6.7	−22	−19	+26	+13	+28	−7
241	+1.7	−6.7	−17	−14	+20	+10	+21	−5
242	+1.0	−6.8	−13	−11	+14	+9	+17	−2
243	+1.3	−7.2	−21	−9	+23	+3	+21	−10
244	+1.3	−7.2	−21	−16	+23	+12	+26	−2
245	+0.6	−7.6	−27	−24	+35	+11	+36	−2
246	+0.8	−7.5	−21	−12	+22	+9	+22	−8
247	+1.3	−7.5	−19	−18	+22	+14	+26	0
248	+1.4	−7.3	−8	−15	+12	+13	+16	+4
249	+1.6	−7.8	−27	−6	+28	0	+22	−17
250	+0.6	−12.4	−23	−2	+23	−1	+19	−13
251	+0.8	−12.2	−29	+1	+29	−5	+24	−17
252	+1.0	−12.2	−22	−7	+22	+4	+21	−9
253	+0.8	−12.4	−42	−16	+44	+11	+44	−11
254	+1.3	−12.2	−39	−7	+39	+3	+35	−18
255	+1.4	−12.2	−30	−1	+30	−3	+25	−17
256	+3.0	−4.0	−16	−19	+24	+6	+24	−7
257	+3.0	−3.8	−14	−13	+19	0	+17	−10
258	+2.9	−3.8	−12	−23	+24	+11	+25	+6
259	+2.1	−3.0	−19	−15	+24	+3	+23	−7
260	+2.2	−3.2	−7	−17	+15	+10	+18	+1
261	+2.5	−2.9	+1	−7	+4	+6	+6	+4
262	+2.9	−2.7	−1	−12	+9	+7	+12	0
263	+2.5	−1.7	−11	−10	+14	−4	+12	−9
264	+2.7	−1.7	−1	−25	+23	+11	+25	−1
265	+1.4	−1.3	−8	−11	+13	+2	+14	0
266	+1.1	−1.1	−24	−10	+24	−10	+26	0
267	+2.2	−0.2	+8	−10	+9	+9	+10	+8
268	+2.5	+0.2	+11	−10	+10	+11	+13	+6
269	+2.2	+0.2	+7	−19	+19	+6	+20	0
270	+2.5	+0.6	+16	−9	+12	+14	+17	+7
271	+2.5	+1.1	+5	−10	+11	+1	+10	−3
272	+2.5	+1.7	+14	+3	+5	+14	+10	+10
273	+2.4	+1.7	+15	+10	+1	+18	+8	+16
274	+2.7	+2.5	+25	+1	+15	+20	+23	+10
275	+2.7	+2.7	+12	−1	+10	+7	+12	+3
276	+2.5	+2.9	+17	−6	+17	+6	+18	−4
277	+2.2	+2.9	+14	+3	+9	+11	+13	+6
278	+2.4	+3.0	+16	+1	+12	+11	+16	+2
279	+2.5	+3.0	+8	+6	+2	+10	+7	+7
280	+1.6	+2.7	+14	+5	+9	+11	+14	+4
281	+1.3	+2.4	+1	+9	−3	+9	+4	+8
282	+1.3	+2.2	+20	+14	+11	+22	+24	+6
283	+0.2	+1.4	+7	+10	+6	+11	+12	0
284	+0.2	+1.6	+16	+21	+12	+24	+26	+2
285	0.0	+1.4	+16	+2	+15	+4	+14	−8
286	0.0	+1.7	+11	+6	+12	+5	+12	−5

TABLE II—*Continued*

No.	x	y	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
287	−0.′2	+1.′9	+0.″018	+0.″022	+0.″020	+0.″020	+0.″029	+0.″001
288	+1.0	+5.9	+13	+5	+13	+6	+14	−2
289	+1.6	+6.0	+23	+20	+18	+25	+29	+8
290	+1.6	+6.2	+21	+12	+18	+16	+24	+5
291	+1.9	+6.2	+24	+13	+20	+19	+27	+4
292	+2.4	+6.8	+25	−1	+24	+6	+25	−2
293	+2.4	+6.8	+27	−2	+26	+8	+26	−6
294	+2.7	+7.0	+20	−1	+19	+5	+19	−5
295	+2.7	+7.0	+21	+5	+17	+13	+21	+2
296	+3.7	+7.8	+20	+4	+17	+12	+20	+4
297	+3.7	+8.0	+9	+5	+5	+9	+9	+5
298	+3.3	+12.4	+25	−13	+27	−6	+24	−14
299	+3.5	+12.2	+14	−11	+17	−7	+13	−13
300	+3.8	+12.2	+28	+1	+26	+10	+28	−1
301	+5.9	+12.1	+3	−31	+18	−26	+8	−30
302	+4.9	+9.1	+16	−9	+18	−1	+17	−7
303	+4.9	+8.9	+32	−19	+37	−2	+32	−19
304	+4.9	+8.7	+28	0	+24	+14	+28	+3
305	+4.9	+8.7	+9	−9	+12	−5	+9	−9
306	+4.6	+7.0	+20	−20	+27	−7	+24	−15
307	+4.5	+6.8	+20	−9	+22	+4	+22	−3
308	+4.6	+6.8	+29	−6	+27	+12	+30	−1
309	+4.3	+6.7	+18	−10	+20	+2	+20	−3
310	+4.3	+6.4	+21	−17	+27	−2	+24	−12
311	+4.9	+5.9	+13	−16	+20	−5	+17	−11
312	+5.1	+6.0	+11	−17	+19	−7	+17	−11
313	+5.2	+6.2	+15	−13	+20	0	+19	−7
314	+5.4	+6.2	+8	−9	+12	−1	+11	−5
315	+7.0	+6.4	+25	−11	+25	+11	+27	+3
316	+7.2	+6.2	+4	−11	+11	−4	+9	−7
317	+7.2	+6.4	+18	−19	+26	+2	+25	−7
318	+7.3	+6.4	+28	−25	+38	+4	+37	−9
319	+7.0	+5.9	+14	−22	+26	−4	+24	−10
320	+7.0	+5.9	+17	−17	+24	+3	+23	−5
321	+5.9	+5.1	+9	−16	+17	−4	+16	−9
322	+5.9	+4.9	+16	−18	+24	0	+23	−7
323	+6.2	+4.9	+17	−16	+23	+4	+23	−4
324	+6.8	+4.3	+17	−11	+19	+7	+20	0
325	+6.8	+4.1	+13	−16	+20	+2	+20	−3
326	+6.8	+4.1	+14	−15	+20	+5	+21	0
327	+5.2	+2.7	+24	−15	+24	+15	+28	+5
328	+5.4	+2.7	+24	−19	+29	+11	+30	−1
329	+5.2	+2.5	+19	−16	+23	+9	+25	0
330	+4.3	+0.8	+11	−6	+7	+9	+9	+7
331	+4.5	+0.8	+8	−4	+6	+7	+7	+6
332	+4.8	+0.5	+11	−14	+15	+10	+17	+7
333	+3.3	−0.2	+9	−18	+17	+10	+19	+6
334	+3.3	−0.3	+17	−13	+10	+19	+16	+15
335	+4.0	−0.3	+8	−8	+7	+8	+9	+7
336	+4.1	−0.3	+11	−10	+8	+12	+11	+9
337	+4.1	−0.5	+28	−1	−2	+28	+6	+27
338	+4.0	−1.4	−16	−9	+13	−13	+9	−16
339	+5.1	−1.3	+7	−17	+15	+11	+18	+6
340	+4.9	−1.4	+4	−18	+15	+10	+18	+4
341	+4.9	−1.6	−2	−20	+20	+3	+20	−3
342	+5.1	−1.6	+1	−13	+12	+6	+13	0
343	+4.9	−2.1	−22	−18	+25	−13	+18	−22
344	+5.1	−2.1	−1	−20	+19	+6	+20	−2
345	+5.1	−2.1	−4	−16	+16	+2	+16	−3
346	+4.8	−3.2	−10	−22	+24	+4	+23	−9
347	+4.6	−3.3	−9	−17	+18	+3	+18	−8
348	+4.6	−3.5	−12	−18	+22	−1	+19	−11
349	+4.8	−4.9	−17	−17	+24	−1	+21	−12
350	+4.8	−5.1	−18	−24	+30	+4	+28	−11
351	+4.6	−5.4	−20	−11	+22	−8	+18	−14
352	+5.9	−5.2	−12	−20	+23	+4	+22	−7
353	+5.4	−6.7	−15	−24	+26	+11	+28	−3
354	+5.7	−6.7	−27	−18	+32	−4	+28	−16
355	+5.6	−6.8	−26	−22	+34	0	+30	−16
356	+5.7	−7.8	−25	−20	+32	+1	+30	−12
357	+5.7	−7.8	−28	−14	+31	−6	+25	−19
358	+6.7	−6.5	−6	−23	+21	+11	+24	0

TABLE II—*Continued*

No.	x	y	μ_α	μ_δ	Rotational	Radial	Stream	Transverse
359	+9′.9	−5′.2	−0″.029	−0″.024	+0″.035	−0″.015	+0″.031	−0″.022
360	+8.7	−4.8	−16	−29	+33	0	+31	−11
361	+9.2	−4.8	−17	−18	+24	−8	+20	−14
362	+9.2	−4.3	−22	−28	+35	−7	+31	−17
363	+6.2	−3.5	−2	−17	+16	+5	+17	−3
364	+5.6	−3.2	−3	−28	+26	+10	+28	−4
365	+5.9	−2.9	−5	−20	+20	+5	+20	−5
366	+6.2	−0.2	+5	−16	+15	+7	+16	+4
367	+6.4	−0.2	+95	−98	....	....	....	....
368	+6.4	0.0	+2	−21	+21	+2	+21	−2
369	+5.4	+0.2	+2	−18	+18	+2	+18	−1
370	+5.4	+0.3	+5	−16	+16	+5	+17	0
371	+10.8	−2.5	−11	−28	+29	−6	+29	−10
372	+11.1	−2.5	−6	−29	+29	0	+29	−3
373	+11.4	−2.2	−13	−23	+24	−10	+24	−11
374	+11.3	−2.1	−11	−21	+22	−8	+22	−9
375	+9.4	−0.2	+10	−17	+16	+11	+18	+9
376	+9.4	+0.2	+8	−13	+13	+8	+14	+5
377	+9.1	+0.3	+20	−21	+22	+20	+24	+17
378	+9.1	+0.5	−1	−24	+24	−3	+23	−5
379	+10.2	+1.1	+3	−22	+22	+2	+22	+2
380	+10.2	+1.4	+5	−19	+20	+2	+20	+2
381	+10.5	+1.1	+2	−12	+12	0	+12	+1
382	+10.7	+2.1	+3	−14	+14	+1	+14	0
383	+10.7	+2.4	+5	−22	+23	+1	+23	0
384	+10.5	+2.4	−5	−16	+14	−9	+14	−9
385	+10.8	+4.0	+1	−25	+24	−7	+23	−9
386	+11.0	+4.1	+2	−13	+13	−3	+13	−4
387	+9.9	+5.6	+13	−19	+23	+1	+23	−2
388	+9.9	+5.6	+26	−17	+28	+14	+30	+8
389	+10.0	+5.7	+16	−19	+24	+6	+25	−1
390	+10.0	+5.9	+11	−34	+35	−8	+32	−15
391	+10.5	+6.2	+7	−21	+22	−3	+21	−7
392	+10.7	+6.2	+6	−24	+24	−7	+23	−10
393	+10.8	+6.0	+6	−35	+33	−12	+31	−17
394	+11.0	+5.7	+16	−19	+24	+6	+25	+1
395	+11.0	+6.2	+20	−20	+27	+8	+28	+1
396	+11.4	+6.2	+6	−35	+34	−11	+32	−16
397	+11.8	+6.5	+21	−18	+26	+10	+27	+4
398	+11.9	+6.2	0	−19	+17	−8	+16	−10
399	+8.9	+6.7	+8	−26	+26	−9	+20	−18
400	+6.5	+9.2	+0.017	−0.009	+0.019	+0.001	+0.019	−0.004

a) The mean rotational component is +0″.020±0″.001; the mean radial component, +0″.003 ±0″.001. There seems, however, to be a considerable increase of motion with distance from the center. We have

$r=\ <3'$	$\mu_{\text{rot.}}=+0''.012$	$\mu_{\text{rad.}}=+0''.006$	$n=$ 26
$3'-6'$	+0.015	+0.006	100
6−9	+0.021	+0.005	103
9−12	+0.022	+0.002	98
12−15	+0.024	−0.004	69
>15	+0.024	−0.015	3

These rotational components correspond to hypothetical periods of from 60,000 to 240,000 years.

b) The mean stream component is $+0''.020 \pm 0''.001$, with a transverse component $-0''.003 \pm 0''.001$. With the increasing distances from the center we find

$r = <3'$	$\mu_{\text{stream}} = +0''.013$	$\mu_{\text{trans.}} = +0''.002$	$n = 26$
$3'-6'$	$+0.017$	$+0.001$	100
$6-9$	$+0.022$	-0.003	103
$9-12$	$+0.023$	-0.004	98
$12-15$	$+0.023$	-0.009	69
>15	$+0.021$	-0.019	3

With these measures on M 33 we have reached the end of the work which can at present be done advantageously on the internal motions in spirals. Only two or three other spirals were photographed with the 60-inch reflector between 1910 and 1912, and these objects do not show images sharp enough to promise any further advance in our knowledge of internal motions. Since it seems wise to defer further measures until plates taken with the 100-inch Hooker telescope become available, it is appropriate to summarize and discuss the results thus far obtained.

Seven spirals have been measured, viz., M 33, 51, 63, 81, 94, 101, and N.G.C. 2403, all of which definitely show an internal motion, best interpreted as a motion outward along the arms. The question as to the reality of the displacements, which are very small, may be answered first.

a) The instrument with which the photographs were taken was for most of the plates the 25-foot arrangement of the 60-inch reflector. That the displacements are instrumental is very improbable; there is no reason why the old plates should differ in appearance from the new ones in such a way as to produce a displacement of the nebular points with respect to the comparison stars corresponding to rotational or stream motion. It is true that in practically all cases the comparison stars were in the mean brighter than the nebular points. This might give rise to a magnitude error, but such an error could produce only a bodily shift of the nebular points with respect to the comparison stars, or a radial shift due to curvature of the field and the smaller mean distance of the nebular points from the center than of the comparison stars.

But relative motion along the arms of the spirals cannot conceivably be caused by the instrument. Moreover, it would be strange if the instrument produced a left-handed twist for the left-handed spirals, M 51, 81, and 101, and a right-handed twist for the right-handed spirals, M 33, 63, 94, and N.G.C. 2403. Further, three of the plates measured were taken with the 36-inch reflector of the Lick Observatory, and two with the 80-foot arrangement of the 60-inch reflector. Last of all, similar motions have been found in M 51 by Kostinsky, Lampland, and Schouten, from observations with three other smaller instruments.

b) The possibility that the displacements are due to a difference in quality in old and new plates is also extremely small. A difference in quality of the images or in the density of the plates might cause a slight difference in the bisections, because of asymmetry in the images of the nebular points. As a whole, the older plates taken by Ritchey are of extremely good quality, and many of the newer plates are equally good. Of the Lick photographs of M 101, the 1908 plate was considerably better than that of 1899. Of the two plates of M 33, taken at the 80-foot focus of the 60-inch reflector, the later plate is also of considerably better quality. As far as the density of the photographs is concerned, the exposure times of the new plates taken with the 60-inch reflector have all been made so nearly equal to those of the old plates, that no difference in the density can be seen; neither the quality nor the density can therefore account for the displacements found.

c) In measuring the plates, three machines were used: the Zeiss stereocomparator, the new stereocomparator built in our instrument shop, and an auxiliary instrument used for some test measures. For the last, two plates of M 101 were mounted side by side on the moving stage of an ordinary measuring machine, which was fitted with an extra microscope, one for each plate, thus permitting differential measures. It is clear that defects in the optical system of the stereocomparator could never reveal themselves as a rotatory motion of the nebular points, without equally affecting the comparison stars. Moreover, the effect of such defects would be eliminated in the reductions. Further in the case of M 33, 81, 101, and N.G.C. 2403, three or more plates were measured. As an

extra precaution, care was taken in all these cases to measure the plates partly with the older plate in the left-hand, and partly in the right-hand plate-carrier of the stereocomparator. This precaution would also eliminate any influence due to a possible curvature in the rails, either horizontal or vertical, that guide the plate-carriers. Finally, several other plates measured for proper motion in both stereocomparators have never shown any rotational effects whatsoever.

d) The measures were practically all made by the author. Mr. Nicholson, however, was kind enough to make enough check measures on M 101, both with the Zeiss stereocomparator and with the auxiliary machine, to avoid any doubt as to the results. Lately Mr. Lundmark has carried out measures on M 33, and his results seem, as a whole, to corroborate those by the author. Finally, there are the measures by Kostinsky, Lampland, and Schouten mentioned under (*a*).

If, then, the results obtained in all seven spirals measured are to be taken as real displacements of the nebular points with respect to the comparison stars, as apparently must be done, there are only two possible explanations: either the comparison stars show a vortex-motion around the spirals, which to say the least is very improbable, or the spirals show a rotatory or stream motion. The beautiful work of Jeans has given us reason even to anticipate such motions. He has shown that, in a highly compressible mass of rotating gas, as a gaseous nebula undoubtedly is, we must expect shrinking as a result of radiation. The rotation must become faster and faster, until the nebula takes on the shape of an oblate spheroid; the spheroidal shape, however, is soon departed from, and at a certain critical speed a sharp equatorial edge is developed. With still further contraction and accordingly higher speed of rotation, the particles which form the sharp edge are left behind. It does not matter if they are thrown off at all points equally or not. Neither rings of gas nor jets of gas, which must appear at two antipodal points when other heavenly bodies are present, can be stable; they must tend to form condensations or nuclei. In the sky we find plenty of examples of development such as just described. Three stages are shown in Plate XX, N.G.C. 4486, 3115, and 4565, examples

PLATE XX

a

b

c

a) N.G.C. 4486; *b*) N.G.C. 3115; *c*) N.G.C. 4565

of the nearly spherical nebula,[1] the lenticular figure, and the spiral seen edgewise. That N.G.C. 4565 is really a spiral cannot be doubted. The universe has revealed so many spiral nebulae inclined at different angles to the tangential plane of the celestial sphere that it is quite certain M 33 and N.G.C. 4565 belong to the same class of objects. That the spirals show rotation around the smaller axis has been proved spectroscopically in six or seven spirals whose planes are inclined to the celestial sphere. No object tested thus far has shown a lack of rotation. It thus seems only logical to interpret the displacements observed in those spirals which are viewed perpendicularly to their planes as motions corresponding to the rotation found spectroscopically. In only two of the spirals measured has it been possible to detect spectroscopic motion as well. Some of the others form so nearly an angle of 90° with the line of sight that no spectroscopic results can be expected, while others are so faint that no spectroscopic results have as yet been secured. For M 81 Wolf[2] has observed rotation in the central nebulous part within which it has been impossible to secure measures from the photographs. For M 33, however, it has been possible to observe both the radial velocity and the motion perpendicular to the line of sight for the same point, viz., the bright knot 10′ *nf* the nucleus. Taking into account the probable inclination of the nebula with respect to the tangential plane of the celestial sphere, we can gain some idea of the order of the parallax of the nebula; the result is $\pi = 0\overset{''}{.}0005$.

Happily there are other means for obtaining an idea of the distances of spiral nebulae. Jeans has shown that mathematical theory not only predicts that condensations will form in the arms of the nebula, but also predicts how far apart these condensations will be. The comparison of these calculated mean distances with the mean distances as they appear in the sky provide a second means of estimating the distances of the nebulae. In this way Jeans[3] estimated

[1] Although it cannot be decided whether this particular nebula is a spherical object or a disk perpendicular to the line of sight, the large percentage of round or nearly round nebulae found among the structureless objects makes it probable that many of these are spherical.

[2] *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 162, 1914.

[3] *The Nebular Hypothesis and Modern Cosmogony: Being the Halley Lecture delivered on 23d May 1922*, Oxford, 1923.

the parallax of the Andromeda nebula to be 0".0006; of M 101, 0".0011; and of M 51, 0".0065.

We have further possibilities of estimating the mean parallax of the larger spiral nebulae. Curtis[1] gives 0".033 as the average annual motion of sixty-six large spiral nebulae. In a recent paper[2] I have shown that Curtis' results cannot be due to the influence of accidental errors as was once assumed by Curtis himself. Comparing this mean proper motion with the mean radial velocity of about thirty objects (600 km/sec.) for which the line-of-sight motion has been determined, we derive a mean parallax of 0".00026. Finally, we may quote again the values recently derived for the mean parallax of sixty-seven and eighty-two spirals whose motions were determined by Curtis and Lundmark, respectively. Using 600 km/sec. for Campbell's K-term, the mean parallaxes are 0".00013 and 0".00015, respectively.

All this material seems to point to parallaxes for the larger spiral nebulae between a few ten-thousandths and a few thousandths of a second of arc. With such values the diameters of the spirals range from a few light-years to several hundred light-years. Since our present estimates of the Milky Way system vary, according to different authorities, from 20,000 to 300,000 light-years, it is clear that the present material indicates that the spirals, while enormous in size as compared with our solar system, are not at all comparable with the Milky Way system.

In concluding, I wish to express my sincere thanks to Messrs. Ritchey, Pease, Humason, and Duncan, who have secured the necessary photographs to carry out this work, and to Miss Davis and Mrs. Marsh, of the Computing Division, who have assisted in the numerous reductions involved.

Mount Wilson Observatory
March 1923

[1] *Publications of the Astronomical Society of the Pacific*, **27**, 217, 1915.

[2] *Mt. Wilson Contr.*, No. 243; *Astrophysical Journal*, **56**, 208, 1922.

THE VACUUM-SPARK SPECTRUM OF SILICON

By R. A. SAWYER and R. F. PATON

ABSTRACT

Vacuum-spark spectrum of silicon, λ 6700–λ 2100.—Silicon electrodes less than a millimeter apart were mounted in a suitably designed brass box with a quartz window, the best obtainable vacuum was produced, and a highly condensed spark was obtained by the use of 70,000 volts. Lines of iron, aluminum, calcium, and oxygen and the strongest lines of hydrogen, nitrogen, copper, zinc, and titanium appeared as impurities. The wave-lengths of 227 lines attributed to silicon in the region λ 6700–λ 2100, together with the measurements of other observers, are given in Table I. An additional 75 lines, listed in Table II, are given as doubtful, since they were faint lines appearing on only one plate. In the intervals λ 6700–λ 5500 and λ 4070–λ 3400 the wave-lengths are believed to be accurate to within 0.2 A and in the intervals λ 5500–λ 4070 and λ 3400–λ 2100 to within 0.1 A.

I. INTRODUCTION

Silicon is one of the most widely distributed of all the chemical elements, but is difficult to isolate in a pure state. Moreover the slight conductivity of pure silicon renders its use with ordinary light sources very difficult. It is not surprising, then, that most of the earliest spectroscopic work on silicon was done with its compounds only. Kayser[1] has summarized all of the important work on silicon up to 1912. The first work Kayser mentions dates back as far as 1859, when Plücker investigated the light given off by an electrical discharge in various gases and vapors in a Geissler tube. Using silicon chloride, Plücker found four lines which he attributed to silicon. Of those who followed, Rowland was the first to point out the presence of silicon in the sun, and Lockyer identified silicon lines in the spectra of many of the stars. Up to 1912, however, none of the work on silicon seems to have been particularly satisfactory.

The most recent work on the spectrum of silicon was done by Sir William Crookes[2] in 1914. He succeeded in obtaining some rather pure samples of silicon, and using a condensed spark between

[1] *Handbuch der Spectroscopie*, **6**, 478–494.

[2] *Proceedings Royal Society*, Series A, **90**, 512–520, 1914.

electrodes of this silicon, photographed the spectrum with a prism spectrograph. The rapid oxidation of the silicon made it very difficult to maintain the spark and necessitated long exposures. His work covered the region λ 6500 to λ 2100 and he published the wave-lengths of 43 lines, including most of the lines previously recorded. This investigation was the most thorough work that had been done on silicon up to that time. Since the work of Crookes, Fowler[1] has mentioned observing four lines in the visible region of the spectrum, not previously observed; and McLennan[2] has extended the investigation into the extreme ultra-violet, cataloguing some seventeen additional lines.

A comparison of the results of all the work that has been done on silicon brings out the fact that no two observers have the same list of lines, and also shows that even in the lines that have been recorded by several observers, the disagreement in wave-length seems almost unaccountably large. This disagreement may be partly accounted for, however, by the fact that many of the silicon lines are especially broad and hazy when produced at atmospheric pressure.

The importance of having accurate and complete results on the spectrum of silicon can well be realized when one considers the wide terrestrial distribution of this element, as well as the fact that its presence in many of the stars has been definitely established. Complete and accurate data on the spectrum of silicon are also desirable to throw new light on its spectral series, the details of which have not yet been completely worked out. With the discovery of the vacuum-spark, a new source of light became available that seemed to have particular advantages in this problem. The use of the vacuum-spark in extending the spectrum into the extreme ultra-violet,[3] had given results which led to the expectation that it might give new information in the rest of the spectrum. With this in mind, it was decided to photograph the vacuum-spark of silicon in the region covered by Crookes, hoping that the results might give more accurate information concerning the wave-lengths of some

[1] *Monthly Notices*, **74**, 196–197, 1916.

[2] *Philosophical Magazine*, **30**, 482–484, 1915.

[3] *Astrophysical Journal*, **52**, 286–300, 1920.

of the lines and even add to the already known list. In adapting the vacuum-spark to silicon, it was found necessary to use high voltage and extremely good vacua; but once obtained, the spark was brilliant, of a reddish yellow color, and resembled the vacuum-spark of carbon.

II. DESCRIPTION OF APPARATUS AND MANIPULATIONS

A 100,000-volt closed-core transformer gave the necessary voltage. Energy was supplied to the primary from a 110-volt power circuit, and controlled by a heavy rheostat in series with the primary. To prevent overheating the transformer and the silicon electrodes, a commutator switch was placed in the primary and adjusted so that the current was automatically turned on forty times a minute for about one half a second at a time. One terminal of the high voltage transformer was connected directly to one of the electrodes; the other terminal was grounded. By grounding the second electrode, the circuit was completed with only one line charged to a dangerous voltage.

In order to have a large amount of energy available when the spark passed, a capacity consisting of five glass condensers, immersed in oil and connected in series, was placed in parallel with the gap. This arrangement gave a capacity of the order of magnitude of one one-hundredth of a microfarad, while the voltage over any one condenser was not sufficient to puncture the glass dielectric.

The vacuum was obtained with two mercury diffusion pumps designed to work in series, and a small oil pump as a pre-pump. The vapor pressure of the mercury at room temperature was too great to permit a sufficiently high vacuum to be obtained. To reduce this vapor pressure, a glass trap was sealed in between the spark box and the pumps. When this trap was immersed in liquid air, the pressure due to any vapors present was sufficiently reduced so that vacua could be obtained that would remain non-conducting for sparking voltages of roughly 70,000 volts with a spark-gap length of less than one millimeter. It was found that, with the pumps working well, such a vacuum could be obtained in about five minutes. The pressure was then of the order of magnitude of one ten-thousandth of a millimeter of mercury or less.

Since the spark box was heated somewhat by the spark radiation, and the electrodes themselves heated to a white heat the instant the spark passed, frequent adjustments of the spark were necessary. To accomplish these adjustments without opening the box, the electrodes were mounted eccentrically in insulating plugs ground into brass tubes soldered to opposite ends of the box. The electrodes could then be rotated inside the box without affecting the vacuum. One electrode was mounted on a nut which ran on a shaft that could be rotated from the outside in such a way as to move the electrode in or out as desired, and thus regulate the distance between the electrodes. The box was made with one removable side. Leakage was prevented by the use of a rubber gasket and a rubber stopcock grease prepared for the purpose. This removable side made it possible to renew the electrodes and clean the windows through which the spark was photographed.

The electrodes themselves consisted of small pieces of elementary silicon mounted in brass clamps. Pure silicon is very hard and brittle, and the electrodes could be shaped to rough points only by chipping. The silicon was very kindly furnished by the Carborundum Company of Niagara Falls, who also furnished that used by Crookes and McLennan.

With the electrodes adjusted so that they were all but touching, a spark could be obtained with a comparatively poor vacuum. This spark was very feeble; for most of the energy available was dissipated by conduction through the gas remaining in the box. With the best vacuum that could be obtained, the distance between the electrodes could be made somewhat larger, but never more than about a millimeter for silicon electrodes. The spark under these conditions was extremely brilliant. The electrodes became white-hot almost instantly, and some of the silicon was vaporized, eventually fogging the windows through which the spark was photographed. If the power was kept on for more than a second at a time, occluded gases from the electrodes and their insulators were given off in sufficient quantity to stop the spark. In spite of the very refractory nature of silicon, the electrodes were worn away rapidly, and frequent adjustments and renewals were necessary.

To photograph the spark, two spectrographs were used, both of which were designed and built during the course of this research.

The region λ 6700 to λ 4070 was photographed by a glass spectrograph of the Littrow type. A 3-inch telescope objective lens with a focal length of roughly 24 inches was used. It was a cemented doublet and gave a reasonably flat field. The refracting angle of the prism was 60°, and its face about an inch and a half across. The entire region λ 6700 to λ 4070 was photographed on a 5-inch plate. Exposures with this instrument extended sometimes over 15-hour periods, making it necessary to house the entire spectrograph to prevent shifts due to changes in temperature.

The region λ 4100 to λ 2100 was photographed by a quartz spectrograph of a type first described by Fabry and Buisson.[1] In this instrument, the slit was mounted directly over the middle of the prism. A platinized concave mirror of 32-inch focal length served to collimate the light from the slit. The prism consisted of a cemented doublet of right and left quartz—the whole having an oblong face 1.75 inches high and a refracting angle of 60°. The light, after passing through the prism, was focused on the plate by a quartz lens whose focal length was 35.7 inches for the "D" lines. The use of only one quartz lens has the advantage of making the focal plane of the instrument more nearly perpendicular to the axis of the lens, since the collimating mirror insures parallelism of the beams of every wave-length incident on the prism. The inclination of the plate is due wholly to the variation with wave-length of the focal length of the camera lens. Thus, in this instrument, the photographic plate is at an angle of 45° to the axis rather than at 30°, as in an instrument of similar power but using a single quartz collimating lens. A shorter plate is thus used and yet the increased sharpness of the lines due to the lesser inclination more than offsets the shortness of the spectrum. With the quartz instrument, the whole region from λ 6000 to λ 2000 was photographed on a 10-inch plate. Only the region λ 4100 to λ 2000 was sufficiently dispersed, however, for satisfactory measurements. The copper line at λ 1944 has been photographed with this instrument, but no lines below λ 2100 were observed in the silicon spectrum.

For the region λ 6700 to λ 4070, radiation from the spark was focused on the slit with a lens. In order to be sure that the radia-

[1] *Journal de Physique*, **9**, 929, 1910.

tion from the iron comparison arc would enter the slit at exactly the same angle as that from the vacuum-spark, the arc was placed beyond the spark box and on the extension of the line joining the slit and the silicon spark gap. A short focus lens formed an image of the arc between the spark electrodes. This insured identity of source for the known and unknown spectrum. The iron arc was used as a standard, as very complete data on the wave-lengths of these spectral lines are now available. These data were taken from results obtained at Mount Wilson[1] and at Bonn,[2] Germany. For the quartz instrument, no external source comparison was necessary as it was found that the iron occurred as an impurity in the silicon. An iron comparison spectrum was photographed on the plate, however, to help in identifying the spark lines.

For the visible region, a special type of panchromatic plate, manufactured by Ilford, London, England, was found most satisfactory. Ordinary Seed plates were used in the ultra-violet.

In order to guard against any possible error due to changes in the optical set-up during any one exposure, the iron comparison arc radiation was admitted to the spectrograph at the beginning, at the end, and eight to ten times at intervals during the exposure. This precaution was not necessary, of course, with the ultra-violet plates, as sufficient standard lines existed as impurities in the silicon spark itself.

III. MEASUREMENTS AND RESULTS

The plates were measured on a large engine kindly made available by the Detroit Observatory of the University of Michigan. The reductions were made by using the Hartmann interpolation formula, and the final wave-lengths expressed in International Angstroms. The results show that the probable error for the region λ 6700 to λ 5500 is about 0.2 angstroms; from λ 5500 to λ 4070 less than 0.1 angstroms; from λ 4070 to λ 3400 about 0.2 angstroms; and from λ 3400 to λ 2100 less than 0.1 angstroms. It is unfortunate that only two plates were obtained with the quartz instrument and the

[1] *Astrophysical Journal*, **53**, 260, 1921.

[2] *Zeitschrift für Wissenschaftliche Photographie*, **12**, 207–235, 1913; and **19**, 149–157, 1919.

possible error is therefore larger in the region λ 4070 to λ 2100. The results from λ 6700 to λ 4070 are the mean observations from eight plates and are correspondingly more reliable.

In general, conclusions as to the accuracy of the present work were drawn from the variation in wave-lengths of the lines between individual plates, and from the accuracy with which determinations were made on impurity lines of known wave-length. It is of interest to note, in considering the accuracy, that interferometric measurements of wave-length have been made by Fabry and Buisson on three ultra-violet silicon lines.[1] The following table shows the comparison of their results with the present work.

Fabry and Buisson	Vacuum-spark	Intensity	Difference
2528.516	2528.50	5	0.016
2506.904	2506.90	4	0.004
2435.159	2435.17	5	0.011

Such an agreement as this leads to the belief that the error estimates given above are, at least for the strong lines, conservative.

Altogether over 700 lines have been measured in the spark spectrum of silicon, and of this number some 400 have been identified as due to the elements which occurred as impurities in the source. This leaves a list of more than 300 lines that have been ascribed to silicon. Of this number, 75 are classified as "doubtful" lines (see Table II). These lines are mostly very faint and have been observed on one plate only. Although the possible error in their wave-lengths may be somewhat larger than for the rest, and in many cases their actual existence may be in doubt, still they have been listed, and evidence is in favor of retaining them. The photographs were taken under varying conditions as to exposure time, voltage used on the spark, length of spark gap, plate developing, and kind of plate; and many other lines, equally faint, have been measured and found to be due to impurities. The main list (Table I) contains only those lines concerning which there can be little doubt. All possible impurities have been very carefully checked, and where there was any reasonable doubt, the line in question has been eliminated. There still remain in this list some 224 lines where previously less than 80 had been observed.

[1] *Astrophysical Journal*, 27, 169, 1908.

The wave-lengths are given in International Angstrom Units, whereas those by other observers are in the Rowland System. In general, the wave-lengths agree fairly well. In some cases, however, there exist variations of more than an angstrom. Such disagreement is hard to explain, and no such disagreement has been found in the results from the various plates measured during the course of the present work. The lines in most cases are sharp. General fogging of the plate due to band spectra of nitrogen or carbon, such as occurs in the ordinary spark in air or in the carbon arc, is entirely absent. Faint lines that under ordinary conditions would be obliterated, are, under the conditions of the vacuum-spark, measurable.

Of the impurities that appeared, only a very few can be attributed to remnants of gases in the vacuum chamber. The hydrogen line, $H\alpha$, showed faintly on only a few plates; $H\beta$, on two plates; $H\gamma$, and the remaining hydrogen lines, not at all. A few of the strongest mercury lines were recorded, and five of the strongest nitrogen lines. The strongest copper and zinc lines appear, coming probably from the brass clamps in which the silicon was mounted—there was usually a little sparking from the clamps. The lines which appeared due to impurities in the silicon itself are more numerous. In the visible, a few of the strongest iron lines appeared, but in the ultra-violet practically the entire spark spectrum of iron was observed. The appearance of the iron spectrum made unnecessary the use of a separate iron arc comparison, and eliminated any possibility of lack of identity in position of the silicon and the comparison source. Titanium, which Crookes mentioned as the principal impurity, appeared not at all in the visible, while only a few of the very strongest lines showed faintly in the ultra-violet. The entire spectrum of aluminum appeared in the visible, together with all but a few of the fainter arc lines in the ultra-violet. Of the two new aluminum lines observed by Shallenberger,[1] the one at λ 4150 came out strongly on all the plates, but the other one, at λ 2907, showed not at all. All of the strong oxygen lines appeared faintly, denoting probably a trace of oxide in the sample. One boron line showed, and a few carbon lines. The sodium D

[1] *Physical Review*, **19**, 398–399, 1922.

lines appeared extremely faint. H and K of calcium appeared strong, together with the calcium line at λ 4227. Most of the other strong calcium lines appeared on some of the plates. The resonance lines of magnesium, barium, and strontium, described by De Gramont[1] as "raies ultimes," were also photographed. The character of the impurity lines showed that while arc lines certainly appear in the vacuum-spark, the spectrum which it gives tends to emphasize the spark lines.

There is, of course, the possibility that some of the new lines found are due to impurities and not to silicon. To guard against this possibility, the vacuum-spark spectra of aluminum and of carbon were photographed and checked against the silicon plates. Only the aluminum line, λ 4150, however, was thus eliminated. The vacuum-spark spectra of iron, calcium, magnesium, and titanium were photographed through most of this region by Miss Carter,[2] who did not report any new lines. As the principal impurities occurring in the source used in this work are thus covered, the possibility that any of the new lines are due to impurities seems slight.

In comparing the results obtained, with those of previous observers, the number of new lines found is surprising. With but one or two exceptions, all the lines that have ever been found, either in the arc or spark of silicon, have been photographed. A few of the lines ascribed by some to silicon have been assigned to impurities. Special note is made of these lines in Table I. The intensities are estimated arbitrarily, but give some insight into the nature of the lines, and agree in the main with others. Strangely enough, though, the most intense line in the ultra-violet, λ 2541.91, has been missed by some observers, and no one before found it to be the strongest.

Time has not permitted a study of the lines for possible series, but the spectrum certainly contains many doublets and some triplets of constant frequency difference. The most conspicuous is the set of doublets having a frequency difference of 60 waves per centimeter. Sixteen of these doublets have been observed. Among the triplets, the difference $-74-34-$ occurs five times with

[1] *Comptes Rendus*, **171**, 1105–1109, 1920.

[2] *Astrophysical Journal*, **55**, 162, 1922.

TABLE I

VACUUM-SPARK SPECTRUM OF SILICON

Wave-Length in I.A. Units	Intensity	Rowland	Exner and Haschek	De Gramont and C. de Watteville	Eder and Valenta	Crookes
2122.33	0					
2123.77	1			2123.0	2124.17	2124.163
2135.90	2					
2148.81	1					
2179.37	2					
2189.62	2					
2192.22	3					
2208.01	2	2208.06	2208.06	2208.8	2208.1	2208.048
2210.92	3	2210.94	2210.97	2211.0	2210.9	2210.987
2211.76	2	2211.76	2211.84	2212.0	2211.8	2211.839
2216.71	3	2216.76	2216.75	2217.2	2216.76	2216.882
2218.11	3	2218.15	2218.13	2218.7	2218.15	2218.227
			2218.97	2219.5	2219.5	
2228.86	2					
2287.06	3					
2299.86	2					
(1)	1		2303.15	2303.8	2303.3	
2308.21	1					
2325.35	2					
2334.48	1					
2338.97	2					
2346.80	2					
2349.54	1					
2350.26	0					
2353.19	1					
2356.35	3				2356.9	
2357.20	2					
2358.02	2					
2363.82	2					
2371.02	0					
2416.54	1					
2419.80	1					
2432.23	2					
2435.17	5	2435.25	2435.27	2435.6	2435.25	2435.212
(2)	2	2438.86	2438.87	2439.4	2438.86	2438.911
2443.47	1	2443.46	2443.47	2443.5	2443.46	2443.484
(2)	1		2443.91			
(2)	1		2446.63		2446.0	
2449.70	2					
2452.12	1	2452.22	2452.23	2452.5	2452.22	2452.219
2480.04	1		2478.41		2479.8	
2481.10	0					
2483.29	1					
2486.28	2					
2495.79	1					
2500.90	1					
2503.64	1					
2506.90	4	2506.99	2507.01	2506.8	2506.99	2507.055
2514.34	4	2514.42	2514.41	2514.3	2514.42	2514.406
2516.08	5	2516.21	2516.26	2516.0	2516.21	2516.131
2517.48	3					

TABLE I—*Continued*

Wave-Length in I.A. Units	Intensity	Rowland	Exner and Haschek	De Gramont and C. de Watteville	Eder and Valenta	Crookes
2519.22.......	3	2519.30	2519.30	2519.2	2519.30	2519.276
2524.11.......	4	2524.21	2524.21	2524.3	2524.21	2524.110
2528.50.......	5	2528.60	2528.60	2529.0	2528.60	2528.585
2532.41.......	1	....	2532.45	2533.0	2533.2	....
(2)	1	....	....	2535.0	2534.7	....
2541.91.......	10	....	2541.90	....	2541.89	2541.970
2559.20.......	4	....	....	....	....	....
....	....	....	2568.8	2569.0	2568.8	....
2570.78.......	1	....	....	....	....	....
2580.35.......	1	....	....	....	....	....
2593.71.......	1	....	....	....	....	....
(2)	4	2631.39	2631.38	....	2631.39	2631.370
2637.92.......	1	....	....	....	....	....
2640.89.......	2	....	....	....	....	....
2645.59.......	1	....	....	....	....	....
2649.47.......	0	....	....	....	....	....
2650.73.......	2	....	....	....	....	....
2658.29.......	0	....	....	....	2659.0	....
2672.43.......	0	....	....	....	2673.3	....
2675.25.......	2	....	....	....	....	....
....	....	....	....	....	2677.4	....
2682.42.......	2	....	....	....	....	....
2687.75.......	2	....	....	....	....	....
(*)	....	....	....	....	2689.8	....
2695.28.......	2	....	....	....	....	....
2697.20.......	1	....	....	....	....	....
2701.53.......	1	....	....	....	....	....
2716.25.......	1	....	....	....	....	....
2813.61.......	1	....	....	....	....	....
2831.40.......	2	....	....	....	....	....
2858.14.......	1	....	....	....	....	....
2866.28.......	1	....	....	....	....	....
2869.73.......	1	....	....	....	....	....
2873.10.......	1	....	....	....	....	....
2881.70.......	7	2881.70	2881.73	2881.7	2881.70	2881.690
2899.52.......	2	....	....	....	....	....
2904.01.......	2	....	....	....	....	....
2905.59.......	2	....	....	....	....	....
2924.04.......	2	....	....	....	....	....
2976.42.......	1	....	....	....	....	....
2987.99.......	2	2987.77	2987.77	2987.8	2987.77	2987.750
3002.37.......	1	....	....	....	....	....
3012.55.......	1	....	....	....	....	....
3034.37.......	0	....	....	....	....	....
3043.70.......	2	....	....	....	....	....
3086.44.......	5	....	3086.6	3087.2	....	3086.479
3093.28.......	4	....	3093.6	3094.5	....	3093.694
3096.92.......	3	....	....	....	....	....
3103.80.......	1	....	....	....	....	....
3106.14.......	1	....	....	....	....	....
3130.48.......	3	....	....	....	....	....
3147.01.......	1	....	....	....	....	....
3149.67.......	4	....	....	....	....	....

TABLE I—*Continued*

Wave-Length in I.A. Units	Intensity	Rowland	Exner and Haschek	De Gramont and C. de Watteville	Eder and Valenta	Crookes
3165.78	4					
3185.28	3					
3188.85	1					
					3191.1	
3192.69	1					
3196.12	2					
3199.42	2					
3203.87	2					
3210.27	3					
3230.43	2					
3234.05	3					
3241.80	4					
(1)	2					3247.684
3258.45	2					
3270.33	1					
3313.85	0					
3333.48	1					
						3438.444
3464.14	2					
3470.68	1					
3481.55	2					
3486.79	4					
3532.02	0					
3537.70	0					
3548.24	1					
3563.61	1					
3570.05	1					
3576.15	2					
3590.77	3		3591.0			
3702.01	2					
3713.23	2					
3762.42	2					
3774.74	2					
3791.13	3		3791.8	3791.5	3791.1	
3796.18	4		3796.5	3796.0	3795.9	3796.364
3806.60	6		3806.90	3807.0		3806.802
(2)	1				3826.7	
(2)	1				3834.4	
3853.01	2		3854.02		3854.00	3853.812
3856.09	4		3856.19	3856.0	3856.20	3856.193
3862.51	4		3862.80	3862.5	3862.75	3862.743
3870.64	1					
3891.55	2					
3905.37	3	3905.66	3905.71	3905.5		3905.726
3924.44	3					
3991.62	2					

(1) This line apparently due to copper.
(2) Lines in these positions attributed to iron.
(*) A line here too faint to measure accurately.

TABLE I—*Continued*

Wave-Length in I.A. Units	Intensity	Rowland	Exner and Haschek	De Gramont	Lunt	Crookes
4016.28	1					
			4021.0			
			4030.1			
4058.49	1					
4088.88	6				4089.0	4089.016
4102.62	0	4103.09	4103.2			
			4103.7			
4116.15	6				4116.35	
4128.11	8		4128.1	4128.2	4128.20	4128.189
4130.96	10		4131.0	4131.3	4131.08	4131.192
4183.67	0					
4190.92	2		4191.1		4191.00	
4198.25	2				4198.43	
4212.66	1					
4236.45	1					
4277.95	1					
4314.32	1					
4328.40	1					
4338.57	2					
4372.33	1					
4377.80	1					
4494.02	1					
4552.50	20		4552.75	4552.5	4552.82	4552.841
4567.66	16		4567.95	4567.0	4567.82	4568.123
4574.66	12		4574.90	4574.5	4774.86	4574.823
4619.60	1					
4631.22	2					
4632.94	1					
4638.36	1					
4654.08	4					
4665.76	1					
4673.45	0					
4683.10	2					
4709.20	1					
4716.71	4					
4730.52	1					
			4764.20			
4776.58	1					
4800.43	1					
4813.28*	3					
4819.57*	4					
4828.84*	6					
4837.97	1					
4842.35	1					
4883.51	1					
4907.50	1					
4921.86	1					
4943.16	1					
5041.17	6		5043	5045.5	5042.4	5042.715
5056.10	8		5057.3	5060.0	5057.1	5057.832
5092.00	1					
5101.42	0					
5113.70	1					
5182.13	1					

TABLE I—*Continued*

Wave-Length in I.A. Units	Intensity	Rowland	Exner and Haschek	De Gramont	Lunt	Crookes
5185.64	1					
5193.21	1					
5196.62	1					
5202.85	2					
5219.05	1					
5240.63	0					
5295.66	0					
5417.36	1					
5438.58	1					
5456.61	1					
5469.18	1					
5472.90	1					
5494.82	0					
5576.23	0					
5589.18	0					
5593.82	0					
5632.72	0					
5639.13	1					
		5645.83				
5669.63	2	5665.78				
		5684.71				
5688.83	1	5690.65				
5694.68	0					
5701.26	1	5701.32				
5706.23	1	5708.62				
5716.08	1					
5739.20†	4					
		5772.36				
5785.45	0					
5800.25	1					
5806.30	1					
5845.06	1					
5867.33	2					
5914.7	1					
		5948.77		5948.?		
5957.80	2			5960.3	5960.3	5961.6
5979.20	2			5978.9	5981.3	5982.0
6329.71	1					
6339.39	2					
6347.01	6	6342.2	6347.1	6342.2	6346.9	6346.962
6363.10	1					
6371.09	4	6369.7	6371.4	6369.7	6372.2	6371.032
6662.3	1					
6671.2	1					

* Observed also by A. Fowler, "Silicon Lines in the Spectra of B-type Stars," Royal Astronomical Society, *Monthly Notices*, **76**, 196–197, 1916.

† Observed also by A. Fowler (*ibid.*).

a possible sixth. Several other frequency differences have been observed, but unless series can be found relating them, their significance may be questioned in many cases. For this reason they are omitted.

In conclusion it may be said that, so far as is known, the present work is the only extensive work on the spectrum of silicon which has been published in International units. The discovery of so large a number of new lines shows that for an element with which it is very difficult to get results when the usual light sources are used the vacuum-spark is a very powerful source.

TABLE II

VACUUM-SPARK SPECTRUM OF SILICON

(A list of doubtful lines observed on one plate only)

Wave-Length in I.A. Units	Intensity	Wave-Length in I.A. Units	Intensity
2125.77	0	3420.32	0
2468.19	1	3423.66	0
2482.58	1	3430.51	0
2505.03	0	3499.61	1
2546.69	0	3651.66	2
2566.33	1	3661.97	0
2578.22	1	3670.49	1
2619.27	1	3681.38	1
2634.35	1	3779.47	1
2669.26	1	3837.65	1
2672.43	1	4141.04	0
2678.31	0	4304.15	0
2686.34	1	4321.81	0
2690.64	1	4324.40	0
2704.84	2	4598.21	0
2722.63	1	4599.62	0
2724.98	1	4624.91	0
2785.45	1	4627.35	0
2793.75	1	4657.20	0
2855.41	0	4659.17	0
2856.76	0	4900.48	0
2875.10	1	4949.72	0
2887.68	2	4958.19	0
2938.98	0	5105.98	0
2945.72	1	5108.94	0
2952.83	2	5206.66	0
2980.33	1	5392.75	0
2996.97	1	5448.46	0
3053.02	2	5451.91	0
3121.99	1	5466.70	0
3266.95	2	5540.16	0
3276.10	2	5622.36	0
3281.64	1	5796.71	0
3283.69	1	5813.50	0
3402.96	0	5827.16	0
3413.41	0	6079.23	0
3416.94	0	6099.73	0

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A SPECTROSCOPIC METHOD OF DERIVING THE PARALLAXES OF THE B-TYPE STARS[1]

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ABSTRACT

Spectroscopic method of determining the absolute magnitudes of B-type stars.—The correlation between spectral type and absolute magnitude found to exist in the case of stars of the A-type of spectrum has been extended to those of the B-type. The spectrograms of 300 such stars have been classified as accurately as possible, according to the Harvard system, and *the character of the lines* has been indicated as nebulous or sharp. With the aid of absolute magnitudes derived (1) from trigonometric parallaxes, (2) from the moving clusters Pleiades, Perseus, Orion, and Scorpio-Centaurus, (3) from mean values obtained by Plummer, Charlier, Kapteyn and others, the correlation between type and absolute magnitude has been established for these stars. The agreement found in this way is satisfactory. The mean difference between spectroscopic absolute magnitudes and those for the four moving clusters is -0.2, and for the trigonometric values $+0.4$. The *principal uncertainties* arise in the case of the O-type stars and the early B-stars with sharp and diffuse lines.

The mean spectroscopic absolute magnitudes for 299 of the stars have been compared with the reduced proper motion $0.2\,m+\log\mu$, the stars being divided into 12 groups for this purpose. The curve found in this way is nearly a straight line except for the stars of extremely small proper motion. This result yields valuable evidence for the accuracy of the method and indicates that the reduced proper motion can be used to derive the absolute magnitudes of these stars with a considerable degree of precision.

In a recent publication[2] we showed that the individual parallaxes of the A-type stars could in most cases be determined with a satisfactory degree of accuracy from their spectral types. A close correlation was found to exist between absolute magnitude and the subdivision of the A-type of spectrum based upon the Harvard system of classification. It was stated that similar considerations could probably be applied to B-type stars and the purpose of this communication is to show the results of such an application to about 300 stars of this type for which we have obtained spectrograms at Mount Wilson.

The first step in this investigation, as in the case of the A-type stars, was an accurate determination of spectral type. The Harvard system was followed as closely as possible and use was made of spectrograms of typical stars for reference purposes. The results

[1] *Contributions from the Mount Wilson Observatory*, No. 262.

[2] *Mt. Wilson Contr.*, No. 244; *Astrophysical Journal*, **56**, 242, 1922.

show, in general, close accordance with the spectral types of the Revised Draper Catalogue. The principal differences come in types B5 and B8, and it is evident that, if the successive subdivisions from B0 to B9 are to be considered as equal steps, the interval from B5 to B8 is too large as compared with similar intervals preceding and following. Thus the difference in type between B2 and B5, or between B8 and A0 is greater than between B5 and B8. The Harvard observers have classified no stars in this interval and it seems probable that the spectra listed by them as B5 could be classed advantageously as B7 and the other subdivisions revised to correspond. We have in addition adopted provisionally in the case of the Harvard stars of type Oe5 the nomenclature suggested by H. H. Plaskett,[1] using the notation O5, O6, etc., to indicate the dark line stars immediately preceding B0. The spectra have further been characterized by the letters "n" and "s" to indicate nebulous or diffuse and sharp lines, respectively.

The correlation of these spectral types with absolute magnitude was first attempted through the use of trigonometric parallaxes and those derived from moving clusters such as the Pleiades, Orion, Perseus, and Scorpius. A satisfactory relationship was obtained in this way, but it seemed desirable to make use in addition of the important statistical investigations of Plummer,[2] Charlier,[3] Kapteyn,[4] and others on the mean magnitudes of the B-type stars. Accordingly, their values, which, as is well known, show clearly the decrease in absolute magnitude with advancing spectral type, were combined and assigned high weight in the construction of our reduction curves.

Since many of the trigonometric parallaxes of the B-type stars are negative, it is not possible to compute the absolute magnitudes of the individual stars in all cases. Accordingly, the procedure adopted has been to divide the stars into groups with a small range in spectral type and to compute the average absolute magnitude for each group on the assumption that the sum of the errors of the

[1] *Publications of the Dominion Astrophysical Observatory*, 1, No. 30.

[2] *Monthly Notices of the Royal Astronomical Society*, 73, 174, 1912.

[3] *Meddelanden från Lunds Observatorium*, Serie II, No. 14.

[4] *Mt. Wilson Contr.*, No. 147; *Astrophysical Journal*, 47, 255, 1918.

trigonometric parallaxes for the stars in each group is zero. The formula used is one derived by Strömberg and has the form

$$M = 5 + 5 \log \Sigma\pi_0 - 5 \log \Sigma 10^{-0.2\,m}$$

where M is the required absolute magnitude, π_0 the observed trigonometric parallax, and m the apparent magnitude.

The final values upon which our curves are based are as follows:

MOVING CLUSTERS AND TRIGONOMETRIC PARALLAXES

B0.5n	−2.8;	B1.8n	−1.4;	B3.3	−0.6;	B5.3n	−0.1;	B8.2n	+0.3
		B1.7s	−1.9					B8.4s	−0.1

MEAN ABSOLUTE MAGNITUDES FROM STATISTICAL INVESTIGATIONS[1]

O5 to O9 −1.8; B1 −2.4; B4 −0.9; B8.5 0.0; A0 +0.4; A2 +1.3

In addition to these values those furnished by the curves for the A-type stars are available for stars of type B8 or later. These are in good agreement with the results of Plummer and have been adopted directly.

Two outstanding difficulties are the absolute magnitudes of the O-type stars and the question of the difference in absolute magnitude between stars of the early types with sharp and diffuse lines. The material is insufficient for a definite answer to either. The investigation of Gyllenberg[2] on stars of Harvard type Oe5 would indicate that their average absolute magnitude is considerably fainter than the probable value for B0 and B1 stars. Accordingly, we have made a turning point in our curves at B0 which would bring them into agreement with Gyllenberg's result at about O8. The values for these stars must, however, necessarily be uncertain.

Our investigation of the A-type stars showed that the average absolute magnitude of those showing sharp lines was about 0.7 magnitude brighter than that of stars with diffuse lines. This applies to spectra as early as B8. In the case of stars of types

[1] These include the results of Plummer, Charlier, Kapteyn, Dyson, Gyllenberg, and Malmquist.

[2] *Arkiv for Matematik, Astronomi och Fysik, K. Svenska Vetenskapsakademien*, **16**, No. 24.

B0 to B5, however, the question is very doubtful on account of the small amount of material available for study. There is some indication from a number of stars in Orion of average type about B2 that the stars having sharp lines are somewhat brighter than those with diffuse lines. We have made a partial compromise by drawing two curves, one for stars of each class of lines, which come together at B0. The procedure is somewhat arbitrary, but probably no very serious error is introduced since the number of stars with sharp lines between types B0 and B5 is small. It is quite possible that some of the stars in Orion which have sharp spectral lines and great intrinsic brightness, such as γ Orionis, may resemble the c-class of stars of high luminosity to which β Orionis of type B8 belongs. In the absence of other data, however, they have been included in our list.

The values resulting from our reduction curves are as follows:

TABLE I

Spectrum	Diffuse	Sharp
O5–O9	$-2^{\mathrm{M}}5$	$-2^{\mathrm{M}}5$
B0	−3.1	−3.1
B1	−2.4	−2.6
B2	−1.5	−2.0
B3	−0.9	−1.5
B4	−0.6	−1.2
B5	−0.5	−1.1
B6	−0.3	−0.9
B7	−0.1	−0.8
B8	+0.1	−0.6
B9	+0.5	−0.2
A0	+0.9	+0.2
A1	+1.3	+0.6
A2	+1.7	+0.9

These values have been used in the determination of the absolute magnitudes of 300 stars of the O and B types for which we have spectrograms at Mount Wilson. The list is given in Table II. The c-stars have been included, although it may prove that as in other types these stars require separate treatment.

TABLE II

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
5	5.5	0″.010	B9n	B8	+0.5	+0″.010	
18	5.5	.034	B9n	B8	+0.5	.010	
52	6.0	.028	B9s	B9	−0.2	.006	−0.022 t
67	5.4	.017	B5n	B3	−0.6	.006	
68	5.4	.016	B8n	B9	+0.3	.010	
118	5.1	.021	B8n	B5	0.0	.010	
124	5.9	.028	B3s	B3	−1.5	.003	
159	5.4	.022	B9s	B8	−0.2	.008	
169	5.4	.014	B9n	B9	+0.5	.010	
224	5.5	.039	B9s	B9	−0.2	.007	
263	5.5	.038	B9s	B9	−0.2	.007	+0.010 g
265	5.5	.027	B8n	B8	+0.3	.009	
284	5.8	.028	B9s	B8	−0.2	.006	
340	5.8	.025	B9n	B9	+0.5	.009	+0.017 t
347	6.2	.011	B9n	B9	+0.5	.007	
379	5.4	.053	B9s	B8	−0.2	.008	+0.013 g
412	5.5	.035	B3n	B3	−1.2	.005	+0.009 g
419	3.4	.043	B5n	B3	−0.5	.017	−0.001 t +0.011 g
425	5.0	.010	B8s	B8	−0.6	.008	
457	5.6	.010	cB6s	B5p	−0.8	.005	
459	5.0	.041	B9n	B8	+0.2	.011	
488	6.4	.013	B7s	B3p	−0.8	.004	
507	6.4	.015	B2s	B1p	−2.0	.002	
519	6.7	.013	cB3s	B0	−1.6	.002	
529	6.1	.005	B5s	B5	−1.0	.004	
535	7.0	.009	B8s	B8p	−0.6	.003	
544	6.2	.012	B1n	B2	−2.8	.002	
546	5.5	.019	B8n	B5	+0.2	.009	
641	5.3	.022	B6n	B5	−0.6	.007	
648	5.5	.045	B5n	B5	−0.4	.007	
666	5.3	.054	A0n	A2	+0.7	.012	
678	5.4	.043	B8n	B5	0.0	.008	+0.010 g
682	5.5	.017	B9n	B9	+0.5	.010	
692	5.6	.016	B8s	B5	−0.6	.006	
699	5.8	.006	B9s	B9	−0.2	.006	
715	5.6	.038	B9n	B9	+0.5	.010	+0.009 g
731	5.5	.039	A1s	A0	+0.8	.011	
740	5.4	.038	B7n	B5	0.0	.008	+0.009 g
742	5.3	.045	B4n	B3	−0.8	.006	+0.011 g
744	5.1	.041	B4n	B3	−0.6	.007	+0.010 g
767	5.3	.036	B5n	B3	−0.4	.007	+0.009 g
780	4.9	.032	B4s	B5	−1.0	.007	+0.008 g
783	5.6	.049	B6n	B5	−0.2	.007	+0.011 g
790	4.7	.044	B3n	B5	−0.8	.008	+0.010 g
791	5.3	.033	A0n	A2	+0.9	.013	

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
801	5.1	0″.021	B9n	A0	+0.5	+0″.012	
802	5.5	.040	B8s	B8	−0.4	.007	+0.010 g
807	5.8	.011	B1n	B3	−2.4	.002	
833	5.6	.032	B8n	B5	0.0	.008	+0.008 g
838	3.1	.046	B5n	B5	−0.4	.020	+0.006 t +0.011 g
839	5.0	.015	B1s	B2	−2.3	.003	
841	5.5	.019	B8n	B8	+0.3	.009	
852	3.8	.053	B8n	B8	+0.2	.019	+0.014 g
855	5.6	.057	B9n	B8	+0.5	.010	
856	4.4	.049	B7n	B5	−0.1	.013	+0.013 g
857	5.1	.017	B9n	B8	+0.5	.012	
860	4.0	.054	B9s	B5	−0.2	.014	+0.014 g
861	5.8	.044	B9n	B8	+0.3	.008	+0.011 g
865	4.2	.059	B5n	B5	−0.4	.012	+0.015 g
869	3.0	.052	B8n	B5p	+0.3	.029	+0.013 g
872	5.5	.059	B8n	B8	0.0	.008	
877	3.8	.053	B9n	B8	+0.5	.022	+0.014 g
879	5.2	.053	B8n	B8p	+0.1	.010	+0.013 g
886	5.7	.011	B3n	B3	−0.9	.005	
893	5.5	.012	B7n	B8	−0.1	.008	
896	4.9	.005	B9n	B9	+0.5	.013	
898	5.3	.039	B4n	B5	−0.6	.007	+0.010 g
904	5.5	.020	B2n	B3	−1.2	.005	
910	3.0	.039	B1n	B1	−2.0	.010	−0.008 t
913	4.0	.018	O8n	Oe5	−2.4	.005	−0.004 t
926	5.2	.024	B4n	B5	−0.8	.006	
956	5.4	.013	B5s	B3	−1.0	.005	
992	5.5	.014	B4n	B3	−0.7	.006	
1015	5.4	.041	B9n	B9	+0.5	.010	
1039	5.4	.019	B6n	B6	−0.4	.007	
1070	5.7	.012	B9n	B9	0.0	.007	
1084	5.3	.018	B7n	B5	−0.4	.007	
1097	5.4	.027	B8s	B9	−0.7	.006	
1107	4.3	.023	B4n	B5	−0.8	.010	+0.018 t
1139	4.4	.009	B0s	B0	−3.0	.004	−0.026 t
1147	3.8	.007	B1s	B3	−2.3	.006	0.000 t +0.007 g
1159	3.9	.004	B3s	B3	−1.5	.008	+0.003 t +0.007 g
1163	4.7	.145	A0n	A0	+0.9	.017	
1165	5.7	.024	B9n	B8	+0.5	.009	
1177	5.6	.067	A0n	B9	+1.1	.013	
1204	3.3	.081	B3s	B3	−1.5	.011	+0.017 t
1216	5.5	.011	B3s	B3	−1.5	.004	
1249	5.8	.029	O9s	B0p	−3.0	.002	
1262	3.7	.017	B7s	B5	−0.8	.013	+0.007 g
1274	5.1	.045	B5n	B3	−0.4	.008	+0.011 g

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
1283	5.6	0".010	B3n	B3	−0.9	+0".005	
1284	4.6	.005	B2s	B3	−2.0	.005	+0.007 g
1293	5.9	.015	B9n	B9	+0.7	.009	
1295	5.6	.015	B2s	B3	−2.0	.003	+0.007 g
1301	3.4	.006	B1s	B1	−2.3	.007	+0.007 g
1303	1.7	.020	B2s	B2	−2.0	.018	+0.021 t +0.007 g
1304	1.8	.180	B9s	B8	−0.2	.040	−0.002 t
1307	6.2	.009	B3n	B3	−0.8	.004	
1310	5.7	.019	B9s	B9	0.0	.007	
1314	4.7	.014	B3n	B2	−0.9	.008	−0.015 t +0.007 g
1318	5.5	.031	B9n	A0	+0.5	.010	
1320	5.4	.039	B9n	A0	+0.5	.010	
1328	5.7	.013	B4n	B3	−0.8	.005	
1332	5.5	.009	B2s	B3	−2.0	.003	+0.007 g
1339	2.5	.004	B1n	B0	−2.4	.010	−0.018 t +0.007 g
1339C	6.9	.003	B6n	....	−0.4	.003	
1340	4.6	.012	B1s	B3	−2.6	.004	+0.007 g
1346	5.6	.006	B4n	B3	−0.6	.006	
1349	5.4	.007	B3n	B2	−0.9	.005	
1354	5.3	.032	B3n	B3	−0.9	.006	
1357A	3.7	.011	O8n	Oe5	−2.1	.006	
1357B	5.6	.011	B2s	....	−1.8	.003	
1361	5.6	.004	B2s	B1	−1.8	.003	+0.007 g
1363	5.4	.005	O8n	Oe5	−2.1	.003	−0.016 t
1364	4.6	.002	B2s	B3	−1.4	.006	
1365	5.2	.020	B6n	B1	−0.3	.008	
1366A	2.9	.005	O9n	Oe5	−2.4	.009	
1366B	7.3	.005	B9s	....	−0.2	.003	
1370	1.8	.002	B1n	B0	−2.4	.014	+0.008 t +0.007 g
1375	3.0	.028	B4n	B3p	−0.6	.019	
1382	5.8	.024	B3n	B1	−1.0	.004	
1389	3.8	.001	O9s	B0	−2.8	.005	+0.007 g
1398	2.0	.011	B0n	B0	−2.8	.011	−0.020 t +0.007 g
1398B	4.2	.011	B1n	....	−2.4	.005	
1399	5.0	.014	B4n	B3	−0.8	.007	+0.007 g
1423	5.7	.059	B9n	A2	+0.3	.006	
1435	2.2	.006	B1n	B0	−2.4	.012	+0.029 t +0.007 g
1507	4.7	.015	cB2s	cB2p	−2.3	.004	
1517	5.6	.123	B4n	B3	−0.8	.005	+0.074 t
1523	5.4	.007	B5s	B3	−1.0	.005	

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
1567......	5.8	0".041	B2s	B2	−2.0	+0".003	
1568......	5.3	.021	B8n	B9	+0.3	.010	
1572......	5.4	.017	B9s	B9	−0.2	.008	
1578......	6.3	.013	B4s	B2	−1.0	.003	
1609......	2.0	.007	B1s	B1	−2.3	.014	+0.012 t
1706......	4.7	.008	O8n	Oe5	−2.4	.004	−0.004 t
1739......	5.2	.012	B8n	B8	+0.1	.010	
1751......	5.7	.023	B8s	B8	−0.4	.006	
1754......	5.3	.019	B6n	B5	−0.2	.008	
1807......	5.8	.031	B3s	B3	−1.8	.003	
1817......	3.1	.009	cB1s	cB5p	−2.8	.007	
1899......	4.9	.016	O9s	Oe	−2.8	.003	
1905......	6.5	.029	B9n	B8	+0.5	.006	+0.006 t
1906......	5.6	.042	B8n	B8	+0.1	.008	+0.012 t
1944......	3.1	.066	B9n	B8	+0.5	.030	+0.022 t
1955......	6.0	.042	B3n	B3	−0.8	.004	
1998......	5.7	.016	B2s	B3	−2.0	.003	
2019......	5.8	.024	B3n	B5	−0.9	.005	
2045......	5.3	.027	B8n	A0	0.0	.009	
2159......	5.5	.014	B3s	B3	−1.5	.004	
2353......	5.7	.098	A0n	A0	+0.7	.010	
2451......	5.5	.032	B9n	B8	+0.5	.010	
2465......	5.3	.052	A1s	B8	+0.6	.011	
2492......	5.5	.040	B9n	B9	+0.7	.011	
2600......	5.0	.031	B4n	B3	−0.6	.008	
2698......	1.3	.247	B9n	B8	+0.5	.069	+0.055 t
2712......	6.1	.021	A0n	A0	+0.9	.009	
2724......	5.4	.074	A0n	A0	+0.9	.013	
2748......	6.5	.008	B2n	B3	−1.2	.003	
2788......	5.2	.052	B9n	B9	+0.5	.011	
2792......	5.0	.054	B5s	B5	−1.1	.006	
2797......	5.5	.048	B8n	B9	+0.1	.008	
2798......	5.8	.015	B9n	B9	+0.5	.009	
2804......	3.8	.009	B1s	B0p	−2.3	.006	+0.039 t
2839......	6.9	.013	B9n	A2	+0.5	.005	
2866......	5.4	.049	B8n	B9	+0.3	.010	
3045......	5.8	.011	B3n	B3	−0.8	.005	
3055......	4.8	.061	B9n	B9	+0.5	.014	
3138......	5.3	.020	B3n	B3	−0.9	.006	
3190......	3.4	.110	A0n	A2	+0.9	.032	
3309......	5.0	.135	B9n	A0	+0.5	.013	
3329......	6.0	.054	A0n	A3	+0.7	.009	
3392......	5.1	.027	B9n	B9	+0.7	.013	
3428......	6.3	.039	A0n	A0	+0.7	.008	
3476......	1.2	.055	B3n	B2	−0.9	.038	

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
3546......	6.2	0".053	A0n	A0	+0.9	+0".009	
3604......	5.2	.064	B6n	B8	−0.4	.008	
3653......	5.6	.044	B9s	B9	−0.4	.006	
3724......	2.6	.049	B3n	B3p	−1.0	.019	+0.013 g
3756......	5.8	.026	A0n	B9	+0.9	.010	
3820......	6.4	.029	A0n	A0	+0.7	.007	
3875......	6.3	.029	B9s	B9	−0.2	.005	
3915......	5.4	.051	B9n	B9	+0.5	.010	
3944......	5.6	.028	B4n	B3	−0.8	.005	
3946......	6.1	.016	B9n	B9	+0.5	.008	
3955......	5.2	.028	B6s	B5	−0.8	.006	
3988......	5.1	.014	B6n	B8	−0.2	.009	
4008......	5.4	.055	B5n	B8	−0.4	.007	
4019......	4.8	.042	B4n	B3	−0.6	.008	+0.011 g
4028......	5.2	.099	A0n	A2	+0.9	.014	
4033......	5.1	.037	B4n	B3	−0.8	.007	+0.010 g
4034......	4.7	.034	B3n	B3	−0.8	.008	+0.009 g
4037......	5.4	.046	B6n	B5	−0.2	.008	+0.012 g
4038......	5.4	.047	B4n	B3	−0.8	.006	
4041......	5.9	.034	B9n	B8	+0.5	.008	
4043......	5.9	.046	B5n	B5	−0.4	.005	+0.012 g
4058......	5.4	.052	B6n	B8	−0.2	.008	
4086......	2.9	.031	B0n	B1	−2.8	.007	{−0.019 t, +0.008 g}
4087C....	5.1	.034	B3n	....	−1.2	.005	+0.009 g
4097......	5.6	.065	B9n	A0	+0.5	.010	
4115......	4.7	.047	B2n	B3	−1.2	.007	+0.012 g
4117......	4.3	.034	B2n	B3	−1.6	.007	+0.015 t
4151......	5.5	.009	B9n	B8	+0.5	.010	
4178......	5.2	.024	B4n	B3	−0.8	.006	+0.006 g
4184......	5.5	.026	A0n	A3	+0.9	.012	
4198......	4.9	.028	B3n	B3	−0.9	.007	+0.008 g
4213......	5.0	.042	B9n	cB8p	+0.7	.014	
4291......	6.4	.011	A1n	A0p	+1.3	.010	
4338......	6.2	.038	B9n	B3	+0.5	.007	
4345......	5.6	.008	B3n	B3	−0.9	.005	
4368......	3.2	.024	B5s	B5	−1.1	.014	
4427......	5.4	.016	B5n	B5	−0.5	.007	
4442......	5.7	.108	A0n	A2	+0.9	.011	
4468......	5.7	.023	B9n	A0	+0.5	.009	
4527......	5.7	.006	B2n	B3	−1.5	.004	
4548......	3.9	.014	B2s	cB5p	−2.0	.007	
4552......	4.4	.026	B9n	A2	+0.5	.017	
4562......	5.1	.018	B3s	B3	−1.5	.005	+0.002 t
4573......	5.8	.026	B6n	B8	−0.2	.006	
4576......	6.2	.014	B0s	B1	−2.8	.002	

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
4612	5.4	0″.005	B0n	cB0	−2.6	+0″.003	
4613	6.0	.010	B1n	B1	−2.8	.002	
4620	5.4	.009	B6n	B5	−0.2	.008	
4640	6.5	.010	B9n	B8	+0.5	.006	
4668	6.0	.007	B8n	B8	0.0	.006	
4685	5.2	.035	A0n	A0	+0.9	.014	
4687	5.8	.031	B3s	B3	−1.5	.003	
4702	5.4	.007	B9s	B8	−0.2	.008	
4721	5.5	.024	B8s	B8	−0.4	.007	
4740	5.0	.022	B8n	B5	+0.2	.011	
4772	5.8	.014	B3n	B2	−0.9	.005	
4777	7.8	.020	B8n	B3	0.0	.003	
4783	5.0	.024	B6s	B5	−1.0	.006	
4794	5.5	.011	B3n	B3	−0.9	.005	
4813	5.6	.018	B8n	B8	+0.3	.009	
4816	5.4	.025	B7s	B5	−0.8	.006	
4821	5.8	.002	B6s	B8	−0.6	.005	
4842	5.2	.025	B3n	B3	−1.2	.005	
4873	5.1	.009	B6n	B5	−0.3	.008	
4883	5.4	.017	B3n	B3	−0.9	.005	
4917	5.4	.013	B4n	B	−0.6	.006	
4936	4.1	.129	B9n	B8	+0.5	.019	
4942	4.9	.015	B8s	B5	−0.4	.009	
4954	5.7	.023	B8n	B8	+0.1	.008	
4974	5.8	.037	A0n	A	+0.9	.010	
4987	5.4	.014	B8n	B9	+0.3	.010	+0.006 t
5003	5.0	.004	B4n	B	−0.7	.007	
5073	6.3	.007	O8n	Oe5	−2.6	.002	
5083	5.5	.015	B0n	....	−2.7	.002	+0.009 t
5087	5.8	.022	B5n	B3	−0.4	.006	
5088	6.5	.029	B8n	B3	+0.1	.005	
5102	4.9	.012	B5s	B3	−1.1	.006	
5105	4.8	.051	A0n	A3	+0.9	.017	
5113	5.4	.024	B5n	B3	−0.5	.007	
5122	5.4	.031	B9n	B8	+0.5	.010	
5130	6.0	.020	B7n	B5	−0.1	.006	
5156	5.1	.008	B4n	B3	−0.7	.007	
5160	6.2	.038	B9n	A	+0.5	.007	
5188	5.0	.094	A0n	A	+1.1	.017	
5210	5.8	.011	B2n	....	−1.9	.003	+0.008 t
5240	5.2	.016	B8s	B8	−0.6	.007	
5307	5.9	.025	B4n	B3	−0.7	.005	
5316	6.5	.009	B4n	B5	−0.6	.004	
5322	5.6	.017	B8s	A	−0.6	.006	
5325	5.4	.006	B3s	B3	−1.2	.005	

TABLE II—*Continued*

Boss No.	m	μ	Spectrum M.W.	Spectrum H.D.	M	Spec. π	Group or Trig. π
5389......	5.8	0″.005	cB8s	cB8p	−0.6	+0″.005	
5393......	4.0	.025	B9n	A	+0.5	.020	
5405......	5.5	.018	B8n	B8	+0.1	.008	
5414	5.2	.004	B5n	B3	−0.4	.008	
5465......	5.4	.015	B9s	B8	−0.2	.008	
5474......	5.0	.028	O9n	Oe5	−2.4	.003	
5512......	5.8	.011	B0n	B	−3.0	.002	
5516......	5.2	.004	B3n	B3	−0.8	.006	
5525......	5.4	.018	B8n	B5	+0.1	.009	
5532......	3.3	.012	B1s	B1	−2.3	.008	−0.011 t
5550......	6.3	.026	A0n	A	+0.9	.008	
5573......	6.0	.052	A1n	A	+1.3	.011	
5585......	5.6	.038	B9n	A	+0.7	.009	
5627......	5.0	.001	B3n	B3	−0.8	.007	
5629......	5.5	.015	B6n	B3	−0.2	.007	
5641......	5.6	.036	B9n	A2	+0.5	.010	
5653......	6.0	.006	B4s	B5	−1.2	.004	
5681......	5.6	.036	B9n	A	+0.5	.010	
5687......	5.2	.011	O9n	Oe5	−2.8	.003	
5706......	5.4	.026	B7s	B5	−0.8	.006	
5738......	6.2	.028	B9n	A	+0.3	.007	
5757......	5.4	.019	B6s	B5	−1.0	.005	
5796......	5.7	.011	B0n	....	−3.1	.002	
5810......	4.5	.013	B3s	B3	−1.2	.007	
5856......	5.2	.021	B2n	B2	−1.8	.004	
5879......	5.7	.037	A0n	B9	+1.1	.012	
5933......	3.6	.032	B7n	B5	0.0	.019	+0.004 t
5967......	5.3	.024	B8n	B8	+0.3	.010	
5973......	5.2	.017	B9n	A3	+0.5	.011	
6075......	5.3	.016	B9n	B9	+0.5	.011	
6128......	5.4	.033	A1n	A	+1.5	.017	
6142......	6.0	.006	B0n	Bp	−3.0	.002	+0.030 t
W B 3^h 147	6.0	....	B9n	B9	+0.5	.008	+0.038 t
Bond 708 .	6.3	.020	B2s	B1	−1.6	.003	
4 Gemin. .	6.7	.014	B9n	B9	+0.5	.006	+0.005 t

A comparison of the results given by this method of reduction with those derived from moving clusters and from trigonometric parallaxes shows a satisfactory degree of accordance. Using the values of Rasmuson for moving clusters, except in the case of the

Orion cluster where a parallax of $+0''.007$ has been assumed, we find the following results for the absolute magnitudes:

	No.	Cluster	Spec.
Pleiades	8	$-0^M.1$	$+0^M.1$
Perseus	20	0.0	-0.5
Orion	17	-2.0	-2.0
Scorpio-Centaurus	11	-0.5	-1.0

The very remarkable case of Voûte's star of large proper motion and parallax, Boss 1517, enters into the results for trigonometric parallaxes. The spectrum is B4n. The comparison for stars with positive trigonometric parallaxes, including and excluding this star, is as follows:

No.	Trig.	Spec.
24	$-0^M.2$	$-1^M.0$
23	-0.4	-1.0

The difference is that to be expected from the effect of selection of the stars with positive trigonometric parallaxes, for which alone absolute magnitudes may be computed. This necessarily involves a selection of the positive errors in the determinations. A better comparison would be furnished by the use of the formula already employed in which the sum of the errors of the trigonometric parallaxes is assumed to be zero. For 34 stars with trigonometric parallaxes the mean absolute magnitude derived in this way is -1.4, as compared with the spectroscopic value of -1.0. It is of interest to note that of the trigonometric parallaxes (absolute), 24 are positive, 9 negative, and one is zero. The simple mean of these trigonometric parallaxes is $+0''.011$ and that of the spectroscopic parallaxes is $+0''.012$.

The catalogue which is given in Table II contains our results for the absolute magnitudes and parallaxes of stars of types O8 to B9 with the addition of a very few A-type stars. The successive columns give the Boss number, visual apparent magnitude, total proper motion, mean spectral type from the determinations of Adams and Joy, spectral type according to the Revised Draper Catalogue, absolute magnitude and parallax derived by the spectroscopic method, and values of the parallax as obtained from moving clusters or trigonometric determinations. The absolute magnitudes have been computed separately for the types of Adams and Joy

and then combined; in some cases, accordingly, there will be slight differences from those furnished directly by the mean spectral types. For stars of right ascension later than 18 hours the Harvard spectral types are from the earlier Harvard publications.

A valuable check on the accuracy of the absolute magnitudes derived by this method is furnished by the relationship of M to total proper motion. For such a comparison, especially when a wide range in apparent magnitude is involved, the best procedure is doubtless to use the reduced proper motion as has been done in the case of other stars by Lundmark and Luyten.[1] For each star the value of $0.2\,m+\log\mu$ has been computed, and the mean value of M has been derived for groups of stars within narrow limits of this quantity. For very small values of the proper motion the uncertainty is, of course, considerable.

TABLE III

No.	$0.2\,m+\log\mu$		Mean M
	Limits	Mean	
12	<8.50	8.19	−2.01
11	8.50–8.74	8.65	−1.49
23	8.75–8.99	8.90	−1.37
23	9.00–9.14	9.07	−1.18
43	9.15–9.29	9.22	−0.80
38	9.30–9.39	9.34	−0.47
27	9.40–9.49	9.45	−0.48
32	9.50–9.59	9.55	−0.25
32	9.60–9.69	9.65	−0.15
29	9.70–9.79	9.74	+0.13
18	9.80–9.94	9.86	+0.21
11	>9.95	10.07	+0.61
299			

The results are shown graphically in Figure 1. The curve is very nearly a straight line except for the stars having extremely small proper motions.

The same conclusion may, therefore, be drawn for the B-type stars as for the A-type stars discussed previously, that the dispersion in absolute magnitude is small for stars of the same spectral subdivision with the same character of spectral lines. Occasional exceptions no doubt occur, as illustrated by β Orionis and such a star as Boss 1517, which is characterized by a large proper motion and

[1] *Lick Observatory Bulletin*, No. 339.

high radial velocity. Whether the B-type stars with bright lines are also exceptional in their behavior is less certain; but, since the reduction curves are based entirely upon the stars showing absorption lines, it has seemed preferable not to include material which is not of a homogeneous character.

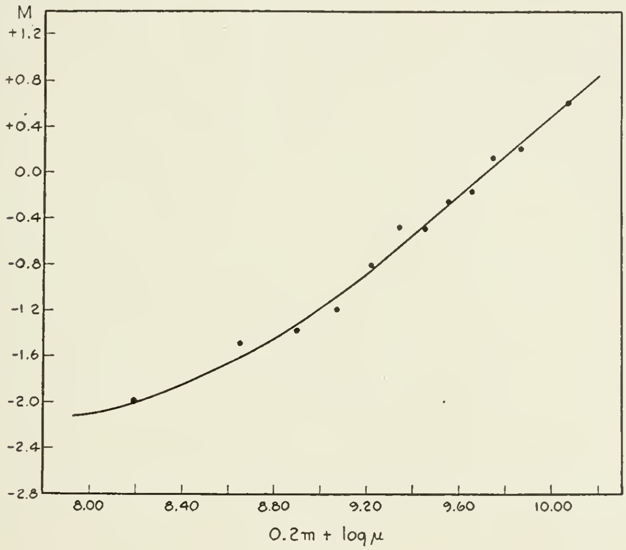

FIG. 1

An interesting publication by D. L. Edwards on "Spectroscopic Parallaxes of the Hotter Stars" has appeared while this investigation was in progress. The method used is to measure the relative intensities of the helium lines λλ 4144 and 4388 against the hydrogen lines Hδ and Hγ, respectively, and to determine the absolute magnitude from the correlation found to exist between these two quantities. To some extent this is a correlation with spectral type and in the case of the stars listed by Edwards, comparison of types with absolute magnitudes derived from the parallaxes which he uses does, in fact, show the existence of such a relationship. The successful development of this method should be of great value in showing the dispersion in absolute magnitude for stars of the same spectral subdivision.

MOUNT WILSON OBSERVATORY
March 1923

NOTICE TO CONTRIBUTORS

There is occasionally published in the *Astrophysical Journal* a Standing Notice (for instance, on pages 179–80 of the number for September 1917). This is principally intended to guide contributors regarding the manuscript, illustrations, and reprints. This notice contains the following paragraph:

> Where unusual expense is involved in the publication of an article, on account of length, tabular matter, or illustrations, arrangements are made whereby the expense is shared by the author or by the institution which he represents, according to a uniform system.

The present sheet has been printed for amplifying further that paragraph.

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As to illustrations, the arrangement cannot be quite as specific, but it may be generally assumed that not more than three half-tone inserts can be allowed without payment by the author. The cost of paper, presswork, and binding for each full-page insert is about \$8.00, aside from the cost of the half-tone itself. In the matter of zinc etchings, considerable latitude has to be allowed, as in many cases diagrams take the place of more expensive tables. It may be assumed, however, that it will seldom be possible for the *Journal* to bear an expense of over \$25 for diagrams and text illustrations in any one article.

Contributors should notice that since January, 1917, it has been impossible to supply any free reprints of articles.

Reprints of articles, with or without covers, will be supplied to authors at cost. No reprints can be furnished unless a request for them is received before the *Journal* goes to press.

Every article in the *Astrophysical Journal*, however short, is to be preceded by an abstract prepared by the author and submitted by him with the manuscript. The abstract is intended to serve as an aid to the reader by furnishing an index and brief summary or preliminary survey of the contents of the article; it should also be suitable for reprinting in an abstract journal so as to make a reabstracting of the article unnecessary. For details concerning the preparation of abstracts, see page 231 in the April, 1920, number of the *Journal*.

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From a photograph made in 1917 by Harris and Ewing

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME LVIII JULY 1923 NUMBER 1

EDWARD EMERSON BARNARD

BY EDWIN B. FROST

Edward Emerson Barnard was born at Nashville, Tennessee, on December 16, 1857, the son of Reuben and Elizabeth Jane (Haywood) Barnard. His father had died before his birth and his mother was obliged to support herself and her two young sons. Those were hard times for poor people in that section of our country, and they were still harder after the Civil War came on, a few years later, bringing tragedy to all on the scene of conflict. The lad's memory of the flashes of artillery at the Battle of Nashville never left him. Later in his youth he survived an attack of cholera, when that plague raged beyond its accustomed habitat.

His mother was a woman of character, and did not lose her taste for culture in the struggle with poverty. The name, Emerson, given to the young son in honor of our American philosopher, was an evidence of this. She inspired the lad with a desire to know good literature, although his opportunities for any regular education in school were most meager; in fact, Edward Barnard attended the common school only two months in his life. His mother had a taste for art, also, and partially supported herself by modeling wax flowers. That she had impressed upon her son character and self-reliance was indicated by her statement that the lad, when less than nine years old, could be depended upon to do a task in which many

other boys had failed. A photographer in Nashville had on the roof of his studio a ponderous enlarging camera which had to be kept pointed at the sun. Most of the lads had lacked the patience necessary in the human substitute for a driving clock, and went to sleep at the task. Edward Barnard justified his mother's confidence; he worked in that studio, in various capacities, for seventeen years. His duties, doubtless often monotonous, were fitting him to be a pioneer in the photography of the heavens.

As a boy, he had watched with wondering eyes the starry skies above him, as he lay upon an old wagon-box in the yard; and he had, indeed, learned to know the stars, but not their names or their constellations. A young man employed in the studio, who had mechanical skill, one day picked up in the street the small objective of a broken spyglass, and, making for it a paper tube, constructed a small telescope for the young apprentice. With this, Edward Barnard studied the stars further, but still without the means of identifying them, until chance brought into his hands, somewhat later, a volume of the works of Thomas Dick, who enjoyed a considerable reputation as a writer on astronomy as well as on theology. It seems that Barnard found his first star map here, and was delighted to learn the conventional names of the objects with which he was already so familiar. Later, he put together a better instrument for which he purchased lenses of $2\frac{1}{4}$ inches aperture.

The young man was now supporting himself and his mother by his daily work in the studio, but his passion for astronomical observation had already developed. Through rigid economy he gratified this passion by the purchase, in 1876, of a 5-inch telescope, equatorially mounted, from John Byrne, of New York. The purchase price, $400, represented some two-thirds of a whole year's earnings, and thus gives sufficient evidence of his determination to acquaint himself with the science of astronomy. In 1877 the American Association for the Advancement of Science held its annual session at Nashville, and the youth of twenty, who was becoming locally well known for his zeal as a star-gazer, joined the Association. His friends persuaded him to bring his telescope and meet the president of the Association, Simon Newcomb. This distinguished astronomer, already eminent at the age of forty-two, whose researches were in

the domain of theory rather than in observation, advised young Barnard that to accomplish anything important in astronomical research he must first be well grounded in mathematics. This advice might have discouraged a less ardent seeker for knowledge of the stars, but it inspired Barnard to do just what was advised. He applied himself diligently to elementary mathematics and other common items of education which he had been obliged to neglect in his youth, and from his own slender earnings hired a tutor for some branches of his study.

In January, 1881, while still an employee at the studio, he married Miss Rhoda Calvert, who was born in Yorkshire, England, and had come to Tennessee a few years earlier with her brothers, who were artists and who also had work in connection with the studio. His marriage greatly influenced his subsequent career, as his wife most unselfishly encouraged and helped him in his efforts to obtain a better education and to overcome some cultural deficiencies of which he was conscious. She proved a true helpmeet in every way, caring for the household in a most prudent manner and taking over all responsibility for his now invalid mother. This made it possible for him to improve every opportunity that presented itself for his advancement.

On May 12, 1881, Barnard discovered his first comet in the morning sky near Alpha Pegasi. He found it again on the next night, but could not afterward locate it, and inasmuch as he did not send out any announcement to the astronomical world, and it was not seen by any other astronomer, this comet was never assigned a place in the formal records of astronomy—no number was given it and it was not counted by Mr. Barnard himself. Of course, from what was later known of his reliability and skill as an observer, there could be no question as to the certainty of his observation, but at that time he was unknown among astronomers.

This accidental discovery, however, developed his interest in the search for comets, and he began systematically to sweep the sky for them. His diligence was rewarded on the night of September 17 of the same year, when he found a comet in the constellation Virgo. He announced the discovery to Dr. Lewis Swift, so that it was observed by other astronomers and received the name, "Comet

1881 VI." Mr. H. H. Warner, who had established at Rochester, New York, the private astronomical observatory of which Dr. Swift was the director, had taken much interest in astronomy and offered a prize of $200 for each unexpected comet discovered by an American observer. The award was made to Mr. Barnard for the discovery of this comet, and it happened opportunely, for at that time the young man was building a little house for his bride and his mother, and the burden of paying notes on borrowed money as they came due was a serious one. Five times in all Barnard received this award for the discovery of a comet, and it meant to his family the possibility of owning their modest home. That dwelling is still known in Nashville as the "Comet House." Few, indeed, are the astronomers whose keen eyesight and extraordinary diligence in the quest for celestial discovery have literally provided them with a roof to sleep under. It was very little, however, that he slept under that roof when the sky was clear.

I am describing these circumstances in this detail because they may seem almost incredible to some of our contemporary European astronomers who have reached positions in our science in university or governmental observatories after passing through a very definite and uniform course of study and training. That inborn genius can find a way of achieving its ideal in America, we may call to witness our lately departed friends, Burnham and Brashear, as well as Barnard.

A year later, on September 13, 1882, the second recorded comet (1882 III) was discovered with the 5-inch telescope. A rather dramatic incident occurred a month later, October 14, of which we have a printed record in Barnard's own words. He had been searching for comets through the early part of the night and had set his alarm clock for a later hour, in order to get some much-needed rest. He says that when the alarm clock sounded he had been dreaming of discovering a wonderful field of comets, big and little, with long and short tails, in his field of view. After he awoke, he began sweeping in the neighborhood of the Great Comet of 1882, and to his astonishment saw twelve or fifteen small comets, of varied appearance, in the vicinity. He had obtained positions for seven or eight of these when the dawn came. He announced his discoveries to Dr. Swift,

but this astronomer did not distribute the information. Speaking of this omission, Mr. Barnard has said: "Whether he thought that I was trying to form a comet trust, or had suddenly gone demented, has never been clear to me." These comets were undoubtedly real, and fragments of the Great Comet. Schmidt, of Athens, reported one such object which he observed on October 8, another was observed by E. Hartwig on October 9, and still another by Brooks, of Phelps, New York, on October 21. They were never separately announced in the list of comets.

During this time, young Barnard was working in the studio by day, for a livelihood; and studying, by himself or with a tutor, on cloudy and moonlit nights. In 1880 he planned to write a booklet on Mars, and solicited subscriptions from friends in Nashville to cover the cost of publication. We have been unable to find a copy of this, which was planned to be a duodecimo; and it is doubtful whether it was ever actually printed. In 1883, we find the young amateur conducting an astronomical column in a journal known as the *Artisan*, which was published twice every month at Nashville. In the same year, friends who had perceived the genius of the young man offered him a fellowship in Vanderbilt University at Nashville, giving him an opportunity to devote his time exclusively to his studies and to make use of the 6-inch equatorial of the small observatory of the university. The stipend was only about $300, together with a house on the university campus, near the observatory, and it might have been regarded as a venturesome step for a young man with a wife and a mother dependent upon him to give up his work at the studio and endeavor to live on the sum provided. But his wife bravely counseled acceptance of the offer, and said, "We will get along somehow"; and they did. Barnard became enrolled as a special student in the school of mathematics, at the same time having the care of the observatory. Later, he received an appointment as instructor in practical astronomy, continuing his studies in mathematics, physics, and chemistry, and some of the modern languages.

The young astronomer's first trip into the outer world was made in 1884, when he attended the meeting of the American Association for the Advancement of Science at Philadelphia, visiting en route

the observatories at Cincinnati, Allegheny, Washington, Harvard, Albany, and Princeton, and seeing for the first time those cities, together with New York and Boston. Every economy had to be practiced, and he was accustomed to avoid unnecessary hotel bills by traveling in day coaches at night. His record of this trip, in daily postal cards and letters to his wife and his mother, is very humorous and interesting. He had formed, in advance, mental pictures of the prominent contemporary astronomers, and many of them turned out to be quite different from his anticipations. Thus, he had expected that Professor E. C. Pickering would be a formal and distant dignitary, as might befit a native of Boston and the director of the Harvard College Observatory. To his surprise, he found that "Professor Pickering is comparatively a young man, and strongly resembles a simple countryman. Had anyone shown him to me on the street and told me that was the famous director of the Harvard College Observatory, I should not have taken his word on oath. I should have been positive there was a mistake. However, he is the most unassuming man that you can imagine, and I admired him very much, indeed."

The meeting at Philadelphia in the section of astronomy was honored by the presence of John Couch Adams, of Cambridge, and Robert Ball, then Astronomer Royal for Ireland. To his great regret, Mr. Barnard, who was suffering from neuralgia, could not stay through the evening address of the latter, but he tells of the meeting of the section on another day, when an American astronomer (now deceased), who had specialized on the theory of the motion of the moon and who thought he had found the right basis for a correct theory, was giving a paper on his favorite topic. The situation is described by Barnard as follows:

> Now this same problem has been more or less Adams' hobby, and it was interesting to watch the twitching of his fingers during the progress of the paper. When it was finally through, and criticisms were in order, he quickly rose and in the most rapid manner proceeded to show that the gentleman was altogether in error, and went through the tedious formula with the ease of a master, and showed here and there where certain assumptions were erroneous, and where a false step had been made. It was quite a picture to see him. He has a peculiar affection, causing him, when excited, to breathe through his nose with a sound like escaping steam at almost every sentence; especially was this noticed when he used a word that began with "s."

Barnard arrived at Nashville, thoroughly worn out with his travels; but, after having received a most friendly welcome and recognition from the astronomers whom he had at last met in person, he could henceforth feel that he was one of the fraternity.

In 1887, at the age of thirty, he received the degree of Bachelor of Science. Meanwhile, he had discovered seven more comets—the last on May 12, 1887—namely: 1884 II, 1885 II, 1886 II, 1886 IX, 1886 VIII, 1887 III, 1887 IV. Of these comets, 1884 II was periodic, with a return expected every 5.4 years, but it escaped detection in subsequent years. In 1887 he published in the *Astronomische Nachrichten* a parabolic orbit for Comet 1887 III.

In 1883 he had independently discovered the Gegenschein, while sweeping the skies, and had become extraordinarily familiar with the objects in the sky which could be seen with a small telescope. He had given special attention to the planet Jupiter and was an independent discoverer of the red spot, although some months after it had been announced by C. W. Pritchett, of Glasgow, Missouri, in July, 1878. Barnard was a careful delineator of what he saw and many of his early sketches of Jupiter were published in the first volumes of the Astronomical Society of the Pacific.

In 1887 the Lick Observatory was nearing completion, and Professor E. S. Holden had been chosen director, functioning ad interim as the president of the University of California. He corresponded with Barnard, and in that summer offered him a position on the staff then in process of formation. Barnard accepted the opportunity to join this finely equipped institution, and in September he and his wife started on their interesting journey to California. They found temporary lodgings in San Francisco, until they should be able to make their home on Mount Hamilton. There were delays in the completion of the observatory, and the regents of the university were unable to make provision for the astronomers until the trustees of the Lick estate had formally turned over to them the completed observatory. Mr. Barnard, again finding himself in need of a salary which was not forthcoming, obtained work in copying legal papers in the office of a lawyer. Early in the next spring the Lick trustees invited him to come to the mountain and make an inventory of the observatory's equipment. Finally, on June 1, 1888, Director Holden was able to write, with evident relief after the try-

ing delay: "The observatory begins its active existence tonight." S. W. Burnham and J. M. Schaeberle were senior astronomers, J. E. Keeler and E. E. Barnard, juniors; and work began enthusiastically. Mr. Barnard became very warmly attached to these members of the staff. We must remember that this was his first opportunity to be regularly associated with astronomers of considerable experience, and it was of great importance to him. In turn, his associates highly appreciated Barnard's ability as an observer and his tremendous capacity for work.

The 12-inch telescope was assigned to Barnard, together with the comet-seeker, and very soon Director Holden began to make use of Barnard's technical knowledge of photography. On September 2, Barnard discovered Comet 1889 I, and in October, Comet 1888 V. He also observed their positions assiduously with the excellent equatorial and filar micrometer. He further observed nebulae and planets, and in 1889 made a notable observation of the eclipse of Japetus by the ring system of Saturn. He could see that the sunlight illuminated the satellite through the crape ring, thus indicating that the ring was quite transparent, and supporting the view that it was made up of small particles. During that year he discovered Comets 1889 II and 1889 III (which has a computed period of 128 years), but his most important work was the beginning he made during that summer in photographing the Milky Way with the Willard lens, which became a famous instrument in his hands. This was a portrait lens of 31 inches focal length, which had been used by some photographer and had received its name from the dealer who sold them in New York. This camera was strapped to the $6\frac{1}{2}$-inch equatorial, which served as a guiding telescope. Barnard's long exposures with this instrument brought out the wonderful richness of the star clouds and other features of the Milky Way as they had never before been revealed. They thrilled him and his associates with their significance and beauty, and later the entire scientific world shared in this appreciation of them.

Barnard was the first to observe the return of d'Arrest's Comet in 1890, and, in the following year, of Comets Encke and Tempel-Swift. In 1891 he discovered Comets 1891 I and 1891 IV. In 1892 he made the first discovery of a comet by photography, finding on

his plate taken on October 12, Comet 1892 V, for which a period of 6.5 years was computed, but which has never been observed at a subsequent return.

As a junior member of the staff of the Lick Observatory, Mr. Barnard did not receive a regular assignment at the 36-inch telescope, but his friend, Burnham, was always glad to check any important observation for him, or give him the opportunity of examining it with the great refractor. Professor Burnham resigned his position in June, 1892, and resumed work as clerk of the Federal Court in Chicago. Mr. Barnard had naturally been eager for an opportunity to make regular use of the great refractor, but he had been unable to secure this privilege until the first of July, 1892, when he received the coveted assignment for one night per week. On the eleventh night (September 9, 1892), he made with it his brilliant discovery of the fifth satellite of Jupiter. We quote from his own account of his observations in *Astronomy and Astrophysics*, **11**, 749, 1892:

Friday being my night with the 36-inch telescope, after observing Mars and measuring the positions of his satellites, I began an examination of the region immediately about the planet Jupiter. At 12 o'clock as near as may be, to within a few minutes, I detected a tiny point of light closely following the planet and near the 3rd satellite which was approaching transit. I immediately suspected it was an unknown satellite and at once began measuring its position-angle and distance from the 3rd satellite. On the spur of the moment, this seemed to be the only method of securing a position of the new object, for upon bringing the slightest trace of the planet in the field the little point of light was instantly lost.

I got two sets of distances and one set of position-angles, and then attempted to refer it to Jupiter but found that one of the wires of the micrometer was broken out and the other loose. Before anything could be done the object rapidly disappeared in the glare of Jupiter. From the fact that it was not left behind by the planet in its motion, I was convinced that the object was a satellite. A careful watch was kept at the preceding limb of the planet for the reappearance of the satellite, but up to daylight it could not be seen.

Though positive that a new satellite had been found, extreme caution suggested that it would be better to wait for a careful verification before making any announcement.

The following night with the 36-inch belonging to Professor Schaeberle, he kindly gave it up to me, and shortly before midnight the satellite was again detected rapidly leaving the planet on the following side. That morning I had

put new wires in the micrometer, and now began a series of careful measures for position. As I have said, the satellite was so small that no trace of Jupiter could be admitted into the field for reference in the measures. It was necessary, therefore, to bisect the satellite, with the planet out of the field, and then by sliding the eyepiece bring the limb of Jupiter into view and bisect it. This method did not permit any measures from the polar limbs of Jupiter. Following the satellite thus, it was seen to recede from the planet to a distance of some 36″ from the limb when it gradually became stationary. Remaining so for a while it began once more to approach the planet and rapidly disappeared in the glow near the limb. The measures, repeated as rapidly as possible, thoroughly covered the elongation, and gave the means of approximating to its period.

The following morning a telegram was sent out announcing the discovery. Subsequent observations have thoroughly confirmed the discovery.

On account of its extreme closeness to the planet it is difficult to say just what its magnitude is. Taking everything into account, I have provisionally assigned it as thirteenth magnitude. I hope to be able to settle definitely this question by observing some little star near Jupiter and then afterwards determining its magnitude when the planet has left it. Until this is settled, any estimate of the actual size of the satellite must be the merest guess, but it will probably be found to not exceed 100 miles in diameter, and perhaps less than that.

After the first few observations I inserted a piece of smoked mica in the eyepiece, and using this as an occulting bar, the measures were made with ease and accuracy. Careful measures thus made from the polar limbs for the Jovicentric latitude of the satellite show that its orbit lies sensibly in the plane of Jupiter's equator and that consequently the satellite is not a new addition to the Jovian family, since it would doubtless require ages for the orbit to be so adjusted if the object were a capture.

The reader will note from this extract the element of independence which was a characteristic of Barnard's discoveries. He perceived with a sort of intuition that this was probably a new satellite, in fact, was convinced of it, by his brief observations on the first night. His exercise of great care in making no premature announcement was also characteristic. He furthermore was quick to realize that it would be a matter of general interest whether the satellite had been newly acquired by Jupiter, and his measurements to decide this point were made a few evenings later. We also see an illustration of his readiness to adapt his observing methods to difficult circumstances, in providing a piece of smoked mica for occulting the brilliant planet. His scientific caution and ingenuity

are illustrated by his proposal that the magnitude of the object should be determined by comparison with some star of about the same magnitude, which some night would lie near the planet's position and thus afford a reliable basis for an estimate of brightness.

This first addition to the family of Jupiter, which had received careful telescopic observation for nearly three centuries, brought to Mr. Barnard instant recognition as an observer of the first class. The Lalande gold medal of the French Academy of Sciences was awarded to him a few months later for this notable astronomical feat. Professor Barnard followed this satellite with very careful micrometric measurements for many years after its discovery, seeking to improve our knowledge of its orbit, and he published 14 papers covering his observations of elongations, nearly all of them in the *Astronomical Journal*. The difficulty of observing the object was not because it was so faint, but because of the brightness of the planet. Quite good conditions of seeing were always necessary for observing it, even with the large telescope. So far as is known to the writer, the fifth satellite has never been photographed and the smallest aperture with which it has been observed is that of the 18½-inch equatorial of the Dearborn Observatory at Evanston, with which Professor G. W. Hough observed it on October 15 and November 11, 1892. The discovery of this satellite doubtless renewed interest in the search for other satellites by the use of photography, resulting in the discovery of three further, remote satellites at the Lick Observatory: VI and VII by C. D. Perrine, and IX by S.P. Nicholson; while VIII was discovered by P. J. Melotte at Greenwich.

During the later years at the Lick Observatory, Mr. Barnard gave much attention to careful measurements of the diameters of the planets, including the four largest asteroids. He made a comprehensive study of the dimensions in the Saturnian system, and measured the ellipticity of Uranus. He gave particular attention to the diameters and the appearance of the brighter satellites of Jupiter. These extensive researches were published in a series of papers in various astronomical journals, several of them appearing after he had left the Lick Observatory. He summed up his results in a paper which he published in *Popular Astronomy*, **5**, 285–302,

1897, entitled, "Micrometrical Determination of the Dimensions of the Planets and Satellites of the Solar System, Made with the 36-Inch Refractor of the Lick Observatory." This was later printed in abstract in the *Monthly Notices of the Royal Astronomical Society*, **58**, 216–218, 1898.

It was not only to the study of the Milky Way that Barnard was applying photography with distinguishing success. He studied the comets, and took a great interest in the remarkable behavior of their tails as revealed on his negatives. Swift's Comet (1892 I) was the first to show on Barnard's photographs the extraordinary changes which the tails of comets may undergo. His subsequent photographs of many comets show that these changes are characteristic of the tails of some comets, but not of others. Cloudy weather had interfered with observations of this comet during March, but the photographs taken on April 4 and 5 displayed extraordinary changes in the short interval between them. The significance and value of the photographic records of these capricious changes were instantly appreciated by Barnard, as will be seen from the following quotation from his article written some years later and appearing in the *Monthly Notices of the Royal Astronomical Society*, **59**, 355, 1899:

> This [Swift's Comet of 1892] was the first comet to show to the photographic plate the extraordinary changes to which these bodies are subject. Indeed, if it had not been for the photographic plate we should have known nothing of the extraordinary changes that occurred in this comet and several that have since appeared. . . . .
>
> For the study of the phenomena of the tails of comets, the portrait lens has shown itself most admirably suited. It has added an interest to the physical study of these bodies that did not exist previously; for the most interesting of the phenomena shown by comets must always escape the visual observer and pass unknown, without the aid of the portrait lens and the photographic plate. Unlike the planets, the comets often traverse the entire solar system. They are, therefore, our only means of exploring the regions between the planetary orbits. Instead of ponderous bodies like the planets, they are but flimsy creations of enormous dimensions. They are thus likely to be easily subject to disturbances in their forms that would produce no perceptible effect on their motions. What these influences may be we do not know; probably swarms or streams of meteors, which we know do exist in space, or possibly some other cosmical matter yet unknown. Such objects might be (and possibly have been)

revealed to us by their effect upon the form of the comet's tail as it sweeps through space.

The comet discovered by Holmes in the autumn of the same year (1892 III) was also photographed by Barnard when this round, tailless object, whose motion was almost entirely in the line of sight, was situated very near the great nebula in Andromeda. The motion of the comet among the stars was, in fact, so slight that Barnard, with an exposure of 75 minutes, obtained, on the night of November 21, 1892, a sharp picture of the Andromeda nebula, together with the comet! a circumstance which is not likely to be duplicated. Brooks's comet of the next year (1893 IV) excited Barnard's interest in a high degree by its behavior, which was quite exceptional in those early days of cometary photography. He speaks of his plates of October 21 and 22 as follows (*ibid.*, p. 358):

There is an utter transformation of the comet in this picture. The tail is larger and brighter and very much distorted, as if it had encountered some resistance in its sweep through space. This disturbance seems to have disrupted the north-east edge of the tail. The small side tail has apparently been swept away, while the more distant portion of the main tail is streaming in a very irregular manner. The entire picture is highly suggestive of an encounter with some sort of resistance. Is it possible the tail passed through a stream of meteors such as we know exist in space? Whatever the cause may have been, the appearance of the tail utterly excludes the idea of the phenomenon being due to irregular emission of the matter from the nucleus—an explanation quite satisfactory in the case of Swift's Comet.

In passing, this particular photograph seems to explain at least one of the ancient descriptions of a comet, viz., "a torch appeared in the heavens." The comet, as shown in the photograph, is sufficiently suggestive of a torch streaming irregularly in the wind.

* * * *

[On the next day the tail of the comet] appears a total wreck in this photograph, and is still more suggestive of a disaster. It is very badly broken, and on the south-west side hangs in irregular cloud-like masses. Near the extremity a large gap exists in the tail, as if something had gone through it from the north-east, and a large mass is torn off beyond this break and seems to be drifting independent of the comet.

For nearly thirty years these unexplained caprices of the tails of comets fascinated Professor Barnard, and whenever a new comet appeared in the sky he was filled with suppressed excitement as to its

behavior on the photographic plate. Comet Morehouse (1908 IV) was thoroughly satisfying in this respect, and he obtained no less than 350 photographs of it. He would sometimes take successive photographs of it as long as it could be followed above the horizon, before the interference of the moon or dawn.

The results of Barnard's assiduous campaign at the Lick Observatory, from 1892 to 1895, in the study of the Milky Way and comets by photography, are preserved in Volume XI of the *Publications* of the Lick Observatory. This volume did not appear until 1913—nearly twenty years after the photographs were taken—because of the difficulties which Professor Barnard found in securing satisfactory reproductions of his pictures. His studies of these photographs had been so minute that he recognized details which would have escaped anyone else, so that his standard of excellence of their reproduction became very exacting; indeed, beyond the possibility of the processes of photogravure and heliogravure. The publication of the volume had been made possible by subscriptions for the purpose which Mr. Barnard had secured from California friends of the Lick Observatory and of himself. The collotype process was employed and the reproductions are as satisfactory as could be expected by any such process. But for a number of years the responsibility of issuing this volume was a heavy one for Mr. Barnard. He became discouraged with what he regarded as the impossibility of securing adequate reproductions, and the work lapsed. He even attempted to return the money already expended, to the Lick Observatory, for distribution among the subscribing friends. He was, however, persuaded to resume and, fortunately, was able again to secure the services of the expert in collotype who had begun the work. The volume contains 129 plates, from 92 photographs of the Milky Way and 42 of comets; and it will stand as a monument to the great skill and the untiring zeal of the pioneer in his beginning in this important field of investigation.

A leave of absence was granted Mr. Barnard in the summer of 1893 to make his first trip to Europe. Mrs. Barnard accompanied him, and thus had an opportunity to visit her old home and her relatives in England. The very cordial welcome given to Barnard by his English colleagues also made this a most pleasant visit. He

then went over to the Continent and made the personal acquaintance of astronomers in France and Germany.

Mr. Barnard's residence of nearly eight years in California was full of romantic interest for him. The conditions for his work were very fine, and a clear sky was assured in advance during many months of the year. His residence on the mountain was novel to one who had always lived in a city, and the views of mountain and canyon made a strong appeal to the artistic element in his nature. The life was isolated in winter, but this was broken by visits on Saturday evenings of the winter tourists in California, and acquaintances were established—many of them lasting—with interesting people from different parts of the world. The association with his fellow-observers and their families in the little colony was congenial, and particularly close was his association with Professor Burnham, who, like himself, was an ardent and expert user of the camera. This phase of his life at the mountain was well brought out in Mr. Barnard's biographical sketch of Mr. Burnham, published in *Popular Astronomy*, **29**, 309, 1921. There was an element of the wild in the howl of the coyotes in the canyons and in the occasional deer seen around the mountain. In the gray dusk, one morning, as Mr. Barnard was nearing the door of his cottage, he saw before him the great form of a panther, or mountain lion, standing a few yards away. Each was returning from his night's work, and each silently respected the rights of the other. After a moment, the panther quietly walked on over the mountain. At another time, as Mr. Barnard was watering the flowers around the cottage, the blazing eyes and terror-stricken cries of the little dog that had been accompanying him told him that a rattlesnake had made its strike, so he dispatched the snake and rushed to apply remedies to save the life of the dog, which, as a result, survived his wound for some months.

The free and hearty cordiality of the Californians, and their appreciation and respect for the men of science on Mount Hamilton, was keenly felt; and he occasionally participated in the meetings and activities of the Camera Club and of the Bohemian Club of San Francisco, as well as with the colleagues at Berkeley. Conditions of life on the mountain were comparatively simple and at the start Mr. Barnard's salary was small; but these circumstances were much

improved toward the end of his stay, and his opportunities for the use of the great telescope were increased so that he often worked with it on two and sometimes even three nights a week. However, circumstances into which we need not enter finally led him to desire a change, and in 1895 he accepted an invitation to become a member of the staff of the Yerkes Observatory, then in process of construction as a department of the new University of Chicago. His official title was to be Professor of Practical Astronomy and Astronomer at the Yerkes Observatory, but no duties of giving instruction were involved, beyond an occasional popular lecture in the summer courses at the University. His official connection with the University began on October 1, 1895. As had happened at the Lick, there were unexpected delays in the completion of the Yerkes Observatory, so that for the greater part of a year Professor Barnard lived in Chicago near the Kenwood Observatory, the equipment for which had been presented to the University by Professor George E. Hale, for whom it had been built. This period constituted something of a gap in Mr. Barnard's observational activity, but the time was usefully employed in preparing for publication some of the results of his observations at Mount Hamilton, including the reproductions of the photographs of the Milky Way and comets made there.

In the summer of 1896, Professor and Mrs. Barnard occupied a cottage on the shore of Lake Geneva, and began the construction of the house which was to be their home for the next quarter of a century, on land which they had purchased adjacent to the grounds of the Observatory. In February, 1897, Mr. Barnard went to England to receive the gold medal of the Royal Astronomical Society, but, owing to delay of the steamer by bad weather, he unfortunately did not arrive until the day after the annual meeting of the Society. A special meeting was held on March 2, at which Professor Barnard exhibited and explained some of his most notable photographs taken at the Lick Observatory, and a dinner was given like the one prepared on the evening which he missed. As Mr. Barnard was very keen to begin work with the 40-inch, which was then expected to be ready in the spring, as well as to complete the equipment of his new home, in which he took a great interest, he

sailed for home after a stay in England of less than three weeks, and was back at Williams Bay by the middle of March.

A few weeks later the objective was brought from Cambridge by Alvan G. Clark, and was adjusted by him in its cell on the great instrument. It was first used on the night of May 21, and the tests of its performance were highly satisfactory. There were occasional opportunities during the next week, when the sky was clear, for further tests, and on the night of May 28 Professor Barnard had a narrow escape. He was observing during the latter part of that night, until daylight, and left the dome at dawn. Just before seven o'clock, as the result of faulty connection of the supporting cables, the rising floor fell, involving its almost complete destruction, but fortunately without injuring the telescope itself. Had this happened a few hours earlier, the observer could hardly have escaped death or a serious injury.

This delayed the formal opening of the Observatory until October 21, 1897, and Mr. Barnard had to exercise his patience in waiting for further use of the great refractor. As soon as it was ready for regular work, in the autumn of 1897, Professor Barnard again plunged into observing, having the great telescope at his disposal regularly for two, and often for three or four nights per week. He was, of course, interested in making some tests of the quality of the 40-inch as compared with the 36-inch telescope which he had previously used. He, therefore, observed some of the difficult double stars, such as Schaeberle's companion to Procyon, and Kappa Pegasi, and secured some elongations of the fifth satellite of Jupiter. He studied some of the variable stars in Messier 5 which had recently been discovered by Professor Bailey, finding a couple of additional variables in that cluster. He gave particular attention to the variations in brightness of Bailey's No. 33, as will be mentioned later. He also measured in the daytime the diameters of Venus and Mercury, sometimes under especially fine conditions of seeing.

He began at this time a micrometric triangulation of some of the globular star clusters, measuring in this first year the positions of 95 stars in Messier 5 and a smaller number in Messier 13, comparing

the positions of the latter with measures obtained by Scheiner at Potsdam on photographs taken in 1891. The views of astronomers as to the size of the clusters were quite different then from those of the present, and Mr. Barnard had hoped that precise measures with the large telescope would reveal internal motions within a few years. He extended his triangulations to many other globular clusters until he had finally included in his program twenty clusters. In spite of his ardor and his experience as a photographer, Mr. Barnard still found it difficult to recognize the superior advantages of the measurement of star clusters on photographs with the use of rectangular co-ordinates. He put his trust in the filar micrometer more than in the measuring machine, particularly because he could recognize certain of the cluster stars as triple which were confused in a single image on the photograph then at hand, taken with an instrument of one-fifth the focal length of the great refractor. A very few years later, when remarkably fine photographs had been secured by Ritchey with the 40-inch telescope, through a yellow filter and with the double-slide plate-holder, Mr. Barnard measured some photographs of the clusters, and in subsequent years took some equally good ones himself. The excellent accordance between his measures on the negatives and those he made visually with the micrometer was to him an evidence that astrometric investigations could be satisfactorily made by photography; to us this accordance was a demonstration of Barnard's extraordinary skill at the telescope with the micrometer. Similarly, when his visual determination of the parallax of Krueger 60 was very closely confirmed by Schlesinger's measures on plates he had taken with the 40-inch telescope, it convinced Barnard that good parallaxes could be obtained by the new photographic method; while for us it was again a demonstration of Barnard's great skill as an observer.

It was probably no small disappointment to Mr. Barnard that his measures in the clusters, continued for nearly twenty-five years, yielded so little in the way of proper motions—in fact, it could hardly be asserted that a single one of the cluster stars showed an appreciable motion with respect to its fellows. From what we know now, it would have been better to omit much of this labor by visual methods and to trust to photographic records made from time to

time for the establishment of the motions which certainly must exist, but which will evidently require a long lapse of time for accurate determination. It will be no small task to evaluate these micrometric measures which were expressed in position angles and distances and referred to selected stars in the clusters; thus, in Messier 5, 239 stars were included; in Messier 13, 247; and in several other clusters the number runs over 100. The measures will certainly represent accurately the positions of the selected stars during the score of years that they were under Barnard's observation. Plans were begun some fifteen years ago for the publication of these measures; but they were delayed in the natural hope that with a longer time some evidences of motion would be established. Considerable attention was given during this work to following changes in brightness of some of the variables, and a few new variables were discovered by Professor Barnard in the clusters. He observed, in particular, Bailey's No. 33 in Messier 5, and determined its period with great precision, contributing half-a-dozen papers to the discussion of this star alone during the score of years that he observed it. At first he thought that the period was constant, but later the continued observations showed that it first lengthened, then shortened.

Although Mr. Barnard would naturally not be regarded as a regular observer of variable stars, he nevertheless discovered some ten of these objects, most of them visually, and he followed particular ones for many years; thus, he published three papers on the period and variation of RS Aquarii, which he discovered visually in 1898. He also followed rather closely several especially interesting stars of this sort, discovered by others, which required large optical power when they were near minimum.

The novae were of especial interest to him: he determined their positions micrometrically with great precision with respect to neighboring stars; he estimated carefully their fluctuations in light and noted the change in focus which resulted from their change in spectrum when the stars were too faint to be observed spectroscopically; he examined them minutely with the great telescope, to detect the presence about them of nebulous shells or phenomena of that nature. He, in fact, discovered visually, in the summer of

1892, the nebulous ring about Nova Aurigae. He included in his studies most of the historical novae for which the positions could be determined—all of this, with the 40-inch telescope. He was an independent discoverer of Nova Aquilae on the night of June 8, 1918, the date of the American eclipse. After his return to the Observatory he found that he had photographed the star on fifty-four dates during the preceding twenty-five years, the Willard lens having been used on four dates and the Bruce telescope on the remainder. He then determined the star's brightness on these plates. His observations of this character were very numerous; thus, he contributed no less than fourteen papers or notes to cover Nova Persei of 1901, and eight such papers on Nova Aquilae of 1918.

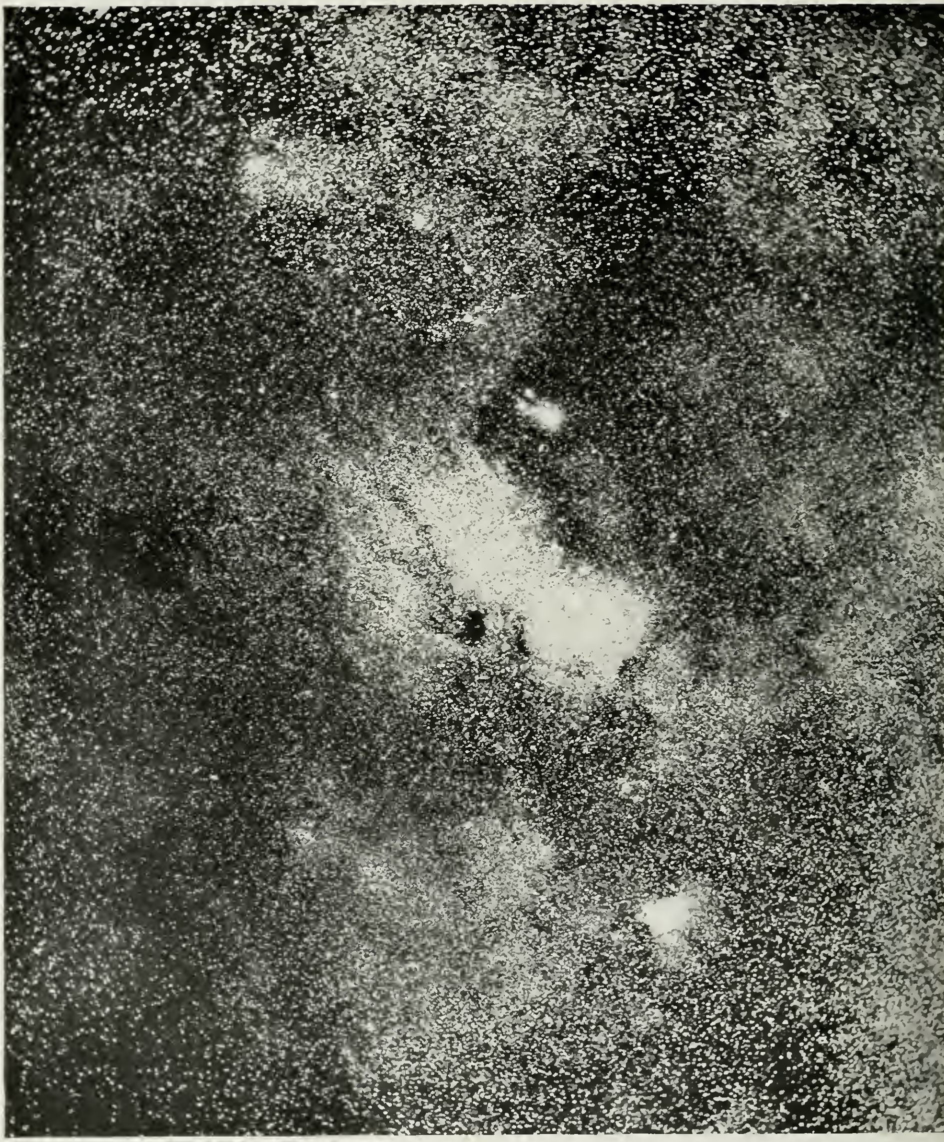
In view of the great range of temperature through which micrometric measurements are made with the 40-inch refractor, extending from $-25°$ F. ($-32°$ C.) to $+100°$ F. ($+38°$ C.), he began, in 1897, a series of control measures of the difference in declination between Atlas and Pleione. These observations were made on 506 nights during the past twenty-five years, and thus constitute a great mass of valuable unpublished material bearing both on the constancy of the telescope and micrometer and that of the stars themselves.

At the Yerkes Observatory he kept up his micrometric observations of the fainter satellites of the planets, which he had begun at the Lick Observatory, and contributed to the *Astronomical Journal* ten papers of observations of Saturn and several of Phoebe, the ninth satellite, which he caught as a very faint object in the opposition of 1904, when the planet was 17° south of the equator. At the oppositions of 1906 and 1912–13, when he had a good ephemeris of the satellite, he observed it several times, and estimated it to be of the fourteenth magnitude. We believe that Professor Barnard's measures with the 40-inch telescope are the only visual determinations of the position that have yet been made of this difficult satellite. He observed visually Perrine's sixth satellite of Jupiter, and published his measures in three papers.

Professor Barnard took part in the campaign for observation of the asteroid Eros, during the opposition of 1900 and 1901, for the determination of the solar parallax. His extensive measure-

2

PLATE II



Star Cloud in Sagittarius (R.A., $18^h 7^m$; Dec.,—$18° 15'$), Photographed with the 10-inch Lens of the Bruce Telescope by E. E. Barnard. Exposure, $4^h 30^m$.

ments appeared in Volume II of the *Publications of the Yerkes Observatory*, pp. 79–116, issued in 1903.

In 1897 Miss Catherine W. Bruce, of New York, at the solicitation of Professor Barnard, had given to the University the sum of $7,000 for providing a photographic telescope of the highest type of excellence with which he could continue his photographic investigation of the Milky Way and comets. Experiments were at once begun with various types of portrait lens, some of them furnished by Mr. Brashear, in order to find which was the most suitable objective for the purpose. At this time the cameras were strapped to a small equatorial, which was later installed for instruction at the University. This task of finding a suitable objective was continued for several years, and in December, 1899, the quest led Mr. Barnard to Europe, for he was determined to secure an objective which would represent the highest quality attainable in optical construction. Several of the leading European firms made small objectives for the test, but choice was made of the 10-inch doublet produced by John A. Brashear, of Allegheny.

The small wooden observatory for the Bruce telescope, having a dome 15 feet in diameter, was erected, in 1904, at a point 350 feet from the great dome and a less distance from Mr. Barnard's own home. The interest accumulated on the Bruce fund was sufficient to pay for the building. Warner & Swasey had provided for the telescope the excellent mounting of a new pattern particularly well adapted for the purpose. Besides the 10-inch doublet, the mounting carried a Voigtländer portrait lens of $6\frac{1}{4}$ inches aperture, which had been refigured by Brashear, and a 5-inch guiding telescope. A description of the Bruce telescope and its building, with reproductions of two photographs of the Milky Way taken with it, was published by Professor Barnard in the *Astrophysical Journal*, **21**, 35, 1905.

Professor Barnard was now provided with equipment for his work, which he had awaited for some years. When the sky was clear and not rendered useless by the obnoxious presence of the moon, Professor Barnard was generally to be found there making a long exposure on some part of the Milky Way, or on a comet, unless he had an assignment with the 40-inch telescope. Plate II

illustrates the remarkable character of Professor Barnard's photographs of the Milky Way. It is reproduced from the article, "Dark Regions in the Sky Suggesting an Obscuration of Light," which appeared in this *Journal* in 1913, **38**, 496–501 (Plate XIX), and depicts the smaller star cloud in Sagittarius. It includes a small black spot which was formerly called a dark hole, but which we now believe to be an intervening mass of obscuring matter.

He was unhampered by any administrative or editorial duties, and free from any engagements in the classroom, so that he was able to gratify to the full his passion for observing. To him, a night at the great telescope was almost a rite—a sacred opportunity for a search for truth in celestial places. Rarely has a priest gone up into the temple with a deeper feeling of responsibility and of service than did this untiring astronomer go up into the great dome. He was usually ready before the sun had set, and impatiently waiting until the darkness should be sufficient for him to "get the parallel" for the thread of the micrometer before he could observe fainter objects. During the day preceding one of his nights, his associates in the Observatory were generally conscious of his keen anxiety for a clear sky, as evidenced by a frequently repeated nervous cough, which was always worse if the prospects for the night were unfavorable.

It was a marvel to all of us that his bodily strength was equal to the tasks which he put upon himself. He was accustomed to get on with very little sleep, and if the night was cloudy he could never trust himself to relax, but was constantly on the lookout for a possible clearing of the sky. Nevertheless, he often appeared in his office by seven o'clock in the morning, and began work on the reduction of his observations of the night before. He was a very painstaking and accurate computer, and it was rare, indeed, that the positions of any celestial objects measured by him required correction for any numerical errors after they were published. From about 1906 he had the valuable assistance of Mrs. Barnard's niece, Miss Mary R. Calvert, who helped in his computations and in his correspondence, and in filing and cataloguing the great number of photographs and reduction sheets which he accumulated.

The nebulous regions of the Milky Way were always of much interest to Mr. Barnard, and he early discovered on his photographs

great nebulous areas which had not been previously suspected. He investigated many cases of nebulous stars, or of stars which seemed to be involved in "nebulosity," a word which he commonly used to describe vague and indefinite nebulous matter, generally of great extent. In some cases the term may represent a real distinction between a gaseous nebula and that which yields a continuous spectrum, sometimes probably due to light from a stellar source reflected by finely divided matter. The following quotation is from one of his early papers, entitled "The Great Nebula of Rho Ophiuchi and the Smallness of the Stars Forming the Groundwork of the Milky Way":[1]

> For many years this part of the sky troubled me every time I swept over it in my comet seeking; though there seemed to be scarcely any stars here, there yet appeared a dullness of the field as if the sky were covered with a thin veiling of dust, that took away the rich blackness peculiar to many vacant regions of the heavens. This was fully fifteen years ago, at Nashville, Tennessee, when I searched for comets with a five-inch refractor.
>
> After going to the Lick Observatory, I still noticed this peculiarity of that part of the sky, and finally found that two small stars north of Antares were involved in nebulosity and that the whole region seemed to be covered with a very weak diluted nebulosity. . . . .
>
> This part of the sky coming within the sphere of my work in photographing the Milky Way, on March 23, 1895, I made a photograph of it with $2^h\ 20^m$ exposure. The resulting negative showed a vast and magnificent nebula, intricate in form and apparently connected with many of the bright stars of that region including Antares and Sigma Scorpii.

Professor Barnard's studies of such objects may be indicated by the titles of some of his later papers on the subject, which are as follows:

"A Great Photographic Nebula near Pi and Delta Scorpii," *Astrophysical Journal*, **23**, 144–147, 1906.

"The Great Photographic Nebula of Orion, Encircling the Belt and Theta Nebula," *Astronomy and Astrophysics*, **13**, 811–814, 1894.

"Diffused Nebulosities in the Heavens," *Astrophysical Journal*, **17**, 77–80, 1903.

"On a Nebulous Groundwork in the Constellation Taurus," *Astrophysical Journal*, **25**, 218–225, 1907.

"On a Great Nebulous Region and on the Question of Absorbing Matter in Space," *Astrophysical Journal*, **31**, 8–14, 1910.

[1] *Popular Astronomy*, **5**, 227, 1897.

"The Exterior Nebulosities of the Pleiades, with a Drawing from the Different Photographs; and on the Appearance of the Involved Nebulosities of the Cluster with the 40-Inch Refractor," *Monthly Notices of the Royal Astronomical Society*, **60**, 258–261, 1900.

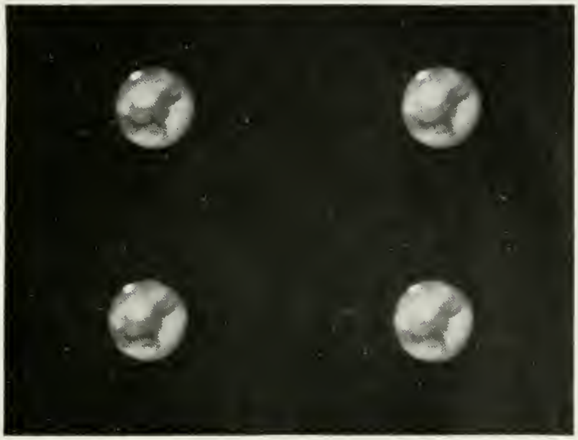
Plate III of this article gives an example of one of his photographs of these nebulous regions, and is taken from his paper entitled, "Some of the Results of Astronomical Photography Pertaining Specially to the Work with a Portrait Lens," which appeared in *Proceedings of the American Philosophical Society*, **46**, 417–429, 1907.

Professor Barnard had early formed a plan for securing a photographic chart of the Milky Way, and he was quick to accept an invitation from Professor Hale to bring the Bruce telescope to Mount Wilson for photographing particularly the southern portions of the Galaxy, in so far as they could be reached from that latitude. The telescope was, accordingly, transported to Mount Wilson, under the auspices of the Carnegie Institution of Washington, in January, 1905, and Professor Barnard spent about nine months on the mountain, engaged in this work. The telescope was back in its own dome at Williams Bay before the end of the year, and for the next seventeen years was always available for Mr. Barnard's use, being very seldom employed by any other observer.

In 1907, the Carnegie Institution undertook to publish the *Atlas of the Milky Way*, when it should be ready, and during several years search was made for the best mode of reproduction of the pictures. Careful experiments were undertaken by experts in photogravure, and with the heliotype process, but the degree of perfection desired could not be quite attained. Finally, Mr. Barnard accepted the suggestion that photographic prints would most faithfully reproduce the wonderful details of the original negatives. Accordingly, with infinite pains, he made positives from the original negatives and then second negatives from which the prints could be prepared. In this way, the contrast in faint regions was increased and details were brought out which might otherwise have been lost. A firm of commercial photographers in Chicago, A. Copelin and Son, personally well known to Mr. Barnard, undertook the task of making

PLATE III

a



b

a) Mars, as photographed by E. E. Barnard with the 40-inch refractor.

b) Nebulous region of Gamma Cygni, photographed with the Bruce telescope by E. E. Barnard. Exposure, $6^h\ 30^m$.

the necessary number of 700 prints from each of the 50 negatives selected to represent the Galaxy. For two years, beginning in May, 1915, Mr. Barnard made frequent trips to the city and personally inspected each of the 35,000 prints, rejecting hundreds and even thousands of those which seemed to him to be lacking, in some detail, the high quality of excellence which he desired to be maintained.

It was very difficult for Mr. Barnard to take time from the reduction and discussion of current observations in order to devote himself to the descriptions of these photographs which were to be a part of the *Atlas*. He was also constantly finding new points of interest, as he studied each photograph in detail, which led him to desire new photographs centering on special regions, or having longer exposures than he had previously given. The publication of this *Atlas* was accordingly delayed, but, fortunately, Mr. Barnard had been prevailed upon to give more time to the completion of the text, and it had been finished, so far as the 50 regions illustrated were concerned. It is to be regretted that he had not written the introduction, which would have summed up his views on the structure of the Milky Way, based upon a personal knowledge more intimate than that possessed by any other person. His notes on the introduction are fragmentary, but they can be used, and it is hoped that the *Atlas* can be published before the close of 1923, in essentially the manner in which Mr. Barnard would have desired it, and accompanied by charts from drawings giving the co-ordinates of the region of each photograph, with a designation of the important features.

During the last decade, Mr. Barnard had taken a special interest in the dark markings in the Milky Way. At first he called them vacancies, and it was only gradually that he was led to the view that they were, after all, in many instances, dark objects projected against the Milky Way and absorbing its light. The titles of some of his papers show this gradual transition in the interpretation of these extraordinary structures. We may cite from his articles on this subject, the following:

"Small Black Hole in the Milky Way," *Astronomische Nachrichten*, **108**, 369, 1884.

"On the Vacant Regions of the Sky" (3 plates), *Popular Astronomy*, **14**, 579–583, 1906.

"Dark Regions in the Sky Suggesting an Obscuration of Light" (2 plates), *Astrophysical Journal*, **38**, 496–501, 1913. This is a paper containing a negative reproduction of the extraordinary dàrk marking near Zeta Orionis which has lately been so successfully shown on a photograph taken by Professòr J. C. Duncan,[1] with the 100-inch reflector at Mount Wilson.

"Some of the Dark Markings of the Sky and What They Suggest" (2 plates), *Astrophysical Journal*, **43**, 1–8, 1916. In this paper, he says:

> An important fáct that may come from our knowledge of the existence of dark nebulae is that their masses must be much greater than would be assumed for the ordinary nebulae, because they are prefectly opaque and must be relatively dense, and hence comparatively massive. If this is so, then, we must take into account these great masses in a study of the motions of the stars as a whole.

In that paper (Plate I) he placed side by side a luminous gaseous nebula and a dark object of very nearly the same shape: the resemblance is striking.

"On the Dark Markings of the Sky, with a Catalogue of 182 Such Objects," *Astrophysical Journal*, **49**, 1–23, 1919.

The last-named paper summed up his studies of these objects, which will doubtless be designated in the future by the numbers which he assigned to them in the catalogue.

A very important question in recent years has been the proper location in our stellar system of the globular star clusters. From his studies of their appearance on his photographs of the Milky Way, Professor Barnard was led to the opinion that the clusters are in some instances obviously projected against the background of the Milky Way. His paper, "On the Comparative Distances of Certain Globular Clusters and the Star Clouds of the Milky Way,"[2] seems not yet to have received quite the attention that it deserves. We may quote from it the following:

> Just as the great star clouds of the Milky Way act as a background against which non-luminous masses may be seen in dark relief, they must also act as a

[1] See *Astrophysical Journal*, **53**, 392, 1921.

[2] *Astronomical Journal*, **33**, 86, 1920.

screen and thus hide any object that is behind them. This gives us a means of inferring the relative distances, etc., of many of the great globular clusters. The rich regions of Sagittarius and Aquila, in which some of the finest globular clusters occur, are specially remarkable for their density. That these clusters are nearer than the great star clouds is evident, for they would not be seen through the star clouds if beyond them.

He cites as particular examples, N.G.C. Nos. 6266 (M 62), 6273 (M 19), 6293, 6304, 6333 (M 9), 6528, 6656 (M 22), and 6712.

Although Professor Barnard had given great attention to the surface markings of the planets, it was not until 1909 that he began experiments in photographing the planets with the large refractor, employing a secondary magnification. It will be understood that the direct images of the planets are so small, even in an instrument of long focus, that the grain of the plates makes it impossible to secure a satisfactory photographic enlargement. This work required great skill and patience, because, as Professor Barnard said:[1] "Better conditions are required for successful work in this direction than for visual observations. One can do much visually under conditions where the best definition is only momentary, but for these enlarged photographs any break in the definition for even a single second during the exposure means injury or total ruin to the image. In all the exposures, though of only a few seconds' duration, it was necessary to guide the telescope to keep the image stationary. This was done by bisecting the polar cap by cross-wires (spider threads) in the focus of the long guiding finder ($61\frac{1}{2}$ feet focus) of the 40-inch telescope." It was intended that a full description of this photographic work on planets should be published in this *Journal*, but Professor Barnard never found time to do this. On Plate III herewith are shown four of these enlarged images of Mars which were taken on the same photographic plate at short intervals, in the hope of catching moments of the best definition. He obtained also pictures of Jupiter and of Saturn, but instants of the finest seeing when such work was in progress were too rare to yield absolutely satisfactory pictures. When visiting Mount Wilson in 1911, Mr. Barnard obtained fine photographs of Saturn with the 60-inch reflector.

[1] *Monthly Notices of the Royal Astronomical Society*, 71, 471, 1911.

We have mentioned before that Mr. Barnard's early interest in the photography of comets and their tails did not abate after the Bruce telescope was put into operation at the Yerkes Observatory. He secured fine series of photographs of all that appeared in our sky, of which may be particularly named: Giacobini's of 1905–1906, Daniel's of 1907, Morehouse's of 1908, Halley's of 1909–1910, Brooks's of 1911, Delavan's of 1914. Of these, Comet Morehouse of 1908 and Brooks of 1911 exhibited the most remarkable activity in their caudal demonstrations, and their eccentricities kept Mr. Barnard almost constantly at the telescope while it was possible to photograph them.

The return of Halley's Comet was awaited with the keenest anticipation by Mr. Barnard. He took many photographs of the region where it might be expected in 1909, and followed it persistently after it was revealed on Professor Max Wolf's plate of September 11 of that year. The records of previous appearances of Halley's Comet had been most carefully studied by Mr. Barnard, but there were many points on which the history was silent or incomplete. He determined to provide against this deficiency for the return of 1910. He kept very full notes on all his observations during the twenty months through which he was able to follow the comet, and embodied these in a long paper appearing in the *Astrophysical Journal* for June, 1914, entitled, "Visual Observations of Halley's Comet in 1910" (**39**, 373–404). In this paper he says: "Halley's Comet at its return in 1910, though a brilliant and interesting object to the naked eye—especially in the month of May—was, nevertheless, a disappointment when considered from a photographic standpoint. It is safe to say that it did not give us any new information concerning these strange bodies." The expected passage of the tail of the comet so close to the earth as to envelop it on May 18–19, 1910, kept Mr. Barnard on the *qui vive*, and the sky was watched throughout the day as well as the night with the greatest of care. Mr. Barnard felt amply rewarded for his pains by the spectacle of the tail in the early morning of the 19th, when he could map it visually for a length of 120 degrees; even on the preceding morning he had been able to record its length as 107 degrees. He last saw the comet a year later, on May 23, 1911, when he secured a

position of it, with some difficulty on account of its faintness, with the 40-inch telescope.

In order to have observations of this comet made in longitudes otherwise unoccupied, the Committee on Comets of the American Astronomical Society, of which Professor Barnard was an active member, secured a grant from the Bache fund of the National Academy of Sciences, which made it possible to send Mr. Ferdinand Ellerman, of the Mount Wilson Observatory, as an observer at Diamond Head, Hawaii. Mr. Barnard spent considerable time in preparing his part of the report of the Committee, which was printed in the *Publications* of the Society in 1915.

It will be understood that in addition to his photographic observations of comets, Professor Barnard was always obtaining their positions with the filar micrometer of the 40-inch telescope, whenever such positions were necessary, upon the first appearance of a comet or after it became too faint for moderate instruments. It may seem a little singular that the large number of long exposures made by Mr. Barnard on the Milky Way did not lead to the discovery of other new comets; but such was the case, and we may well believe that this possibility was not overlooked by one to whom comets had meant so much in his earlier career. In addition to those already mentioned, he was the first to observe, at their predicted return, Encke's Comet in 1914 and Pons-Winnecke in 1921. On a plate taken while he was at Mount Wilson in 1905, he found, some months later, the impression of a comet, which received the name, 1905 f, but was not observed elsewhere.

In spite of Professor Barnard's passion for exact measurement, he still regarded his splendid photographs rather from the point of view of a photographer than from that of an expert in measurement; of course, whenever it was necessary he obtained the positions of comets or other objects on the negatives, but in a general way he had these two distinct attitudes of mind in his work. He was somewhat reluctant to feel that the photographic procedure in astronomy could in many cases supersede the older visual methods for which, in some respects, a much higher degree of expert skill was necessary. His collection of some 1400 negatives of comets contains material on which a vast amount of measurement could be made.

and we trust that this will sometime be done in the study of the peculiar internal motions of comets and their tails. The negatives of the Milky Way and of fields of the sky taken by Professor Barnard at the Yerkes Observatory number nearly 4000, and these constitute a rich field for investigation of stellar motions, for discovery of variable stars, and for statistical studies of the structure of the universe. It is hoped that these plates, which extend over nearly a score of years, may soon be investigated under the "blink" comparator for motions and variables, and it is certain that the full study of this splendid series of photographs will bring to light many important facts.

Occasionally, Mr. Barnard had time to investigate pairs of plates under the "blink" comparator; thus, on comparing a plate taken in May, 1916, with one of the same field he had taken with the Willard lens in August, 1894, he discovered the star in Ophiuchus having a proper motion of 10".3 per year, the greatest proper motion thus far detected. This motion was, in fact, so unexpectedly large as to make the discovery very difficult, but the plates were numerous enough to confirm its reality. The position of the object, familiarly known by our staff as Gilpin, was carefully measured by Mr. Barnard with the filar micrometer. Its parallax was investigated here and at other observatories, and was found to be 0".53, corresponding to a distance of 6.1 light years, thus making this dwarf the nearest star, after the system of Alpha Centauri. With the large scale of the 40-inch telescope, photographs taken a week apart make the proper motion evident and measurable!

Professor Barnard was deeply interested in eclipses of the sun, and he secured with a visual lens some excellent photographs of the corona at the total eclipse of January 1, 1889, at The Willows, a point in California not far from Mount Hamilton.

In 1900, at the station of the Yerkes Observatory at Wadesboro, North Carolina, he was again favored with a clear sky and secured excellent photographs with the horizontal telescope of $61\frac{1}{2}$ feet focus, but here he denied himself the privilege of a direct view of the corona, remaining inside the spacious camera with Mr. Ritchey to assure the accuracy of the exposures and the perfection of the result. They saw the corona only as it was projected on the photographic film.

Mr. Barnard was invited to join the large expedition to Sumatra organized by the United States Naval Observatory for the total eclipse of May 18, 1901. His station was at Padang Padang, and he planned every detail with the greatest of care for photographs of the corona on a very large scale. The duration of totality was very long, nearly a maximum of six minutes. It was tragic that a thick blanket of clouds totally prevented him from making any observations at that time. He was absent from the Observatory for about six months, and this further deprived him of the opportunity of observing Nova Persei when it was bright.

He was greatly interested in the eclipse of June 8, 1918, and made a trip of inspection with the writer in September, 1917, to select suitable stations in Wyoming and Colorado. He went on with the advance guard of our party to our principal station at Green River, Wyoming, six weeks before the date of the eclipse. He took infinite pains in the adjustment of the horizontal telescope used with our coelostat and could be content with nothing but perfection in the focusing of all the cameras for which he had any responsibility. Unfortunately, a great cloud drifted across an otherwise perfect sky on that afternoon, covering the sun until two or three minutes after totality was over. Mr. Barnard had been again interested in studying the conditions for the eclipse of September 10, 1923, and up to within a fortnight of his death we had still hoped that he might be a member of our party.

Lunar eclipses were not neglected by Professor Barnard: he had photographed successfully, at the Lick Observatory, the eclipses of 1894 and 1895, and it was his custom to make with the Bruce telescope many photographs of the different phases of each lunar eclipse which occurred in favorable weather. With that instrument he also kept a photographic record of all interesting conjunctions of the planets and similar occurrences.

The displays of the aurora, which occur frequently in Wisconsin, were a delight to Professor Barnard, and he recorded their details fully and minutely. Two extended papers based on his observations from 1897 to 1909 appeared in this *Journal*, and his notes covering almost another solar cycle of the aurora are still unpublished. He also gave attention to the self-luminous night haze, which his long vigils had given him unusual opportunities to observe,

and he presented two papers on the subject to the American Philosophical Society, one in 1911 and the other in 1919.

We thus find him a keen observer of nature in most of its visible phases. The meteors did not escape him or his photographic plate, nor did the seventeen-year locusts at their regular recurrence. In the growth of the trees which he had planted about his home he had great satisfaction, and he had much pleasure in following the development and planting of the grounds of the Observatory, when that became possible a few years ago.

In his Will, Professor Barnard bequeathed to the University of Chicago his home, as a memorial of his wife, whose death occurred on May 25, 1921. To the Observatory, he left also his scientific books and the medals and awards he had received in recognition of his notable services to science.

Professor Barnard's home had been a center of generous hospitality for a quarter of a century, and nowhere was he more entertaining than as host in his own home. He was full of humor and could tell most amusingly of his experiences in early life and of his travels. It has been the writer's good fortune to make many railroad trips with him, and he was always a most agreeable companion. He was shy and restive in larger companies where he was not well acquainted with the other guests, and was often quite nervous before giving a lecture on a subject with which he was perfectly familiar. After he was well started in an address he quite lost this shyness and would describe the intimate details of his pictures in a charming way. He never spoke more interestingly than in one of his last lectures which he gave one evening, on the subject of comets, at the meeting of the American Astronomical Society at the Yerkes Observatory in September, 1922.

Always mindful of the difficulties that he had to overcome in his early beginnings as an astronomer, Professor Barnard was most generous in giving advice and assistance to all sincere aspirants for knowledge of astronomy who approached him with questions, by letter or in person, and he gave his time freely to such, in exhibiting and explaining his most significant photographs. He willingly took his turn in speaking to the large numbers of visitors admitted to the Yerkes Observatory on Saturday afternoons, and often stayed long

after the closing hour in explaining details to those who had evinced a real interest. He was most kind to other workers in astronomy, and tolerant in expressing opinions of them, even though their views might differ very greatly from his, and though he might regard them as radically wrong. He avoided controversy, and seldom took his pen to oppose the views of others.

Professor Barnard was not a teacher. He had missed the inspiration and opportunity of studying astronomy under some gifted enthusiast. He had, of course, profited by taking part in the work at an institution so well planned and organized as was the Lick Observatory by Director Holden, and had received much benefit from the sane counsel of his seniors there, particularly from Mr. Burnham; but he did not realize from experience the mutual importance of the relation of teacher and pupil, or know the satisfaction of the teacher in having an apt follower in his research to whom he may pass on the acquisitions of his years of study. Mr. Barnard could not bring himself to lose time at the telescope in having a pupil take part in measurements, which he could himself make so much better, and he begrudged the possible loss, in quality, of a photograph if someone less skilled than himself took some part in the guiding. Accordingly, he trained no one to be his successor; he left no disciple who could take up his work after receiving the benefit of his unequalled experience as an observer and of his exceptional knowledge of the heavenly bodies.

Mr. Barnard was striken with diabetes early in the year 1914, and had to undergo the severe privation, by the doctor's orders, of giving up observations with the large telescope for a year. As a result of his obedience, his health was greatly improved, and for the past seven years he kept up his observing most industriously and really beyond the measure of his bodily strength. It was regarded by the director of the Observatory as no small part of his duties to see that such a man should be induced to spare himself as much as possible and to restrict his night work both to save him from exhaustion and to gain time for the reduction and discussion of his great accumulation of observations. But it was almost impossible for Mr. Barnard to keep away from the Bruce photographic telescope when the sky was clear and the moon did not interfere.

He was greatly affected by the death of his wife in May, 1921, after a brief illness and after forty years of married life in which she had devoted herself completely to his comfort. They had no children, and thus Mr. Barnard missed the joys and responsibilities of parenthood, even as he had himself missed the experience of the relation of son to father.

His final illness was of only six weeks' duration, and began rather acutely. The best of medical skill was given him, and up to a short time before his death the specialist was hopeful of his recovery. He died at eight o'clock on the evening of February 6, and simple funeral services were held on the following day in the rotunda of the Observatory, which seemed to us the appropriate place. The interment was at Nashville, after services attended by many friends in his native city and from his Alma Mater. He had always been highly appreciated at Nashville, and one of the interesting evidences of this was the erection, not long ago, by the Nashville Automobile Club, in co-operation with the Nashville Historical Committee, of a tablet at the place in the city where the young enthusiast discovered his first comet in 1881.

Measured by the calendar, his life was but little more than sixty-five years, but, by the number of hours he had spent under the nocturnal sky or in the domes, his period of activity was more than that of many who had passed four score years.

His services to science were recognized by the learned societies. He was vice-president of the American Association for the Advancement of Science in 1898, and delivered an address upon "The Development of Photography in Astronomy." In the same year he was elected a foreign associate of the Royal Astronomical Society. He became a member of the American Academy of Arts and Sciences in 1892, of the American Philosophical Society in 1903, and of the National Academy of Sciences in 1911. He was made a director of the B. A. Gould Fund, under the auspices of the National Academy, in 1914, and at the same time an associate editor of the *Astronomical Journal.* He had been for three years (1892–1894) associate editor of the journal *Astronomy and Astrophysics.* In addition to the medals mentioned heretofore in this article, he received from the French Academy of Sciences in 1893 the Arago

gold medal, and in 1900 the Janssen gold medal; he was the recipient of the Janssen prize of the Astronomical Society of France in 1906, and was awarded the Bruce gold medal of the Astronomical Society of the Pacific in 1917. The last-named society had three times awarded him the Donohoe comet medal. He was given the honorary degree of Doctor of Science by Vanderbilt University in 1893, and that of Doctor of Laws by Queen's University, of Kingston, Ontario, in 1909.

The voluminous character of his contributions to astronomical literature has already been indicated, but we may add that a card catalogue of his writings includes no less than 900 items, without being complete. A bibliography of his articles which appeared since his connection began with the University of Chicago in 1895 has been published in the list of publications of the faculties issued annually by the University of Chicago and also collected in two special volumes of this character covering the first twenty-five years of the work of the University. These contain the titles of 377 articles and 6 book reviews by Professor Barnard, to which number will be added, as time permits their preparation, numerous papers covering his unpublished observations.

His last measurements with the 40-inch telescope were made on the night of December 16, 1922, when he secured the position of Baade's Comet; his last visual observations with that instrument were on December 21, when he made nineteen estimates of the brightness of Nova Persei, referred to thirteen comparison stars; and his last use of that instrument was later on that night when he made a photograph of the cluster, Messier 36, with an exposure of two hours. A photograph of the region of Gamma Leonis, made on the following night with the Bruce telescope, closed his long and untiring work with that instrument. His last visual observation was of the occultation of Venus on the morning of January 13, 1923, which he observed from the window of his sick room. Thus closes the record of the astronomical activity of one of the greatest observers of our time, of whom may be truly said

"Aperuit caelos."

YERKES OBSERVATORY
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INVESTIGATIONS OF PLATE ERRORS WITH THE THERMO-ELECTRIC PHOTOMETER

By HARLAN TRUE STETSON and EDWIN F. CARPENTER

ABSTRACT

Thermo-electric photometer for stellar images.—The improvements in apparatus described in previous articles in this *Journal* have resulted in reducing the time necessary for the measurement of star magnitudes by at least 50 per cent and, at the same time, increasing the sensitivity from 100 to 200 per cent. The probable error of measurement of a photographic star image is ± 0.007 mag., so far as the errors of apparatus and manipulation are concerned.

Tests of uniformity of emulsion on photographic plates.—Tests were made to determine the degree of uniformity in the emulsion of the photographic plates, (*a*) by securing equal exposures of an artificial star scattered widely over the plate, and measuring the resulting images with the thermo-electric photometer; (*b*) by exposing restricted but widely scattered regions of the plate to a uniform source, and measuring the opacities as for extra-focal images. The *sensitivity* of a plate was found to be more nearly uniform over the entire surface when the plates were presoaked for one hour after exposure and before development, and allowed to dry on edge in still air. The *probable error due to non-uniformity for extra-focal work* was found to be 0.03 mag. and for *focal work* 0.02 mag. for a single "image" from the photometric measures. A tendency to systematic changes in sensitivity over the surface is indicated.

In articles previously published in this *Journal*,[1] an apparatus and method were described for measuring the magnitudes of stars from photographic plates by means of a thermopile. Although the thermopile could be used simply for the measurement of plate opacities in extra-focal images, it was pointed out that the principal merits and advantages of the thermo-electric photometer lay in the measurement of focal images, where it appeared a higher degree of precision could be obtained by this method than by other methods applicable to focal images.

As has been pointed out,[2] the most serious drawback to a considerable gain in precision in photographic photometry seems to lie in the errors inherent in the plate itself. Previous investigations have left considerable uncertainty as to the actual order of accuracy which the plate may be relied upon to render in photometric work. In any investigation of plate errors, it appears at once that the method of measurement must exceed in accuracy the reliability

[1] *Astrophysical Journal*, 43, 253 and 325, 1916.

[2] *Ibid.*, 43, 278, 279.

of the plate, for it is only by such means that we can make reliable deductions which may lead to improvements in the photographic process, or that may render feasible a higher degree of precision in photographic photometry. The thermo-electric photometer, therefore, is peculiarly adapted to such an end.

Before describing the methods and results of the plate tests, mention should be made of some important improvements in the design and construction of the photometer since the publication of the description of the apparatus in the previous account.[1]

IMPROVEMENTS IN THE APPARATUS

The general scheme of the arrangement has remained unchanged, and is that depicted in drawing in the earlier article to which reference is again made. However, by refinements in the optical system and the use of a newer type of galvanometer, a remarkable gain in sensitivity has resulted, which has made possible the elimination of the 200 watt 50-volt lamp as a source, and the substitution in its place of a 20 watt 6-volt lamp of standard make. This lamp contains a filament of tungsten wire in the form of a closely wound helix of small dimensions. Its efficiency in the operation of the photometer depends as before upon the proper orientation of the filaments in such a way that the images formed by the condensing lens shall cover the diaphragm with approximately uniform illumination.

This tenfold reduction in the wattage of the lamp has not only made possible the use of a small portable storage battery in place of the 50-volt battery, but avoids undue heating of the emulsion of the plate during the process of measurement, thereby producing more equable conditions of temperature. In spite of the marked reduction in the energy of the source, it has become possible by virtue of the improved galvanometer to employ diaphragms 40 per cent smaller than had previously appeared practicable, so that for faint star images of the order of 0.1 mm in diameter a diaphragm 0.3 mm in diameter is employed, giving values of δ (the fall in galvanometer deflection for a given star image) 80 per cent greater for a given star image than had been obtained previously with a

[1] *Loc. cit.*

diaphragm 0.5 mm in diameter. The galvanometer now used is a D'Arsonval, type R, manufactured by the Leeds Northrup Company. It has a sensitivity of 2.3 mm per microvolt, an internal resistance of 10 ohms, and a period of 5 sec. when in series with critical damping resistance of 50 ohms. With the outfit as thus described and illustrated in Figure 1, an actual gain in sensitivity of 100 to 200 per cent has been secured.

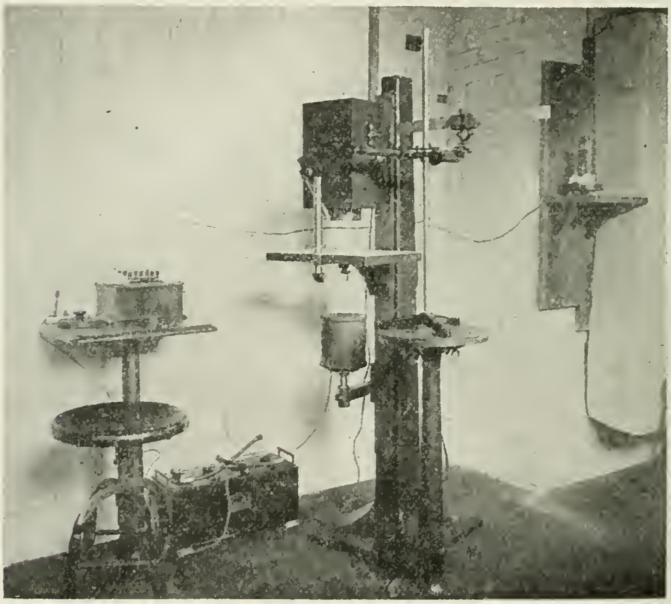

FIG. 1.—The thermo-electric photometer and auxiliary apparatus

More recently, furthermore, the addition of a metallic plate inside the thermopile house just above the thermo elements has facilitated radiation from the cold junctions and reduced from 50 to 75 per cent the time consumed in establishing thermal equilibrium. Recent work shows that the time necessary for the measurement and reduction of a star magnitude, making five settings for each image, is about two and one-half minutes, with a probable error of measurement of 0.007 mag.

By employing three right-angled prisms and an adjustable mirror, it was found possible to combine the microscope for viewing

the plate with the short-range telescope by means of which the centering of the star image upon the thermopile is effective. A small lever just below the eye lens tilts a mirror within the tube into either of two positions, allowing the observer to look directly through to the plate surface at the top of the stage, or by way of the three prisms to view the bottom of the well in the thermopile house for the purpose of centering the star image upon the receiving surface of the thermopile.

In manipulation of thermo-electric apparatus, too great emphasis can hardly be stressed upon securing firm contacts not only in the thermopile-galvanometer circuit but in the battery-lamp circuit as well. Any slight irregularity in current flow interrupts the wattage output of the lamp with an instant effect upon the galvanometer deflections.

DESCRIPTION OF THE PLATE-TESTING DEVICE

In devising an arrangement for testing variations in sensitivity over a photographic plate, two fundamentally different methods may be used: (1) to expose a large number of isolated spots on the plate simultaneously to a uniform field of illumination; and (2) to cover the plate with a series of successive exposures equally timed and of the same illumination. The first method of single exposure guarantees equal exposure times for all parts, but presents no little difficulty in devising a source of light which may be trusted to give the same intensity of illumination on all parts of the plate. In the method of successive exposures, we may arrange for each exposure to take place in the optical axis of the system, and by so doing imprint artificial star images over the entire surface of the film by suitably moving the plate after each exposure. The special precautions necessary in this arrangement are to maintain constant the source of illumination, distance of the plate from the source, and time of exposure.

In the present investigation, it was decided to adopt, first, the second of these methods of procedure, since it made possible the photography of artificial star images free from field corrections, which would very closely simulate astronomical conditions.

The apparatus as adopted in its final form for making the plate tests consists of a small artificial star (pinhole diaphragm) illuminated by a 24-watt lamp, fed by an 8-volt storage battery of 120-ampere-hour capacity. The exposures are produced by releasing a pendulum shutter which uncovers the "star" for about one second. The equality of the exposure times is maintained by the isochronism of the pendulum, and can undoubtedly be relied upon to an accuracy well within 1 per cent.

At a distance of 100 cm from the "star" is an anastigmat lens of 16-cm focus, which serves to form the image on the plate to be tested. The ways that carry the plate are mounted on a rotating disk, the axis of which in turn is mounted so as to allow motion in horizontal ways at right angles to the optical axis of the system. This combination of polar and rectangular co-ordinates was adopted to facilitate both construction and manipulation, and minimizes the errors attributable to variation in focus as the plate is moved to bring different parts into the optical axis for the exposures. A series of depressions on the back side of the metal disk carrying the plate and a series of notches on the horizontal ways serve as convenient stops for spring palls which locate the position of each exposure on the plate as it is rotated in position angle and moved horizontally in the ways. The complete apparatus is set up in the photographic dark room of the laboratory, and is operated in total darkness without inconvenience.

A plate properly exposed and developed shows a series of images uniformly distributed over the surface to within about 10 mm of the edge. These images, having received equal exposures, should then all be of equal size and blackness, and, when measured on the thermo-electric photometer, should yield the same magnitude within the experimental errors, *provided* the sensitivity of the film is uniform over the entire surface. This will not be the case, and the discrepancies which result will therefore be a measure of irregularities in the film sensitivity.

In order to interpret the measured irregularities, there must be impressed upon the film a series of images of known magnitude difference. This is accomplished by making additional exposures

while in the testing machine, and varying the aperture of the camera lens by the use of diaphragms giving approximately half-magnitude intervals. The exact values for the diaphragms were computed through measurement of their diameters.

With such a scale of magnitudes, the plate gamma is readily determined from the thermo-electric measures and therefrom the magnitude residuals for each image become known. A number of "Seed 30" plates (4 in.×5 in.) tested in this way showed probable errors of .05–.08 of a magnitude for commercial plates receiving the ordinary treatment in development. As this error was considerably larger than was anticipated, some plate-glass plates were secured through the courtesy of the Eastman Kodak Company, which, it was hoped, would reduce these errors somewhat. It seemed likely that the concavity of the ordinary plate, which from a series of tests was found to vary from about 0.002 in. to 0.008 in. on the 4×5 size, might be responsible for different thicknesses in the emulsion at different parts of the plate. Furthermore, this concavity introduces a small magnitude error for focal images in that the entire film surface cannot be relied upon to preserve a constant relation to the focal plane of the objective.

Comparison of results obtained with the ordinary and the plate-glass plates, however, did not show startling differences, except where the surface of a commercial plate proved unusually poor. The consistently smaller probable error which results from the use of plate glass shows, however, that for work in photometry of the highest precision, plate glass should be adopted as a necessary requisite for the support of the photographic emulsion.

For the purposes of studying the distribution of plate errors, a chart was made of each plate studied, an example of which is shown in Figure 2. A tendency to systematic distribution of plus and minus errors led to the detection of the source in a small change in focus of about 0.0005 in. (0.0012 cm) as the plate carrier was rotated and shifted in the testing machine. Careful measurements with a surface gauge were taken for the focal setting of each point with a plate in the testing machine; and the small variations in focus, thus determined, were plotted against the mean of the magnitude errors of each "star image" for a series of plates. The results

gave nearly a straight-line relation showing a change of 0.1 mag. for a change in focus of 0.0002 cm. When the allowance was made for these systematic errors, the probable error for a single image was reduced about .02 of a magnitude.

Inasmuch as it seems probable that the errors are chiefly introduced during the drying process, due to unequal draining and evaporation of the wash water, numerous tests were made of plates dried

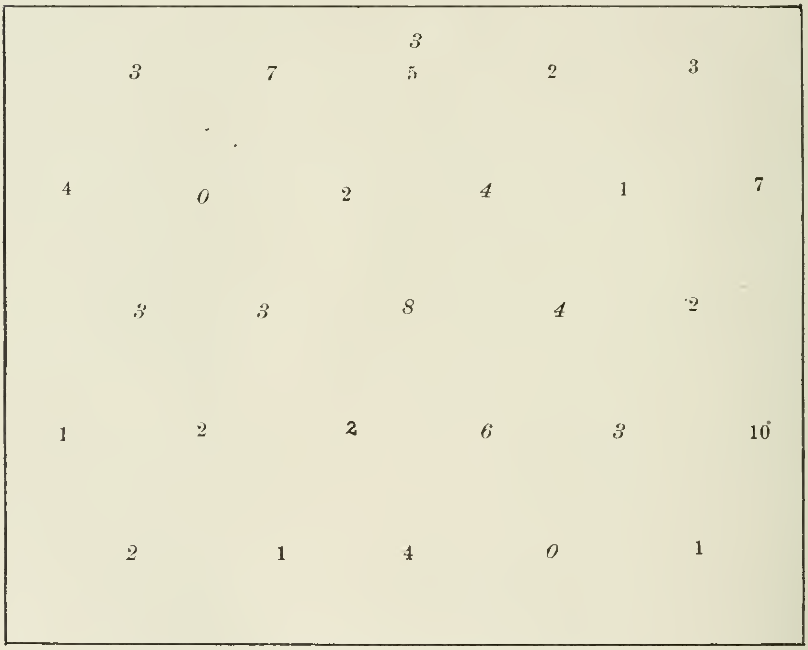

FIG. 2.—Plot showing magnitude residuals for Plate HS 63 P. Roman type indicates positive and *italics* negative residuals. Unit = 0.01 mag.

in different positions, viz., face up, face down, on edge with side down, and on edge with corner down. The result of these tests showed that nothing was to be gained by any different orientation than that usually and most conveniently adopted, i.e., corner down.

To test the effect of speed of drying, plates were dried by force in two ways: (*a*) by use of alcohol, (*b*) by electric fan; and results in both cases showed a considerable increase in the size of errors. Errors were less for plates dried slowly in still air by means of a

drying cabinet consisting of a wooden frame covered with cheesecloth and containing two supporting shelves of coarse wire screening.

Correspondence with Dr. Ross, of the Eastman Kodak Company, led to the experiment of soaking the plates for one hour, preferably in distilled water, just before development. The average probable errors on plates so treated were about one-half that of unsoaked plates. This appears to be an important step. It seems probable that the prolonged soaking relieves certain strains in the gelatin emulsion, resulting in better uniformity both in development and in the subsequent drying.

Test exposures have been made on some plates backed with plate glass and ordinary glass, and including "Seed 27," "Seed 30," "Seed 23," and "Cramer Hi-Speed" emulsions. There seems to be little difference between the fast and slow plates, judging from the plates tested. Experiments were tried with different developers including ferrous oxalate, hydroquinone; and slightly smaller errors have seemed to favor the hydroquinone developer which, as used in the present instance, was compounded as follows:

A		B	
Water (distilled)	100 cc	Water (distilled)	100 cc
Sodium Sulphite	3 gm	Sodium Sulphite	3 gm
Hydroquinone	3.7 gm	Potassium Carbonate	12 gm
Sulphuric Acid	0.4 gm	Potassium Bromide	2 gm

A brief table (I) giving a detailed summary of some of the more recent tests is appended.

The first column gives the serial number of the test plate, the second column the kind of plate—the suffix letter C or P indicating commercial or plate glass, as the case might be. The third and fourth columns give the emulsion number and the kind of developer used. The fifth column gives the probable error for a single image as determined from each plate on the assumption that all errors result from the emulsion itself. The last column gives the same probable error when corrected for the systematic errors of the testing device, due to the slight change in focus as the plate is rotating.

After it appeared that little further variation in the probable error was to be expected from further tests of this kind it was decided to try the first of the two methods mentioned, namely, exposing

simultaneously selected parts of a plate to a uniform source. In doing this, all questions of variation in focus would be eliminated. The plate was accordingly placed in a holder directly behind and in contact with a grid which should give alternate exposed and unexposed regions. The plate was then exposed to a source of illumination rendered as nearly uniform as possible through the use of multiple diffusing screens and was rotated by a motor throughout the exposure so that any lack of uniformity in the illumination should appear as systematic differences in opacity concentric about the geometric center of the plate. Some forty or fifty spots were

TABLE I

Plate No.	Kind	Emulsion	Developer	Prob. Error for Single Image	Corrected Prob. Error
25.......	Seed 30 C	4200–12–53	E.Q.	$\pm 0^{M}.061$	....
27.......	Seed 30 C	4200–12–53	Ferrous Oxalate	$\pm$0.052	....
31.......	Seed 30 C	4035–26–83	Hydroquinone	$\pm$0.037	$\pm$ .024
32.......	Seed 30 P	4239–4–4	Hydroquinone*	$\pm$0.047	$\pm$ .021
33.......	Seed 30 P	4239–4–4	Hydroquinone*	$\pm$0.034	$\pm$ .015
38.......	Wellington X C	5872 B B	Hydroquinone*	$\pm$0.032	$\pm$ .021
43.......	Wellington X C	5872 B B	Hydroquinone*	$\pm$0.065†	....
62.......	Cramer Hi.–P	24413 A2EN	Hydroquinone*	$\pm$0.043	$\pm$ .022
63.......	Cramer Hi.–P	24413 A2EN	Hydroquinone*	$\pm$0.034	$\pm$ .026
65.......	Seed 30 P	5266–80–27	Hydroquinone	$\pm$0.050	$\pm$ .036

* Indicates that the plate was immersed in water at approximately 70° F. for about one hour before development.

† This plate was an ordinary glass with an unusually poor surface as found from surface measures.

selected on the resulting plates for measurement of opacity with the thermo-electric photometer. Readings on the clear glass channels between the exposed strips were taken in every instance to eliminate glass and gelatine absorption as usual. A system of standard squares of known magnitude differences as used at the Harvard College Observatory by Professor King was imposed for giving the plate gamma. Measures of opacities in this way give a direct indication of the magnitude of the errors which would be involved in the method of extra-focal measures due to lack of uniformity in plate sensitivity. The probable error for a single "image" was found to be of the order of $0^{M}.035$ or about 50 per cent larger than the results for focal images. Furthermore, decided tendencies to systematic distribution of plus and minus residuals

left little doubt that the major part of the difficulty lies in the manufacture and application of the emulsion to the plate itself and that the use of plate glass probably does not modify the results of extra-focal measures.

CONCLUSION

From the results of the present investigation, the errors in magnitude of star images due to irregularities in different parts of the same plate are of the order of three or four hundredths of a magnitude. These errors are unquestionably but not startlingly reduced by the use of plate glass. It seems unlikely that there is any consistent difference in plates of any of the well-known brands represented, or that the plate errors depend in any direct way on the speed of the emulsion. It does not seem probable that different developers will give very different results, but there is room for further investigation here. It does seem probable that prolonged immersion of the plate in water before development improves the uniformity, and that slow drying likewise favors uniformity.

To summarize results, it appears that the probable error in magnitude of photographic plates may be kept down to the order of about $0^{M}.02$, such results having been attained in the present investigations under the following conditions:

1. Plates soaked in water (65° F.) for one hour between exposure and development.
2. Developed in hydroquinone (modified Wallace formula).
3. Dried in still air, on edge, corner down.

The diameters of the images involved in this investigation were of the order of 0.2 mm.

Measure of opacities of more extended areas, as for extra-focal images, indicated probable errors about 50 per cent larger than for focal images.

Astronomical Laboratory
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Cambridge, Mass.
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THE HYDROGEN BALMER SERIES AND THE IMPOSSIBILITY OF FURTHER CORRECTIONS TO THE QUANTIZING OF HYDROGENIC ATOMS

By ARTHUR E. RUARK

ABSTRACT

Wave-lengths of the Balmer lines H3 to H18.—In *Table I* the values of λ from H7 to H18 are more accurate than any values previously given, but they are not precise enough to yield a definitive value of the Rydberg constant R_H. *Formulas* are obtained for calculating R_H from the wave-length of an unresolved Balmer line. *The main components of* H3 are separated by 0.0584 A.

The relativity correction only is needed to explain existing observations.—(1) The observed structure of the lines does not admit a correction due to *asphericity of nucleus* or electron; Silberstein's theory shows that if the nucleus and electron have radii of about $3 . 10^{-13}$ cm, the alteration of the lines would be unobservable. (2) H. A. Wilson's correction to the *quantizing of the atom*, due to motion of a part of the potential energy with the electron, is not in accord with experiment. (3) H. S. Allen's formula for the correction due to a possible intrinsic *magnetic moment* of the nucleus is extended to the case of elliptic orbits; a magnetic moment of one Weiss magneton in the nuclei of H or He would distort the fine structure of H and He^+ lines by a readily observable amount; therefore it cannot exist.

Much recent experimental work on the hydrogen spectrum has dealt with the fine structure of the first few Balmer lines, so that the relativistic quantizing of the atom might be tested further. It is possible, however, to arrive at a conclusion more easily by accurate measurement of the wave-lengths of higher members of the Balmer series. In recent papers Professor R. W. Wood[1] has described methods by which he photographed twenty-two Balmer lines, using the light from the central part of a long vacuum tube through which moist hydrogen was flowing. In such a tube the secondary spectrum is nearly absent, and the greatest difficulty in extending the Balmer series is the suppression of the continuous spectrum which masks the faint lines of higher order. In the spring of 1922 Dr. Wood obtained an excellent series of plates in the second and third orders of a 7-inch plane grating, having a resolving power practically equal to the theoretical value; it was used under good temperature control in a 20-foot Littrow mount, giving a dispersion of 1.27 A per mm in the second order at 3800 A. The author undertook to

[1] *Proceedings of the Royal Society*, A, **97**, 455, 1920; *Philosophical Magazine*, **42**, 729, 1921; *ibid.*, **44**, 538, 1922.

measure these plates in order (1) to furnish better wave-lengths for the higher members of the series; (2) to determine the Rydberg constant; and (3) to test the necessity of corrections to the theory of hydrogenic atoms, arising in the asymmetry of nucleus or electron, or in the existence of an intrinsic magnetic moment in either of the two, or, finally, in the possibility that part of the potential energy of the atom may be considered to move with the electron, so as to change its apparent mass. These three corrections have been suggested by Dr. L. Silberstein, Professor H. S. Allen, and Professor H. A. Wilson, respectively.

1. MEASUREMENT OF THE BALMER LINES

The grating measurements of Professor Ames[1] were referred to the Rowland system, and cannot be converted to I.A. without introducing error, while the spectrograms of Dyson, of Evershed, and of Mitchell[2] were obtained under eclipse conditions. The only measures suitable for testing the quantizing of hydrogen are as follows:

Paschen, grating, lines 1–4, *Annalen der Physik*, **50**, 901, 1916.

Meissner, interferometer, lines 1 and 2, *ibid.*

Curtis, grating, lines 1–6, *Proceedings of the Royal Society*, A, **90**, 605, 1914; and **96**, 147, 1919.

The probable error of Paschen's work is 0.004 A; that of Curtis' measurements is smaller because he used from nine to fourteen plates for each line. Probable error is not so good a criterion of precision as the average deviation, but it is certain that both sets of data are very accurate.

The author's wave-lengths are referred to tertiary iron standards; only those lines were utilized which do not show appreciable pole effect and which are assigned the same wave-length by Burns, Meggers, and Merrill,[3] and St. John and Babcock.[4] Seldom was it necessary to use a standard more than 8 A from a Balmer line, and the dispersion was linear, within the limits of the desired accuracy, for a distance of 15 A on either side of the line. The

[1] *Kayser's Handbuch*, Vol. 5.

[2] *Astrophysical Journal*, **38**, 407, 1913.

[3] *Bulletin of the Bureau of Standards*, **13**, 245, 1916.

[4] *Astrophysical Journal*, **53**, 260, 1921.

lines appeared of uniform density and the middle was set on, with a single exception: on two plates H3 was well separated. due to short exposure, and here the minimum was set on. Each line was measured from three to eight times in each direction on each plate, to reduce error of measurement as far as possible. The data of Meggers and Peters[1] were used in reducing to vacuum. Table I gives the results: If equal weights are assigned to the data of Paschen, Curtis, and the author, we get the following wave-lengths for H3 to H6 unresolved: 4340.4655, 4101.7346; 3970.0740 and 3889.0575 I.A.

TABLE I

Line	λ Air	λ Vacuum	ν Vacuum	Average Deviation from Mean in A.U.	Number of Plates Used
3	4340.464	4341.6806	23032.5556	.003	2
4	4101.731	4102.8845	24373.0965	.006	2
5	3970.073	3971.1918	25181.3574	.0016	3
6	3889.064	3980.1618	25705.8717	.0017	3
7	3835.397	3836.4813	26065.5513	.0023	3
8	3797.910	3798.9847	26322.8225	.0025	4
9	3770.634	3771.7016	26513.2321	.010	3
10	3750.152	3751.2136	26658.0394	.010	3
11	3734.372	3735.4293	26770.6847	.010	2
12	3721.948	3723.0030	26860.0374	.007	2
13	3711.98	3713.03	26932.17	....	1
14	3703.86	3704.91	26991.20	....	1
15	3697.15	3698.20	27040.20	....	1
16	3691.55	3692.60	27081.19	....	1
17	3686.83	3687.88	27115.86	....	1
18	3682.82	3683.87	27145.37	....	1

H13 to H18 are correct only to about 0.02 A, because the single plate from which they were measured was jarred between exposures. The wave-lengths were adjusted by comparing the value for H12 obtained from this plate with the mean gotten from other plates.

The separation of H3 was measured on one plate and was found to be 0.0584 A. This result will be discussed elsewhere.

2. CALCULATION OF THE RYDBERG CONSTANTS; TEST OF THE THEORETICAL SERIES FORMULA

Let ν be the wave-number; m_0, the rest-mass of an electron; M_H, the mass of the H-nucleus; h, Planck's constant; $-e$, the electronic charge (electrostatic units); n, n', m, m', the azimuthal

[1] *Bulletin of the Bureau of Standards*, 14, 697, 1918.

and radial quantum numbers of the final and the initial orbits, respectively. The Rydberg number for hydrogen is

$$R_H = \frac{R\infty}{1+\frac{m_0}{M_H}} = \frac{2\pi^2 m_0 e^4}{h^3 c\left(1+\frac{m_0}{M_H}\right)} . \tag{1}$$

The constant of Sommerfeld's relativity correction is

$$\alpha = \frac{2\pi e^2}{hc} . \tag{2}$$

The wave-number of any component of a line is

$$\nu = R_H \left\{ \frac{1}{2^2} - \frac{1}{(n+n')^2} + \alpha^2 \left[\frac{1}{2^4}\left(\frac{1}{4}+\frac{n'}{n}\right) - \frac{1}{(m+m')^4}\left(\frac{1}{4}+\frac{m'}{m}\right) \right] \right\} . \tag{3}$$

Curtis found empirical formulae with two or three constants which represent his measurements of the first six lines quite as well as (3); therefore, it seems useless to calculate such formulae in the present case and to compare the results with those obtained by using (3). To test this formula we shall utilize the theory in all its details to compute a value of R_H from each line, and shall determine the average deviation of the results from their mean. First we must assume a value of α^2. It is certain that α^2 cannot deviate from 5.315×10^{-5} by more than 0.007×10^{-5}, and ΔR_H is less than 0.004 if $\Delta\ (\alpha^2)$ is equal to 0.007×10^{-5}; such a variation is negligible.

There are influences which shift a Balmer line from the position predicted by Sommerfeld's theory. Darwin[1] has evaluated the corrections to the relativistic quantizing due to motion of the nucleus, the magnetic forces caused by the motion, and the retardation of potentials. An extra term appears in the coefficient of α^2, such that *all* components of a given line are shifted in frequency by an amount less than 1/7400 of $R_H\alpha^2/16$, which is negligible.

There is a shift of the center of gravity of a doublet due to the narrowing discussed by Oldenberg,[2] if the two components are

[1] *Philosophical Magazine*, **39**, 537, 1920.

[2] *Annalen der Physik*, **67**, 453, 1922.

of unequal intensities. Roughly, the maxima are shifted toward each other by amounts in the ratio $r:1$, if r is the intensity ratio of the components, and the center of gravity will be moved from its proper position by an amount $r-1/r+1$ times the average shift of the components; the average shift of the components is one-half the narrowing of the doublet, which is obtainable if the doublet separation can be observed. For the intensity ratios of the components of H3 and H4 we use values estimated by Professor Birge[1] for usual experimental conditions, viz., 1.2 and 1.1. The observed narrowings are 0.009 A and 0.015 A; it follows that the shifts of mean center for these lines are 0.00041 A and 0.00036 A. The effect is smaller for all succeeding lines, and is negligible for all lines except $H\alpha$ and $H\beta$.

The theoretical formula gives only the positions of single components; we must express the position of the mean center of a Balmer line in terms of R_H, α^2, and the quantum numbers of the components appearing in the fine structure. There are three such components, in the absence of external fields; it suffices to suppose that the fine structure will approximate in position (though *not* in intensity) to that arising from the undisturbed hydrogen atom. The components of the $(M-2)$th Balmer line actually appearing are these: $(3,M-3)\rightarrow(2,0)$; $(1,M-1)\rightarrow(2,0)$ and $(2,M-2)\rightarrow(1,1)$. The first two falls give components comparatively close together, forming the long-λ half of the doublet; the third fall gives the short-λ half. If all six of the possible components could appear, the separation of the outer members of either half would be as follows: H3, 0.006 A; H4, 0.003 A; H5, 0.002 A. Even if all components were to appear with equal strength, the shift of H3 as a whole could not exceed 0.003 A; the effect would be much smaller for all succeeding lines. Birge's intensity computations for usual experimental conditions show that the changes from the normal patterns are much smaller than those mentioned, in fact, the component $(1,M-1)\rightarrow(2,0)$ is weak compared with the other component of the long-λ half of the doublet, and we shall consider all the lines as simple doublets whose halves coincide with the components $(3,M-3)\rightarrow(2,0)$ and $(2,M-2)\rightarrow(1,1)$.

[1] *Physical Review*, 17, 589, 1921.

Next we find the centers of gravity of the doublets. The extreme faintness and narrowness of the H3 doublet (separated only when underexposed) prevented direct measurement of the intensities of its components; a visual estimate showed that it would be nearly correct to use the intensity ratio 1.2, calculated by Birge for an external field of 100 volts/cm. It seemed best to use also the estimates of Birge for H4 and H5, viz., 1.1 and 1.05. Let λ_c and λ_e be the wave-lengths of the components $(3,M-3)\rightarrow(2,0)$ and $(2,M-2)\rightarrow(1,1)$; and let λ_0 be the observed vacuum wave-length of the doublet as a whole. If the intensities of the components are in the ratio $(1+f)$: 1, then

$$\lambda_0=\lambda_c-\frac{1}{2+f}(\lambda_c-\lambda_e)=\tfrac{1}{2}(\lambda_c+\lambda_e)+\tfrac{1}{2}(\lambda_c-\lambda_e)\left(\frac{f}{2}-\frac{f^2}{2^2}+\ \ldots\ \right). \quad (5)$$

With sufficient approximation we have

$$\frac{1}{\nu}\equiv\tfrac{1}{2}(\lambda_c+\lambda_e)=\lambda_0-\frac{f}{4}(\lambda_c-\lambda_c). \quad (6)$$

The values of $\frac{f}{4}(\lambda_c-\lambda_e)$ are 0.0034 A, 0.0015 A, and 0.0007 A for the lines H3 to H5; applying these corrections we obtain $\bar{\nu}$ for these lines: the values are 23032.5758, 24373.1078, and 25181.3631 for H3 to H5, respectively. For all succeeding lines the intensity ratio is unity, and $1/\bar{\nu}$ is identical with λ_0. From equation (3) we get

$$\nu_c=R_H\left\{\frac{1}{2^2}-\frac{1}{M^2}+\frac{\alpha^2}{4}\left(\frac{1}{2^4}-\frac{1}{M^4}\right)-\frac{\alpha^2(M-3)}{3M^4}\right\}$$

$$\nu_e=R_H\left\{\frac{1}{2^2}-\frac{1}{M^2}+\frac{\alpha^2}{4}\left(\frac{1}{2^4}-\frac{1}{M^4}\right)+\alpha^2\left(\frac{1}{2^4}-\frac{M-2}{2M^4}\right)\right\}$$

If we call the expressions in brackets b_c and b_e,

$$\nu=\frac{2\nu_c\nu_e}{\nu_c+\nu_e}=\frac{2R_Hb_cb_e}{b_c+b_e}. \qquad \therefore\quad R_H=\frac{(b_c+b_e)\bar{\nu}}{2b_cb_e}. \quad (7)$$

Table II shows the values of R_H obtained by equation (7).

TABLE II

Line	R_H	Line	R_H	Line	R_H
3	109676.844	7	109677.288	11	109677.572
4	6.983	8	7.292	12	7.372
5	6.807	9	7.542		
6	7.241	10	7.673		

The lines 13 to 18 are not known accurately enough to be used in obtaining R_H. The mean of the foregoing values is:

$$R_H = 109677.26 \pm 0.23; \quad \text{and} \quad R_\infty = 109736.45 \pm 0.23,$$

if we use Birge's "best" value $m_0/M_H = 1/1853$.

The variation of R_H is within the limits of experimental error. In future work the lines 6 to 15 will be the point of attack; they can be obtained with reasonably short exposures in the fourth or fifth order, and much better results may be expected.

3. CORRECTIONS TO THEORY

We have seen that Sommerfeld's relativistic quantizing suffices to explain the gross features of observations on the doublet separations of the Balmer lines and on their wave-lengths. Outstanding discrepancies are within the limits of experimental errors in almost all cases, and it seems hopeless to attempt to find out by further experiment whether deviations from theory actually exist.

Another method of attack is available; we can revise the theory to take account of all perturbations which can reasonably be considered as existent, and can examine whether the revised theory agrees with experiment. The alterations mentioned in the introduction to this paper will be discussed in order.

1. The alteration of energy due to asphericity of the nucleus has been treated by Dr. Silberstein.[1] He developed formulae applicable to the most general distribution of charge within the nucleus; the results will be of importance in the theory of heavy atoms with large nuclei. Dr. Silberstein has attempted to explain the fine

[1] *Philosophical Magazine*, **39**, 46, 1919.

structure of the H-lines solely by the assumption of an aspherical nucleus.[1] However, he found that in order to satisfy Curtis' data on the wave-lengths of the first six lines, it is necessary to assume a nucleus whose largest dimension is about 2.10^{-11} cm; when this is done the line patterns for Hα and Hβ do not resemble the observed structures, unless *ad hoc* assumptions as to the intensities of the numerous members are brought in. Without going into details, the theory shows that the spread of the line pattern is reduced in proportion to the square of the nuclear dimensions, other things being equal. The preponderance of evidence is to the effect that the sum of the radii of H-nucleus and electron is not greater than $3.\ 10^{-13}$ cm; this is about 0.003 of the dimension used by Silberstein; if we adopt this value the width of the line pattern is reduced by the factor 10^{-5} and no observable fine structure would exist. Asphericity of the nucleus cannot cause a shift of any Balmer line by an amount observable spectroscopically; all such shifts from the positions predicted by the simple theory are comprised within the limits of the fine structure. I have carried through the work of showing that asphericity of the electron cannot be detected by observations of the Balmer lines, if the maximum dimension of the electron is less than 3.10^{-13} cm. (In fact this effect could be observed only in very precise measures of the highest frequencies in the X-ray spectra of heavy elements.) The inapplicability of these ideas in the present case does not alter the value of Dr. Silberstein's calculations within their proper domain. Reasoning backward, we may say that since no term arising from asphericity of the ultimate particles is needed to satisfy the data, both nucleus and electron must be very small, if aspherical, or very nearly spherical, if large. This amounts to an independent confirmation of the conclusions drawn from experiments on scattering of alpha and beta particles in light gases.

2. Professor H. A. Wilson[2] has suggested that the potential energy of a hydrogen atom may be regarded as distributed through the space surrounding it, and that this energy will take part to some extent in the motion of the electron. Its presence will modify the motion of the electron, and shifts of spectral lines will result.

[1] *Proceedings of the Royal Society*, A, **98**, 1, 1920.

[2] *Astrophysical Journal*, **56**, 34, 1922.

Roughly, the effect to be expected is a slight narrowing of the fine structure of the H-lines, accompanied by a very small shift. The narrowing is proportional to f, where f is the fraction of $1/2$ of the potential energy which may be considered as moving with the electron. Since these ideas were put forward, Oldenberg's explanation of the narrowing of the doublets has appeared, and seems to leave no room for such a correction. There would be difficulties even if Oldenberg's explanation were left out of account; a readily observable shift of the lines would result if f were as large as 0.2, say, and the fine structure of ionized helium lines would not be like that observed by Paschen.

We shall not raise here the fundamental question whether the energy of an atom in a stationary state is actually spread through the surrounding space or whether this distribution is to be regarded merely as a convenient mathematical artifice, but it is an experimental fact that the motion of the potential energy of the system does not affect its total energy by an observable amount.

3. We shall calculate the correction due to a nuclear magnetic moment. In 1915, H. S. Allen[1] carried through a similar analysis for the circular orbits of hydrogenic atoms. We neglect the relativity correction and the motion of the nucleus, and attribute to the nucleus an intrinsic magnetic moment, M. If we suppose the electron has a magnetic moment of the same order of magnitude as M, a correction arises which is of the magnitude $(m_0/M_H)^3$ compared to that due to the nuclear moment; this is negligible. Likewise, we may neglect the reaction on the nucleus due to the fact that it is exerting force on the electron. Let us take the nucleus as the origin of a polar co-ordinate system, r, θ, ϕ; θ is measured in the plane defined by the nucleus and the tangent to the electronic orbit at any convenient initial time. It will be found that the orbital plane is invariable in position, so ϕ is always $\pi/2$. The electrical potential energy is $-Ee/r$ where E is the nuclear charge. We now calculate the magnetic energy. The magnetic field due to an electron moving with velocity v in a direction (α, β, γ), is in Gauss,

$$H=\frac{ev\sin\psi}{cr^2}, \tag{8}$$

[1] *Philosophical Magazine*, 29, 40 and 714, 1915.

at a distance r from the electron, if ψ be the angle between its direction of motion and the radius vector drawn to the point of evaluation of H. H is perpendicular to the radius vector and the direction (α, β, γ); we call its direction cosines l', m', n'. The potential energy associated with the moment M, having direction cosines l, m, n, in the field H, is, in electromagnetic units,

$$V_M = -MH(ll' + mm' + nn'). \tag{9}$$

We can now write down Lagrangian equations; only the equations for the co-ordinates l and m are needed. If T and V be the kinetic and potential energies of the system we have

$$\frac{d}{dt}\left(\frac{\partial(T-V)}{\partial \dot{l}}\right) - \frac{\partial(T-V)}{\partial l} = 0; \qquad \frac{d}{dt}\left(\frac{\partial(T-V)}{\partial \dot{m}}\right) - \frac{\partial(T-V)}{\partial m} = 0. \tag{10}$$

These equations reduce to the conditions for equilibrium of the nucleus,

$$\frac{\partial V}{\partial l} = 0; \qquad \frac{\partial V}{\partial m} = 0. \tag{10'}$$

These equations are an expression of the tacit assumption that since the nucleus is so small and so massive, we may neglect its kinetic energy of translation and of rotation, and that under these assumed conditions it will orient itself permanently in a configuration satisfying the equilibrium-equations. V_M may be written

$$-MH(ll' + mm' + \sqrt{1 - l^2 - m^2} \cdot n')$$

and equations (10′) become

$$l' - \frac{n'l}{\sqrt{1-l^2-m^2}} = 0; \qquad m' - \frac{n'm}{\sqrt{1-l^2-m^2}} = 0, \tag{10''}$$

and finally, by symmetry, we obtain the equations,

$$l : m : n = l' : m' : n'. \tag{10'''}$$

The axis of the magnetic moment is always normal to the instantaneous orbital plane. The force on the electron is always in the plane of its orbit and $\phi=\pi/2$ always; the problem is two dimensional. Only one of the two possible orientations of the nucleus gives stable equilibrium. We suppose this to be the position actually occupied. Then

$$V_M=-MH=-\frac{Mev\sin\psi}{cr};$$

$v\sin\psi$ is the component of the velocity of the electron perpendicular to the radius vector, which is $r\dot{\theta}$; therefore,

$$V=-\frac{Me\dot{\theta}}{cr}. \tag{11}$$

Let us call W the total energy of the atom in a conservative state.

$$W=\frac{1}{2m_0}\left[(m_0\dot{r})^2+\frac{1}{r^2}(m_0r^2\dot{\theta})^2\right]-\frac{Ee}{r}-\frac{Me\dot{\theta}}{cr}. \tag{12}$$

Since the magnetic force on the electron due to the nuclear moment is always directed along the radius vector, the angular momentum is constant; we apply the quantum condition,

$$p=m_0r^2\dot{\theta}=nh/2\pi \tag{13}$$

and $Me\dot{\theta}/rc$ becomes $nhMe/2\pi m_0 r^3 c$. Equation (12) becomes

$$\frac{1}{2m_0}\left[(m_0\dot{r})^2+\frac{p^2}{r^2}\right]-\frac{eE}{r}-\frac{nh\,Me}{2\pi m_0r^3c}=W$$

or,

$$(m_0\dot{r})^2=2m_0W+\frac{2m_0eE}{r}-\frac{p^2}{r^2}+\frac{nh\,Me}{\pi c}\frac{1}{r^3}.$$

Put $A=2m_0W$, $B=m_0eE$, $C=-p^2$, $D_1=nh\,Me/\pi c$; the quantum condition for the co-ordinate r is

$$J_4=\int m_0\dot{r}\,dr=\int\sqrt{A+\frac{2B}{r}+\frac{C}{r^2}+\frac{D_1}{r^3}}\cdot dr=n'h. \tag{14}$$

The integral is extended over a complete cycle of values of r, and is given on page 479 of the second edition of Sommerfeld's *Atombau* (p. 670 of the 3rd. ed.):

$$J_4 = -2\pi i\left(\sqrt{C} - \frac{B}{\sqrt{A}} - \frac{BD_1}{2C\sqrt{C}}\right)$$

where we are to use the negative value of $\sqrt{C}$, i.e., $-inh/2\pi$. Finally

$$J_4 = n'h = -nh + \frac{2\pi i m_0 eE}{\sqrt{2m_0 W}} + \frac{8\pi^3 m_0 e^2 EM}{cn^2h^2}. \qquad (15)$$

We shall call the last term on the right hP and shall write $E = Ze$, so that

$$P = \frac{8\pi^3 m_0 e^3 ZM}{cn^2h^3}, \qquad (16)$$

$$W = \frac{-2\pi^2 m_0 e^4 Z^2}{h^2(n+n'-P)^2} = \frac{-hcRZ^2}{(n+n')^2}\left[1 + \frac{2P}{(n+n')} + \ldots\right]. \qquad (17)$$

The magnetic correction to the wave number of any spectrum line will be

$$RZ^2B \cdot \left(\frac{1}{n^2(n+n')^3} - \frac{1}{m^2(m+m')^3}\right) \qquad (18)$$

where B is defined as $2Pn^2$. If ρ be the number of Weiss magnetons equivalent to the nuclear moment we have

$$M = \rho \times 1.854 \quad 10^{-21}. \qquad \text{(Electromagnetic units.)} \qquad (19)$$

$$RB = \rho Z \times 109736 \times 1.071 \times 10^{-5} = 1.175\, \rho Z. \qquad (20)$$

A large amount of evidence could now be adduced for the non-existence of a nuclear moment as large as one Weiss magneton; we content ourselves with computing the magnetic shifts for the allowed components of Hα, on the basis of a single Weiss magneton:

Transition	Shift
(3,0) to (2,0)	−.013 A
(1,2) to (2,0)	+.003 A
(2,1) to (1,1)	−.055 A

Thus the two main components of $H\alpha$ approach each other by the amount 0.043 A, which is far greater than the narrowing of the doublet usually observed. The relations are quite similar for succeeding lines, and for lines of the 3-quantum series of ionized helium; in all these cases the fine structure will be distorted by a readily observable amount.

The conclusion is that the hydrogen nucleus and the alpha particle cannot possess intrinsic moments as large as one Weiss magneton, and were there any object in so doing, we could set a still smaller limit to the magnitude. In order to prove this fact directly, it would be necessary to perform the Stern and Gerlach experiment for H-nuclei and alpha particles.[1] The shifts predicted by this theory for lines in the K series of an element of atomic number Z are of the order of magnitude $\rho/Z \times 0.013$ A, which is observable in light elements if ρ is as large as 5; but this effect cannot be separated from other perturbations of much greater size.

I wish to express my thanks to Professor R. W. Wood for the use of his spectrum plates and his advice in connection with their measurement; to Mr. B. Kurrelmeyer for his generous aid in checking calculations; and to Professors Ames, Pfund, and Wood, and Drs. F. L. Mohler, A. Q. Tool, and P. D. Foote, who have kindly given the paper the benefit of their criticism.

Johns Hopkins University
December 10, 1922

[1] *Zeit. f. Phys.*, **9**, 349, 1922.

WAVE-LENGTH STANDARDS IN THE EXTREME ULTRA-VIOLET

By J. J. HOPFIELD and S. W. LEIFSON

ABSTRACT

Extreme ultra-violet spectra of hydrogen, oxygen, nitrogen, helium, and air.—The wave-lengths of 37 strong lines *between* λ 800 *and* λ 1800 occurring in condensed discharge spectra of impure hydrogen, oxygen, nitrogen, helium, and air are determined, with a probable error of less than 0.1 A, from photographs, on specially prepared ultra-violet films, taken with a Rowland concave grating having a radius of curvature of 50 cm.

INTRODUCTION

In the extreme ultra-violet spectra of impure hydrogen, oxygen, nitrogen, helium, and air obtained with a condensed discharge, there occur a number of strong lines which may serve as standards, since they are almost always present. Several observers have published tables of wave-lengths between λ 800 and λ 1800, but their measurements have perhaps only in a very few cases yielded results accurate to 0.1 A. In view of the growing interest in this part of the spectrum and its importance in atomic theory a careful determination has been made of the wave-lengths of a number of easily obtained characteristic lines to serve as standards in the region λ 800 to λ 1800. These, together with some additional lines known accurately from theory, may serve not only as reference standards for interpolation in this region, but also as an aid to establishing other standards in the region λ 300 to λ 800, a region now accessible to spectroscopists. The wave-lengths are expressed in I.A. in vacuum, since both the comparison and the measured spectra were obtained in a vacuum. The standard wave-lengths used were in I.A. reduced to vacuum.

METHOD

From a large number of photographs of the ultra-violet spectra of hydrogen, oxygen, and nitrogen about twenty-five of the best were selected for measurement. These photographs, recorded on specially prepared ultra-violet films, were made with a Rowland concave grating having a radius of curvature of 50 cm. The photo-

graphs cover a range from 0 to 5000 A, and they therefore contain the second, third, and fourth orders of lines whose wave-lengths lie between λ 800 and λ 1800. By making enlarged prints of each film to be measured, the higher orders were easily recognized and marked. A superimposed mercury arc spectrum, giving about twenty distinct lines between λ 2400 and λ 4800, furnished standards. Stiles's values for the mercury arc lines[1] were used, and the wave-lengths were reduced to vacuum by means of the *Bureau of Standards* tables.[2]

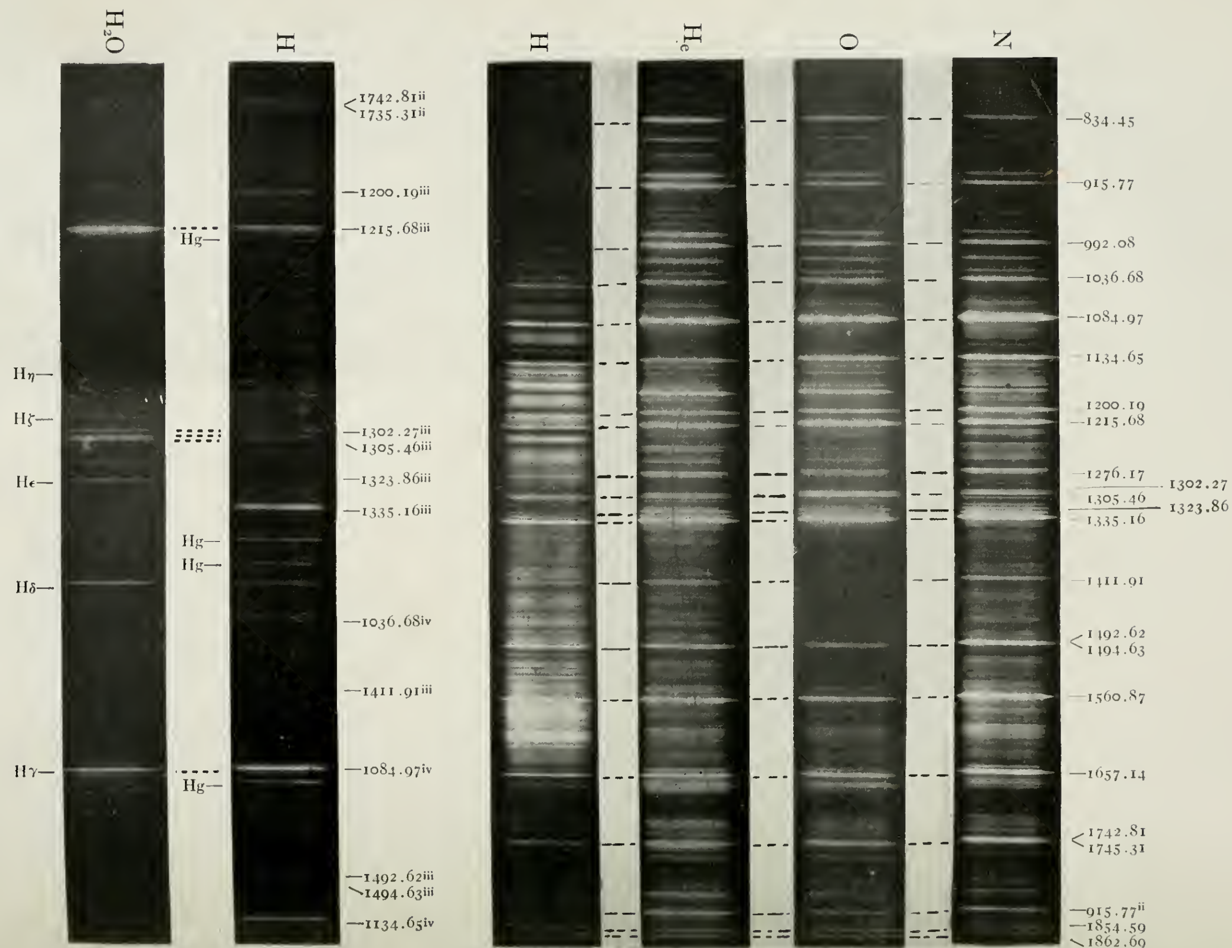
The positions of the lines were measured with a 20-cm Gaertner comparator, owned by the Rumford Committee of the American Academy of Sciences, and kindly loaned to the authors by Professor R. T. Birge. The wave-lengths of the unknown lines were obtained by direct interpolation between two standard lines. Since the photographs were made with the gas in the spectrograph under a pressure of 0.2 mm or less, no correction for dispersion of the medium was made.

Most of the lines were measured in the third order, and in a few cases also in the second and fourth orders. On each of the several films, ten settings of the micrometer were made on each line. The average deviation from the average in ten settings on a line was usually less than 0.005 mm, hence, as calculated from the dispersion of the grating, which is approximately 34 A per mm, the corresponding deviation in the wave-length determination was 0.17 A in the first order and proportionately less in the higher orders. The error from this source would thus average about 0.06 A. The only other source of error that needs to be considered is the possibility of a shifting of the comparison spectrum with respect to the unknown spectrum, owing, first, to asymmetrical illumination of the grating with the mercury arc, and second to shrinkage of the film in the interval between two exposures. Some photographs which showed evidence of a shift were rejected. In the case of most of the films measured, several known lines, such as $H\gamma$ and $H\delta$, were included among the lines measured, and if the wave-lengths computed for these lines in the first order differed by more than 0.2 A from the

[1] *Astrophysical Journal*, **30**, 48, 1909.

[2] *Bureau of Standards Bulletin*, **14**, 731, 1919.

PLATE IV


H₂O
H
H
He
O
N
Hη
Hζ
Hε
Hδ
Hγ
Hg
Hg
Hg
Hg
1742.81ii
1735.31ii
1200.19iii
1215.68iii
1302.27iii
1305.46iii
1323.86iii
1335.16iii
1036.68iv
1411.91iii
1084.97iv
1492.62iii
1494.63iii
1134.65iv
834.45
915.77
992.08
1036.68
1084.97
1134.65
1200.19
1215.68
1276.17
1302.27
1305.46
1323.86
1335.16
1411.91
1492.62
1494.63
1560.87
1657.14
1742.81
1745.31
915.77ii
1854.59
1862.69

correct values, the film was rejected. Care was also taken to measure standard lines along with the unknown lines as a further check on the results. After all the films had been measured once, they were measured a second time after an interval of several months.

RESULTS

The illustrations show how the lines whose wave-lengths have been determined appear in the first order. The two sharp lines at λ 1854.59 and λ 1862.69 are well-known aluminium lines whose wave-lengths have been accurately determined by Eder.[1] They show clearly on most of our photographs.

The higher orders are indicated by Roman numeral exponents, and some of the hydrogen Balmer series lines are also shown.

The wave-lengths in the table are probably accurate to within the ranges signified by A, B, and C, respectively: A indicating a probable error in wave-length of ± 0.05 A or less; B, ± 0.10 A or less; C, ± 0.15 A or less.

The measured wave-lengths are the averages of 80 to 150 determinations obtained from as many as 8 to 15 films, respectively. The value obtained for the resonance line of hydrogen, 1215.68 A, which is the average of 150 determinations, agrees very well with the theoretical value, 1215.67 A, the value found experimentally also by the senior author.[2] The wave-lengths of five additional lines of the Lyman series, of which the last three were recently found by one of the authors,[3] are likewise given in the table. Theoretical values in vacuum are used.

The group of three lines, an oxygen triplet, at λ 1302.27, λ 1304.96, and λ 1306.12, was measured on twelve different photographs in the third order, using Curtis' values[4] for $H\delta$ and $H\eta$ as standards. Spectrum 6 on the plate of illustrations is a part of the spectrum obtained with water vapor under very low pressure. It shows clearly the third orders of the first line of the Lyman series and the oxygen triplet mentioned above, besides five members,

[1] *Zeit. für Wiss. Phot.*, **14**, 144, 1914.

[2] Hopfield, *The Physical Review* (Series II), **20**, 573, 1922.

[3] Hopfield, *Nature*, **110**, 732, 1922.

[4] *Proceedings of the Royal Society of London*, A, **90**, 605, 1914.

TABLE I

Present Work				Lyman*	Millikan, Bowen, and Sawyer†	McLennan and Petrie‡
λ Measured	Intensity	λ of Center of Gravity Computed	Accuracy			
833.35	3	....	......	....	....	....
834.53	5	834.45	B	....	....	....
835.28	4	....	......	....	....	....
915.59	3	....	......	....	....	....
916.03	2	915.77	A	....	....	....
991.57	2	....	......	....	....	....
992.42	3	992.08	A	....	....	....
1036.33	2	....	......	....	....	....
1037.03	2	1036.68	A	1037.0	1036.7	1036.8
1083.97	2	....	......	....	....	....
1084.52	3	....	......	....	....	....
1085.64	5	1084.97	A	1084.9	1085.3	1085.2
1134.31	4	....	......	....	....	....
1135.00	4	1134.65	A	1134.7	....	1134.7
1199.56	3	....	......	....	....	....
1200.25	3	....	......	....	....	....
1200.76	3	1200.19	B	1199.8	....	1199.7
1215.68	10	1215.68	A	1216.0	....	1215.8
1275.08	3	....	......	....	....	....
1276.22	4	....	......	....	....	....
1277.19	3	1276.17	C	....	....	....
1302.27	6	1302.27	A	1302.5	....	1302.9
1304.96	4	....	......	....	....	....
1306.12	3	1305.46	A	....	....	....
1323.86	2	1323.86	B	1323.4	1323.7	....
1334.57	5	....	......	....	....	....
1335.75	5	1335.16	B	....	1335.0	1334.8
1411.91	2	1411.91	B	....	....	1411.6
1492.62	3	1492.62	B	....	....	1492.3
1494.63	3	1494.63	B	....	....	1494.4
1560.46	5	....	......	....	....	....
1561.39	4	1560.87	B	....	1561.3	1560.8
1656.33	3	....	......	....	....	....
1657.08	4	....	......	....	....	....
1658.04	3	1657.14	C	....	1657.6	1656.8
1742.81	5	1742.81	B	1742.7	....	1742.6
1745.31	5	1745.31	B	1745.3	....	1744.9

FIVE THEORETICAL VALUES

1215.67	972.54	937.81
1025.73	949.75	930.76

* *Astrophysical Journal*, **43**, 89, 1916.

† *Ibid.*, **53**, 150, 1921. ‡ *Transactions of the Royal Society of Canada* Sec. III, 1921.

the third to the seventh, respectively, of the Balmer series in the first order. $H\epsilon$ and $H\zeta$ were measured along with the three members of the oxygen triplet. The former differed by an average of 0.03 and 0.04 A, respectively, from the correct values. The average deviation from the mean values for each member of the triplet was 0.03 A. Hence it seems legitimate to claim an accuracy of 0.03 A for these lines.

Since many of the complex lines are resolved for the first time in the higher orders shown on our films, and since they are not ordinarily resolved in the first order with a grating having a focal length of 100 cm or less, their centers of gravity have been calculated, based on the estimated intensity of the members. The groups at λ 915.77, λ 1134.65, and λ 1657.14 are probably even more complex than indicated in the table. The highest orders of λ 915.77 and λ 1134.65 observed indicate that they are composed of three lines. The sixth order of λ 834.45 shows no fewer than five lines. The position of the center of gravity is based on the entire group, of which only the strongest lines are shown in the table.

The similarity of the first three spectra on the plate of illustrations shows that these spectra are probably air spectra with admixture of nitrogen, oxygen, and helium, respectively. They are easily produced by air alone and thus form convenient comparison spectra for this region.

Since the wave-lengths given are believed to be more accurate than any previously published for this region of the spectrum, and since the lines are all easily obtained with a condensed discharge at low pressures, especially in air, it is thought that the results may be of use to other spectroscopists working in the extreme ultraviolet.

Department of Physics
University of California
February 1923

NOTICE TO CONTRIBUTORS

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> Where unusual expense is involved in the publication of an article, on account of length, tabular matter, or illustrations, arrangements are made whereby the expense is shared by the author or by the institution which he represents, according to a uniform system.

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Contributors should notice that since January, 1917, it has been impossible to supply any free reprints of articles.

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Every article in the *Astrophysical Journal*, however short, is to be preceded by an abstract prepared by the author and submitted by him with the manuscript. The abstract is intended to serve as an aid to the reader by furnishing an index and brief summary or preliminary survey of the contents of the article; it should also be suitable for reprinting in an abstract journal so as to make a reabstracting of the article unnecessary. For details concerning the preparation of abstracts, see page 231 in the April, 1920, number of the *Journal*.

THE EDITORS

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WATER-CELL TRANSMISSIONS AND PLANETARY TEMPERATURES

By DONALD H. MENZEL

ABSTRACT

Determination of planetary temperatures from water-cell transmission.—(1) *Theory.* The reflected solar energy depends upon the planet's size, distance, albedo, and phase-angle; the emitted energy depends also upon the effective temperature and the emissivity of the surface. The observed ratio of the two is greatly modified by absorption in the earth's atmosphere and the receiving instrument. The equation connecting the effective temperature with the water-cell transmission is derived. When the whole disk of the planet has been observed the only uncertain factor is its emissivity; when but part of the disk has been observed a further correction (now very uncertain, but capable of accurate measurement) must be applied. (2) *Results of radiometric observations* made by Coblentz at the Lick and Lowell observatories in 1914 and 1921–1922, when reduced by this method, show good agreement and indicate effective noon-day temperatures of 50° C. for *Venus*, −16° C. for *Mars*, and 120° C. for the *moon* (assuming high emissivity in all cases). These are in good agreement with the temperatures which would be maintained by solar radiation on a surface of small heat capacity. For *Jupiter and Saturn* the computed temperatures (−110° C. in both cases) are too high to be maintained by solar radiation, and must be attributed to the internal heat of the planets. These numerical values should be regarded as provisional.

Atmospheric transmission of long-wave radiation as a function of humidity.—The earth's atmosphere (owing mainly to its content of carbon dioxide and water-vapor) acts like a color-screen for long waves, transmitting most of the radiation between 9 μ and 12 μ and very little else. The net transmission of black-body radiation, for temperatures between 400° and 150° K., varies roughly as the cube of the temperature. For the range of humidity likely to be met in practice (0.5 to 4 cm precipitable water), the transmission can be represented by the product of two factors, one depending upon the air-mass and the other upon the total amount of water-vapor along the path. *Tables* of these factors are given for different temperatures of the source.

The accuracy of recent measures of planetary radiation encourages an examination of the relations of the observed data to the

temperatures of the planets. This has been undertaken at the suggestion of Professor Russell, who has prepared the outline of the theory given in section 1. The calculations which follow have been made by the writer.

1. *General theory.*—Much has been written on this subject, and the present summary has small claims to originality. The energy which we receive from a planet is composed partly of reflected solar radiation of short wave-length and partly of long-wave radiation from the planet's warm surface. The former is largely transmitted by a water cell and the latter stopped completely.

Let S be the amount of the former and P that of the latter, and let t' and t be the fractions of these amounts which are transmitted through the earth's atmosphere and the receiving instruments employed. The radiation observed without the water cell will then be $tP+t'S$, while with it, it will be 0.695 $t'S$—the numerical factor being the mean value determined by Coblentz.[1] If W is the "water-cell transmission," we then have

$$W=0.695\,t'S/(tP+t'S)$$

Or

$$\frac{tP}{t'S}=\frac{0.695}{W}-1. \tag{1}$$

In general, the portion of the planet's surface whose energy falls on the receiving thermopile will not be of uniform temperature. We may substitute a hypothetical "gray" surface of equal size and of temperature T and emissivity ϵ such that the radiation from this surface is equal to the integrated radiation from the observed region in amount, and as similar as possible in quality, and write

$$P=ka\epsilon T^4 \tag{2}$$

where a is the apparent area of the observed region and k the constant of Stefan's law (in suitable units).

Let us now assume that the reflected solar energy is unaltered in quality, i.e., that the planet's albedo for all wave-lengths is equal to the visual albedo A. We may set $A=pq$ where q depends only on the law of phase variation and p is the ratio of the radiation reflected at full phase to that emitted by a self-luminous body of the

[1] This factor probably varies somewhat with the air-mass traversed by the sun's rays. Such variations can be taken into account by small changes in t'. Observations bearing on this are especially to be desired.

same size and position, which radiates as much energy from each unit of its surface as the planet receives from the sun under normal incidence.[1]

Taking the solar constant as 1.932 calories per square centimeter per minute,[2] and the constant of Stefan's law as 5.72×10^{-5} ergs[3] or 8.21×10^{-11} calories per square centimeter per minute, it is readily found that the energy reflected from a surface for which $p=1$, at a distance of R astronomical units from the sun, is equal in amount (though not in quality) to that emitted from a black surface at the temperature T_0 where

$$T_0=392^\circ R^{-\frac{1}{2}}. \qquad (3)$$

If b is the whole area of the planet's disk (including the unilluminated portion), the energy reflected at full phase will be $kbpT_0^4$ and at any phase ϕ times that at the full or $kbA\phi T_0^4/q$, where ϕ is less than 1 and depends on the phase-angle α. Finally, if x is the ratio of the mean apparent surface brightness of the observed area to that of the whole disk, S will be xa/b times the quantity just found, or

$$S=kax\frac{A\phi}{q}T_0^4. \qquad (4)$$

From equations (1) to (4) we have

$$\frac{tq\epsilon}{t'\phi Ax}\frac{T^4}{T_0^4}=\frac{0.695}{W}-1. \qquad (5)$$

The factors q, A, and ϕ are known;[4] x can be determined by observation for any given area (though at present very little is known on this matter). It may be much greater than unity for a bright spot or for the illuminated limb of a crescent, or much less for a dark spot or the terminator. For the planet as a whole $x=1$, by definition. The emissivity ϵ can only be conjectured. For a planet with a deep atmosphere it is likely to be near unity, and T will in this case be the temperature of the "effective radiating layer." Any uncertainty regarding values of A for visual light and total radiation can be taken up by changing ϵ. It is therefore permissible, formally, to consider values of ϵ somewhat exceeding unity.

[1] Russell, *Astrophysical Journal*, 43, 178, 1916.

[2] Abbot.

[3] Coblentz.

[4] Russell, *op. cit.*, pp. 179, 190.

The theoretical temperature of the surface, due to solar radiation alone, under idealized conditions, may be computed by the equation

$$T'^4 = \frac{y(1-A)}{\epsilon} T_0^4 , \tag{6}$$

Where ϵ is the emissivity of the surface, A its albedo, and y is the ratio of the mean rate of radiation from the area considered to the rate in equilibrium under a vertical sun. For the center of the full moon y is probably near unity. For a perfectly conducting sphere $y=\frac{1}{4}$. For a point at latitude L on a planet rotating so rapidly that there is no sensible diurnal inequality of temperature, $y=\frac{\cos L}{\pi}$ (provided that there is no convection of heat from one part of the planet's surface to another). On an actual planet y will be greater than this by day and less by night, by an amount increasing with the rotation period, and may be greater in the afternoon than in the morning.

2. *Transmissions.*—The transmission coefficients t and t' now remain to be considered. The depletion in the earth's atmosphere is different for long and short wave-length radiation and must be allowed for in detail.

From the work of Abbot and Fowle[1] there is a wealth of material available for determining the value of t' for the solar radiation. The mean transmission of solar energy for Washington (altitude 0) is 0.701 for unit air-mass, with a factor of 0.797 for each additional air-mass; and the corresponding values for Mount Wilson (altitude 1800 meters) are 0.819 and 0.895. It may therefore be estimated that at Flagstaff (altitude 2200) and at Mount Hamilton (altitude 1280), the respective transmissions are 0.83 and 0.80, and the air-mass factors 0.91 and 0.88. These coefficients vary slightly with the amount of water-vapor present, but, under the actual conditions of observation, this variation is unimportant.

The amount of precipitable water, Q, expressed in centimeters, in the path of the radiation, is derived from an empirical formula,[2]

$$Q = mhe , \tag{7}$$

[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vols. II and III.

[2] Fowle, *Astrophysical Journal*, **37**, 359, 1913.

where m is the air-mass determined from the secant law, e the pressure of aqueous vapor in centimeters at the surface, and h a factor depending on the altitude which, according to Fowle's formula, is 1.6 for Flagstaff and 1.8 for Mount Hamilton. Fowle regards these formulae, which represent mean conditions fairly well, as of little value for individual days. From Mount Wilson experience the actual values are as likely as nót to lie between half and twice those given by the formula, the reason being that the available humidity observations apply only to the air near the earth's surface. In the future a spectroscopic determination of the total water-vapor should be an important part of radiometric measurements.

To obtain t, black-body energy curves were drawn for different temperatures, integrated by mechanical quadratures, and Fowle's[1] values of the transmissions applied to the successive wave-lengths, and also modified for selective absorption and reflection by the fluorite window of the thermopile (which values were kindly furnished by Dr. Coblentz). The results were finally multiplied by 1.1 to allow for the more complete reflection of these long waves than the solar energy by the silvered mirrors of the telescope. Such corrections, strictly speaking, should be applied to both t and t'; but as only their ratio appears in equation (4), it suffices to apply a differential correction to t alone. This factor is rather uncertain, depending on the condition of the mirrors, and, if necessary, could be determined experimentally. The final values for the planetary transmissions for unit air-mass are given in Table I.

They were originally worked out for different amounts of precipitable water and for each air-mass separately. It was then found that the resulting values could be represented, with an average error of one part in one hundred and fifty, by taking the transmission for air-mass 1 and the amount of precipitable water *actually traversed* by the radiation and multiplying this by the factors 0.96, 0.93, or 0.89 for air-masses 2, 3, and 4, respectively. This convenient approximation cannot be applied if the precipitable water along the path is less than about 0.5 cm. For larger amounts, the absorption, even for rays from the zenith, is already nearly complete wherever

[1] Fowle, *Water Vapor Transparency to Low Temperature Radiation*, "Smithsonian Miscellaneous Collections," Vol. 68, No. 8; *Smithsonian Physical Tables* (7th ed.), p. 308.

it takes place at all. A fairly dry atmosphere (0.5 cm to 4 cm precipitable water) acts as a very efficient "color screen" for long waves, transmitting nearly all the radiation between 9μ and 12μ, and hardly anything else. It is fortunate that this is the range of humidity likely to be met with at the best observing stations. Under these conditions the transmission falls off rapidly with the temperature of the planet, so that the energy received at the thermopile varies as the sixth or seventh power of the latter. These circumstances are really very favorable for the determination of planetary temperatures.

TABLE I

TRANSMISSION OF BLACK-BODY RADIATION

Total Precipitable Water in Path	Temperature						
	100°	150°	200°	250°	300°	350°	400°
0.003 cm........	0.002	0.025	0.084	0.170	0.268	0.355	0.440
0.03 cm........	0.001	0.020	0.073	0.150	0.222	0.288	0.342
0.5 cm.........	0.000	0.014	0.050	0.097	0.136	0.168	0.192
1.0 cm.........	0.000	0.013	0.048	0.091	0.126	0.152	0.171
2.0 cm.........	0.000	0.013	0.047	0.089	0.122	0.147	0.163
3.0 cm.........	0.000	0.013	0.045	0.088	0.120	0.142	0.158

There is, however, great need for an extension of the existing data on the transparency of water-vapor to long-wave radiation. Most of the coefficients used for computing Table I were obtained by more or less free extrapolation and should be checked by experiment.

3. *The observations.*—The first measures of water-cell transmissions were made at the Lick Observatory[1] in 1914 by Dr. Coblentz, and the more recent ones at Flagstaff by Drs. Coblentz and Lampland[2]—to whom the writer is much indebted for communication of results in advance of detailed publication.

Table II contains the observational data and the computed temperatures. W is the water-cell transmission, m the air-mass, Q the precipitable water in centimeters along the path, a the phase-angle, and A the albedo. q/ϕ is the phase factor for reflected radia-

[1] *Lick Observatory Bulletin*, No. 266.

[2] Coblentz, *Scientific Papers of the Bureau of Standards*, No. 438; Coblentz and Lampland, *Popular Astronomy*, 30, 551, 1922.

tion and x is the brightness factor defined in §1. T_0 is the black-disk equilibrium temperature from equation (3), while T_1' and T_2' are theoretical temperatures computed according to equation (6), the former for a rapidly rotating planet, in which case $y = \frac{\cos L}{\pi}$, and the latter for the sun permanently on the meridian, $y = \cos L$. If the planet's radiating layer is black for long waves ($\epsilon = 1$), its temperature must lie between these limits. With lower emissivity these computed limits will be multiplied by the factor $\epsilon^{-\frac{1}{4}}$. The observed temperature, T, will be multiplied by a factor approximating $\epsilon^{-\frac{1}{7}}$ (see §2). T is the temperature computed from the water-cell transmissions by equation (5), assuming that the emissivity $\epsilon = 1$. The computed values for the earth are inserted for the sake of comparison. It should be remembered that for planets with an atmosphere, T represents the temperature of the "effective radiating layer." The surface temperature may be considerably higher.[1]

TABLE II

PLANETARY TEMPERATURES

Planet	Date	W	m	Q	a	$\frac{q}{\phi}$	A	x	T_0	T_1'	T_2'	T
Venus	1914, Aug. 19	0.59	5.5	4.7	67°	3.03	0.59	2	460°	276°	369°	330°
	1922, June 15	0.663	3.6	2.0	45	2.16	......	3	.....	.....	.....	310
Mars	1921, Oct. 6	0.67	2.6	2.0*	15	1.30	0.154	1	304	220	293	.....
Equator	1922, June 15	0.470	2.1	1.2	5	1.17	......	.....	323	233	310	268
South	1922, June 15	0.489	.....	.....	.....	......	......	.....	.....	227	303	262
North	1922, June 15	0.511	.....	.....	.....	......	......	.....	.....	223	297	255
Equator	1922, June 18	0.476	2.1	1.8	8	1.20	......	.....	.....	233	310	264
South	1922, June 18	0.523	.....	.....	.....	......	......	.....	.....	227	303	252
North	1922, June 18	0.551	.....	.....	.....	......	......	.....	.....	223	297	242
Jupiter	1914, Aug. 17	0.657	1.7	0.7	2	1.5	0.56	1	174	107	142	157
	1922, June 14	0.682	2.2	1.1	10	......	......	.....	168	103	137	168
Saturn	1914, Aug. 26	0.55	1.6	0.7	0	1.5	0.63	1	130	76	101	172
	1922, June 14	0.60	1.5	0.7	.....	......	......	.....	127	75	100	161
Moon	1914, Aug. 27	0.147	3.6	2.9	80	1†	0.073	1†	392	290	386	400
Earth	....	......	.....	.....	.....	......	0.45	.....	392	254	338	.....

* No humidity data; a guess.

† $\frac{q}{\phi \cdot x}$ assumed equal to 1.

DISCUSSION

Venus.—The quantity x was estimated on the basis of a sketch sent by Dr. Coblentz, showing the region observed. Were it not for the uncertainty in this, it might be possible to judge from the obser-

[1] Compare the discussion by Milne, *Philosophical Magazine*, 44, 872, 1922.

vations whether the period of rotation is long or short. The table shows that the observed temperatures agree fairly well with those computed for a non-rotating or very slowly rotating planet, and as x in the second case may be greater than 3, the agreement may be even better. More data are needed before a decision can be made. Observations of the radiation from small areas of the bright crescent and the dark side of the planet would be of special value, and might show a difference between the sunrise and the sunset terminators.

Mars.—The first isolated observation is very discordant.[1] In the absence of data regarding humidity this has been guessed at, but no reasonable change in the estimate will remove the discordance. The more recent observations give a mean temperature of 257° A. or −16 °C., which is surprisingly high. It may perhaps be explained by assuming that the surface is very little shielded by the atmosphere, and has a low-heat capacity, so that it reaches nearly the equilibrium temperature T'_2 in the early afternoon, cooling greatly at night. A nocturnal fall of 60° C. would be required to maintain the proper average diurnal radiation. The differences between the equator and the higher latitudes are very closely what might be expected, as the sun was 3° south of the Martian equator. The effective latitudes of the observed northern and southern regions have therefore been taken as 33° and 27° in computing T'.

Jupiter and Saturn appear to possess nearly the same temperatures which, although very low, −110° C., are much higher than can be explained by the solar heat alone. They are far from red hot on the surface (as has been often supposed), but radiate about as much heat as if the uppermost layers of the clouds upon their surfaces were composed of carbon dioxide snow, or some similar substance.

The moon.—The observed portion was close to the limb and in a region of very unequal brightness. The area covered by the thermopile is so small that x can hardly be guessed at. It was assumed that $q/\phi x = 1$ (the value for a part of the full moon of albedo equal to the average). The resulting temperature, 127° C., agrees very well with the calculated value.

[1] In a letter Dr. Coblentz states that the seeing conditions were very poor, and that a transmission of 60 per cent would be just as probable as the published 67 per cent. The observed temperature is therefore purposely omitted from the table.

It would be well worth while to observe the moon with a small reflector, using the entire disk on the receiver of the thermopile, and also to determine, with a larger telescope, the distribution of the heat over the surface. It should then be possible to obtain very accurate results. Professor Very[1] in 1889 made a number of observations of the heat distribution on the moon, using glass instead of a water cell to isolate the long-wave energy. An examination of these results shows them to be in agreement with those of this paper (viz., that the moon is very hot), within the accuracy possible at that time. He finds that the brighter parts of the moon's disk show a greater water-cell transmission than the darker areas (as they should do), and that the isothermal curves have nearly the theoretical distribution, approximating circles perpendicular to the line joining the center of the disk with the sun.

4. *Further suggested observations.*—Of the planets which have not yet been observed, Mercury should give a very low water-cell transmission on account of its low albedo and high temperature. Observations would be interesting but difficult.

Uranus should be observable. If it is as hot as Jupiter and Saturn, it should be possible to determine its temperatures with some accuracy. Neptune may be accessible with great telescopes.

SUMMARY

Theoretical conclusions.—The results obtained so far are encouraging and indicate that the method of water-cell transmissions affords a very valuable means of determining planetary temperatures.

The absorption of radiation in the earth's atmosphere does not apparently offer any serious obstacle although a more accurate determination of the absorption of water-vapor for long waves is desirable.

Extending the observations on the planets over widely varying zenith distances will largely eliminate the uncertainty of the water-vapor correction.

To obtain a reliable value of the mean temperature, it is necessary to use a receiver large enough to cover the entire disk.

[1] Very, *The Distribution of the Moon's Heat and Its Variation with Phase,* published by Utrecht Society of Arts and Sciences, 1891; Very, *Astrophysical Journal*, 8, 199 and 265, 1898.

Additional smaller ones are also desirable for the study of the heat distribution over the planet's surface. Either fluorite or rock-salt should be used for the window of the thermocouple.

Numerical conclusions.—Coblentz' observations have been reduced, and indicate that the surface temperature of Venus is 50° C., Mars −16° C., Jupiter and Saturn −110° C., while the moon is very hot, possibly 120° C. These values are subject to some uncertainty, especially the first and last, owing to the difficulty of estimating the correction which has to be applied because only a part of the image fell upon the thermopile.

The computed temperatures of Venus and Mars can be explained if they receive all their heat from the sun and lose it almost as fast as they receive it, cooling greatly at night.

Jupiter and Saturn have higher temperatures than could be maintained by solar radiation alone, confirming the belief that the planets are hot internally.

Further observation on all accessible members of the solar system is highly desirable.

Acknowledgments.—In conclusion, I wish to express my appreciation to Professor Russell, to whom I am indebted, not only for the first section of this paper, but for many conferences concerning the entire problem. I wish also to express my gratitude to Drs. Coblentz and Lampland, who kindly furnished me with all the necessary data in advance of publication; to Dr. Fowle for advice concerning the air transmissions, and to Professor Very for the communication of his early measures of planetary radiation.

PRINCETON UNIVERSITY
February 1923

LIGHT-CURVE AND ORBIT OF CG CYGNI

$\alpha = 20^h 54^m.2$; $\delta = +34° 47'.5 (1900.0)$

By CH'ING-SUNG YÜ

ABSTRACT

Photometric study of the eclipsing variable star CG Cygni.—The *light-curve* of this variable star is determined from a series of 272 extra-focal observations made with the Thaw Photographic Refractor of the Allegheny Observatory during 1922 (Fig. 2). It is found to be an eclipsing system with two minima, the eclipses being partial. *Ellipticity* of the component stars is indicated by the continuous variation between eclipses; there is also definite indication that the system is slightly brighter when the brighter component is approaching us than when it is receding from us. The *range of variation* (photographic) is 1.12 mag.; the *period* is 0.631,139 day. The *orbital elements* and probable dimensions of the system were determined on the assumption of uniform illumination and also on the hypothesis that darkening at the limb exists (Table V and Figs. 4 and 5). The latter solution gives the smaller residuals, the probable error of a normal of weight unity being 0.019 mag. for the "uniform" and 0.014 mag. for the "darkened" solution. The larger and fainter component of the system is of later spectral type than the smaller and brighter star.

Among the variable stars included in the extensive photometric program now being carried on at the Allegheny Observatory was the recently discovered variable star B.D. +34° 4217, which has now received the name "CG Cygni." The variability of this star was discovered by A. Stanley Williams in the year 1905, when comparing two photographs. Observations to determine its elements of variation were not commenced however until October, 1921, when the discoverer made a series of 219 visual observations.[1] While the shape and time of the principal minimum were well determined, observations were too few to represent the true form of other parts of the light-curve, especially the secondary minimum. Williams' results are given in Table II.

Four stars in the neighborhood of the variable were selected as comparison stars. They are designated *a*, *b*, *c*, and *d* in the order of decreasing brightness, and their positions relative to the variable can be seen in Figure 1. Their positions for 1900 with their magnitude differences and the adopted magnitudes are given in Table I. The positions are not exact, but will serve for identification.

For the observations eleven sets of plates were taken on eleven nights with the 30-inch Thaw Photographic Refractor set about

[1] *Monthly Notices*, **82**, 300, 1922.

10 mm out of focus. Each set consists of four "Seed 30" plates. Following the usual procedure of the photometric work at the

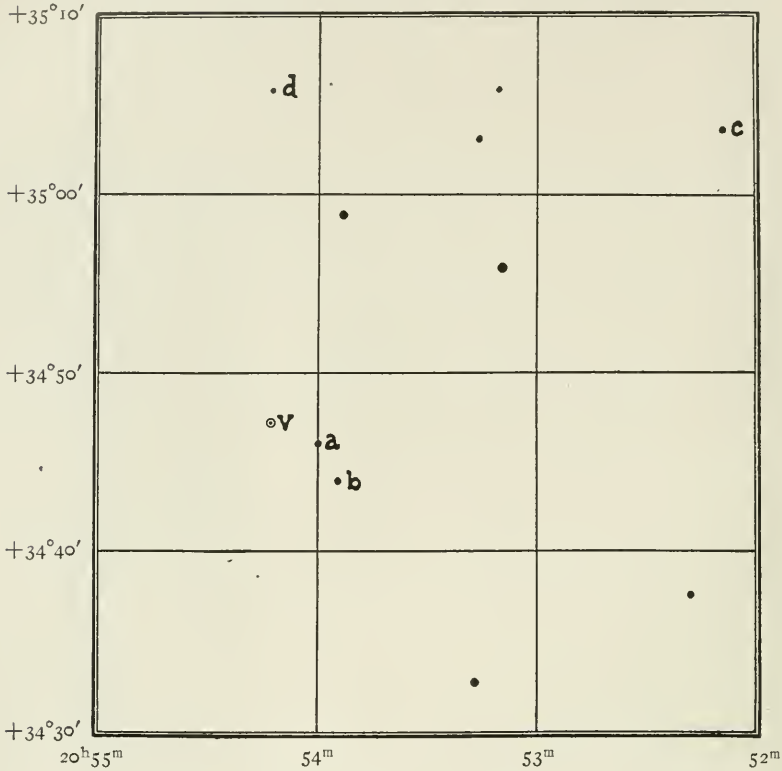


Fig. 1.—Field of CG Cygni

TABLE I

Positions and Magnitudes of Comparison Stars

Star	α	δ	Mag.-Diff.	Adopted Mag.
a............	20^h54^m0	$+34°46'$	0.000	10.100
b............	20 53.9	+34 44	0.313	10.413
c............	20 52.2	+35 04	0.327	10.740
d............	20 54.2	+35 06	0.381	11.121

Allegheny Observatory, the four plates were exposed in succession to one setting of the star; then the position of the star was changed about 2 mm and the process repeated. The average length of a

series of observations was about five hours, which covered a considerable portion of the period of the variable. The time of each exposure ranged from five to eight minutes, depending upon the clearness of the sky. With the exception of passing clouds on a few occasions all plates were taken under favorable conditions. However, it was deemed advisable later to reject one set of plates which were taken on a partially cloudy night; there were only one or two exposures on each plate of this set. The other ten sets of plates were used, making a total of 272 observations, which cover the period at least twice and the minima three or four times.

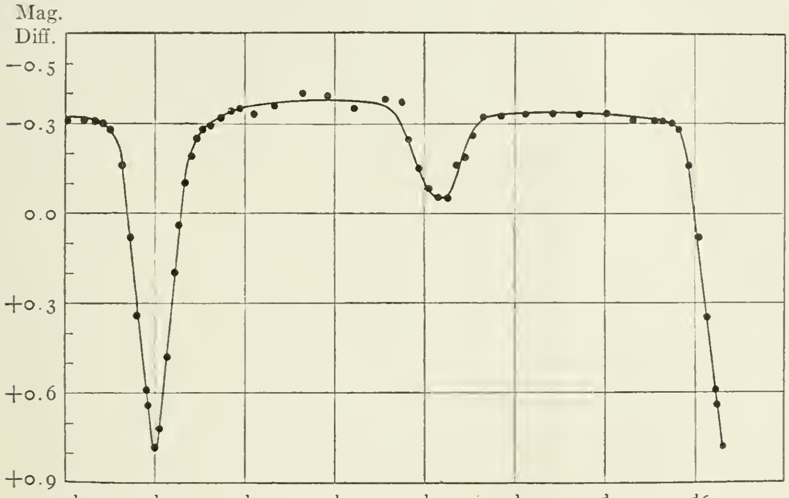


FIG. 2.—Mean light-curve of CG Cygni

The intensities of the star images were measured with a Hartmann microphotometer. Each reading on the variable was compared with the mean s of the comparison stars, and the magnitude-differences (v-s) obtained. These observations were grouped into forty-four normals by taking the means of every 0.005 day during principal minimum, every 0.010 day during secondary minimum, and every 0.030 day between eclipses. The phases have been reduced to the sun. The normals are entered in the second and third columns of Table IV, from which the mean light-curve (Fig. 2) is derived.

By making a preliminary plot of the individual observations, three well-defined minima were found. These indicated that the period is a little shorter than that given by Williams. To fit these, the period as determined by Williams was corrected and slightly shortened, using the original value of the epoch. The new period adopted, and other elements of variation derived from the mean light-curve, are given in Table II, together with the corresponding values obtained visually by Williams.

TABLE II

ELEMENTS OF VARIATION OF CG CYGNI

	Williams (vis.)	Yü (phot.)
Period	$0^d.63118$	$0^d.631139$
Principal minimum	$10^m.42$	$11^m.374$
First maximum	9.93	10.219
Secondary minimum	10.15	10.544
Second maximum	9.94	10.264
Range	0.49	1.115

The adopted formula for phase determination is:

Helio. G.M.T. of Principal Min. = J.D. 2,422,967.4268 + $0^d.631139$ E.

It will be seen by comparing the two sets of results in Table II that a visual range of but 0.49 mag. was found, whereas the corresponding range as found from the photographic data is 1.115 mag. While such differences might be taken as indicating a color-index of roughly +0.3 for CG Cygni, such a deduction is highly uncertain, and the determination of the color-index of this variable must await future color-screen comparisons. The color-index seems definitely greater at primary minimum.

The mean light-curve of CG Cygni (Fig. 2) shows that the system is an eclipsing binary of the "β Lyrae" type, the eclipses being partial. There is a deep primary minimum and a shallower secondary minimum. The former indicates a loss of about 0.6 and the latter about 0.2 of the light of the system. The duration of each partial eclipse is about 0.1 day. The secondary eclipse occurs exactly midway between the primary eclipses, this, together with the equality in the duration of the eclipses, indicates that the orbit

is sensibly circular. Its eccentricity has therefore been assumed as zero ($e=0$). The slight but constant variation of light between eclipses shows that the component stars are elliptical to a small extent. Mutual heating and reflection effects are also indicated to an appreciable amount, as the system is brighter near the secondary eclipse than near the primary. One interesting fact definitely shown by the mean light-curve is that the system is decidedly brighter following the primary minimum than after the secondary minimum, although the positions of the two stars are symmetrically identical in both cases. Since the orbit is sensibly circular, this phenomenon cannot be accounted for by a periastron effect. It will be noticed that the system appears brighter when the bright star is approaching us, and is slightly fainter when the same star is receding from us. Hence it has, in this respect, the characteristics of a Cepheid variable and the "resisting medium" would perhaps explain this discrepancy. There is the difficulty on this theory, however, due to the fact that no effect is observed, when the larger and fainter star is approaching at the same velocity. It will be noticed further that the observations along the first maximum do not fit the smooth curve so well as those along the second maximum do, although more observations were made during the former. The curve at the latter phase is remarkably smooth in spite of the fact that it was covered only twice by observations. The apparently unsteady character during the first maximum is perhaps real, as a result of increased activity of some sort, although errors in observation may account for most of it.

The mean light-curve is "rectified" for the effects discussed above, namely, ellipticity, reflection or radiation effect, and the "resisting medium" effect. The first was corrected by Russell's graphical method, and the corrections (m_e) entered in Table III. The other two effects were small enough to warrant summary treatment and are simply removed by ($-.02 \cos \theta + .022 \sin \theta$). The rectified magnitudes,

$$M = m - m_e - .02 \cos \theta + .022 \sin \theta$$

are given in the fourth column of Table IV, and from these values the rectified curve (Fig. 3) is derived.

The rectified light-curve shows the normal characteristics of the "Algol" type. The drop of the principal minimum is 1.061 mag., and that of the secondary minimum 0.271 mag. The loss of light for the two minima is as follows: $1-\lambda_1=0.624$ and $1-\lambda_2=0.221$.

TABLE III

CORRECTIONS FOR ELLIPTICITY

θ	t	$0.100 \cos^2 \theta$	m_e
0°	.000	0.100	0.057
15	.026	.093	.053
30	.053	.075	.042
45	.078	.050	.028
60	.105	.025	.014
75	.132	.007	.004
90	.158	0.000	0.000

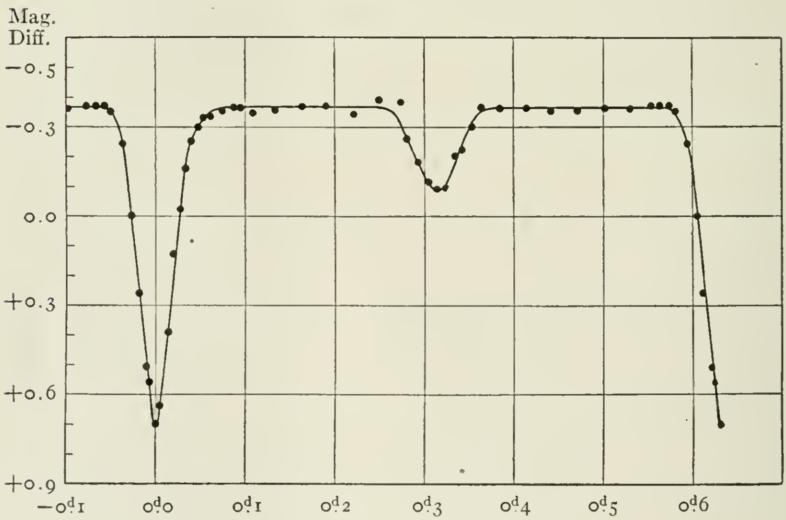


FIG. 3.—Rectified light-curve of CG Cygni

In deriving the orbital elements of CG Cygni, the writer applied the principles of the eclipse theory, as developed by Professor H. N. Russell.[1] Both "uniform" and "darkened" solutions were made. The results are given in Table V, and the system represented in Figures 4 and 5.

[1] *Astrophysical Journal*, 35, 315; 36, 239, 1912.

TABLE IV

NORMAL POINTS, RECTIFIED MAGNITUDES, AND RESIDUALS

No. of Normal Points	Phase t	Mag. $(v-s)$	Corr.	Rectified Mag.	Wts.	$O-C_u$	$O-C_d$
1.......	0^d000	+0.78	−0.08	+0.70	3	.00	.00
2.......	.005	+0.72	−0.08	+0.64	3	+.01	.00
3.......	.014	+0.48	−0.07	+0.41	6	+.03	+.02
4.......	.021	+0.20	−0.07	+0.13	3	−.05	−.06
5.......	.028	+0.04	−0.06	−0.02	5	−.05	−.05
6.......	.034	−0.10	−0.06	−0.16	4	−.06	−.06
7.......	.041	−0.19	−0.05	−0.24	7	−.06	−.04
8.......	.048	−0.25	−0.05	−0.30	3	−.05	−.02
9.......	.053	−0.28	−0.05	−0.33	2	−.05	−.02
10.......	.062	−0.29	−0.04	−0.33	8	−.01	.00
11.......	.075	−0.32	−0.03	−0.35	6	+.01	+.01
12.......	.085	−0.34	−0.02	−0.36	4	.00	.00
13.......	.095	−0.35	−0.01	−0.36	8	.00	.00
14.......	.110	−0.33	0.00	−0.33	12	+.03	+.03
15.......	.134	−0.36	+0.01	−0.35	20	+.01	+.01
16.......	.164	−0.40	+0.02	−0.38	12	−.02	−.02
17.......	.192	−0.39	+0.02	−0.37	10	−.01	−.01
18.......	.223	−0.35	+0.01	−0.34	9	+.02	+.02
19.......	.256	−0.38	−0.01	−0.39	10	−.04	−.03
20.......	.274	−0.36	−0.02	−0.38	2	−.08	−.07
21.......	.282	−0.24	−0.02	−0.26	3	+.01	.00
22.......	.295	−0.15	−0.03	−0.18	4	+.01	.00
23.......	.304	−0.08	−0.03	−0.11	4	+.02	+.01
24.......	.314	−0.05	−0.04	−0.09	5	.00	.00
25.......	.324	−0.05	−0.04	−0.09	3	+.01	.00
26.......	.334	−0.16	−0.04	−0.20	4	−.02	−.03
27.......	.344	−0.18	−0.04	−0.22	3	+.01	+.01
28.......	.354	−0.26	−0.04	−0.30	4	−.01	.00
29.......	.364	−0.32	−0.04	−0.36	3	−.04	−.02
30.......	.385	−0.32	−0.03	−0.35	10	−.01	−.01
31.......	.413	−0.33	−0.03	−0.36	14	.00	.00
32.......	.443	−0.33	−0.02	−0.35	10	+.01	+.01
33.......	.473	−0.33	−0.02	−0.35	16	+.01	+.01
34.......	.503	−0.33	−0.03	−0.36	13	.00	.00
35.......	.532	−0.31	−0.05	−0.36	13	.00	.00
36.......	.555	−0.31	−0.06	−0.37	4	−.01	−.01
37.......	.564	−0.31	−0.06	−0.37	3	−.02	−.01
38.......	.574	−0.30	−0.07	−0.37	2	−.06	−.05
39.......	.582	−0.28	−0.07	−0.35	2	−.08	−.06
40.......	.595	−0.16	−0.08	−0.24	3	−.09	−.07
41.......	.605	+0.08	−0.08	0.00	3	−.05	−.06
42.......	.613	+0.34	−0.08	+0.26	3	+.03	+.03
43.......	.622	+0.59	−0.08	+0.51	3	−.02	−.03
44.......	.626	+0.64	−0.08	+0.56	2	−.06	−.06
1.......	0.631	+0.78	−0.08	+0.70	3	.00	.00

While it is not the primary purpose of this paper to present the relative merits of the "uniform" and the "darkened" hypotheses of the eclipsing theory of variable stars, it would seem from the results obtained that darkening at the limb is strongly indicated for CG Cygni. The average residual, regardless of sign, is 0.016 mag. for the darkened hypothesis as against 0.019 mag. for the uniform one. Securing the "probable error" of a single normal of weight unity from the final residuals by the ordinary formula, this value for the uniform solution is ±0.019 mag., and for the darkened

TABLE V

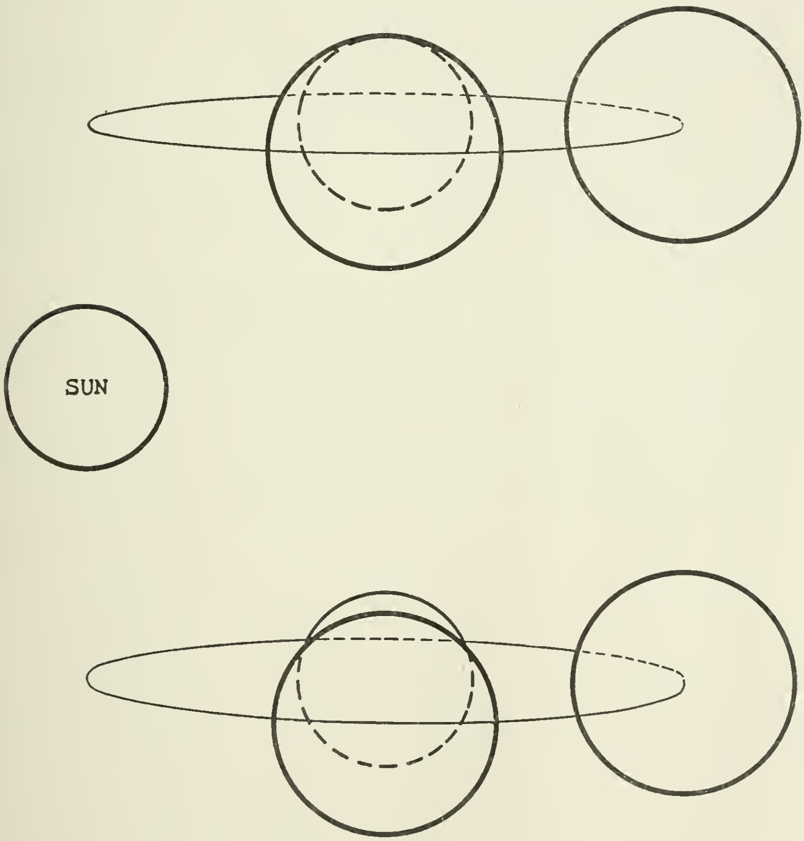
ORBITAL ELEMENTS OF CG CYGNI

		Uniform	Darkened
Ratio of radii of stars	k	.767	.784
Maximum percentage of eclipse	a_0	1.000	.950
Major semi-axis of larger star	a_1	.381	.370
Minor semi-axis of larger star	b_1	.361	.357
Major semi-axis of smaller star	a_2	.292	.290
Minor semi-axis of smaller star	b_2	.277	.280
Eccentricity of meridian section	ϵ	.317	.253
Eccentricity of orbit	e	.000	.000
Inclination of orbit plane	i	85°10′	82°17′
Least apparent distance of centers	$\cos i$	.084	.143
Light of larger star	L_1	.376	.343
Light of smaller star	L_2	.624	.657
Ratio of mean surface intensities	J_1/J_2	1/2.8	1/3.1
Density of larger star (sun = 1)	$\bar{\rho}_1$	.340	.358
Density of smaller star	$\bar{\rho}_2$	.753	.742
Mean radius of larger star (sun's $r = 1$)	$\bar{r}_1$	1.433	1.408
Mean radius of smaller star	$\bar{r}_2$	1.099	1.015
Hypothetical parallax of system	π	0″0044	0″0044

one, ±0.014 mag. The observed secondary minimum is also much better represented by the "darkened" solution.

It was found, by assuming equal masses for the component stars, that the mean radius of the larger star is about one and four-tenths that of the sun. Assuming that its surface brightness is the same as the sun's, the light-emission of the larger star will be about twice as great, and that of the whole system about 5.7 times as great as that of the sun. At maximum the photographic magnitude of the system is 10.22, and the visual magnitude close to 9.9. Taking the absolute magnitude of the sun as +5, and assuming that the system is 5.7 times as bright as the sun, its absolute magnitude will be about

$+3$. Then from the equation $M=m+5+5 \log \pi$, we derive a hypothetical parallax of $0''.0044$, corresponding to a distance of 740 light-years. The distance thus derived is probably trustworthy, though it must be remembered that it depends on the assumptions made above as to mass and intrinsic surface luminosity.



FIGS. 4 and 5.—System of CG Cygni. The upper cut is for the uniform solution; the lower for the darkened solution.

CG Cygni is too faint for spectroscopic study with existing instruments. The larger color-index at primary minimum would indicate that the spectrum of the fainter star is of a more advanced type than that of the brighter star.

SUMMARY

1. Visual observations of the variable star CG Cygni, amounting to 219, were made by Williams, and its epoch and period determined.

2. The writer has taken 272 photographic observations, the reduction of which gives the following new period, the epoch being that given by Williams:

$$\text{Principal Min.} = \text{J.D. } 2{,}422{,}967.4268 + 0^{d}.631139\ E \quad \text{G.H.M.T.}$$

3. The mean light-curve shows that CG Cygni is an eclipsing system of the "β Lyrae" type, the eclipses being partial.

4. At primary minimum when over nine-tenths of the smaller and brighter component is covered by the larger and fainter one, the star is 1.115 mag. fainter than at maximum. The magnitude at maximum is 10.219.

5. The spectrum of the system is unknown, but there is definite indication that the fainter star is later in type than the primary.

6. The component stars are elliptical, but the ellipticity is small. The orbit is sensibly circular.

7. Mutual heating and reflection effects are noticeable. The system is 0.045 mag. brighter at first maximum when the bright star is approaching us than at second maximum when the same star is receding.

8. Two methods of solution were employed to determine the orbital elements of the system. The agreement of the computed curves in each case with the observed one proves that the light-variation of CG Cygni is satisfied by the eclipse theory, and the slightly better representation of the "darkened" solution indicates that darkening at the limb exists.

9. The smaller star emits twice as much light as the larger one, while its surface brightness is 3.1 times as great. The fainter star exceeds the brighter one in linear dimensions by 27 per cent. The distance between centers is 2.7 times the radius of the fainter star. The larger and the smaller components are, respectively, .358 and .742 as dense as the sun on the equal-mass assumption.

10. CG Cygni is probably 740 light-years away, having a parallax of $0''.0044$.

ACKNOWLEDGMENTS

I wish to express my indebtedness to Dr. H. D. Curtis, director of the Allegheny Observatory, who suggested this investigation as a thesis in partial fulfilment of the requirements for the degree of Master of Science at the University of Pittsburgh, placed at my disposal the 30-inch Thaw Refractor and all other instruments and supplies that made the undertaking of this investigation possible, and looked over part of my computed materials. My thanks are also due to Dr. F. C. Jordan, who supervised and directed the work, set aside for me about a dozen nights from his busy photometric schedule, and also checked part of my computations. I am indebted also to Mr. Z. Daniel for photographing a set of plates for me.

Allegheny Observatory
Pittsburgh, Pa.
February 1923

EXCITATION STAGES IN THE OPEN ARC-LIGHT SPECTRA

PART III. LEAD, MERCURY, THALLIUM, MAGNESIUM

By B. E. MOORE

ABSTRACT

Excitation stages in low-current arc spectra of Pb, Hg, Tl, and Mg.—In the present study *photographic methods* were used in contrast to visual methods of the previous studies. It is found that the *true arc* begins to discharge from a minute point or negative polar tip. This tip discharge never forms without ionization; and when the pole is a bivalent element there are always lines belonging to the Rydberg 4N limit, lines which are theoretically associated with a return of the second valence electron to the nucleus, i.e., at the pole tip, the temperature and velocities of the electrons are always high enough to produce appreciable *dissociation* of the atom. For the element *lead* all of the observed lines are easily produced and show an exceptionally low ratio of pole to middle intensity. This indicates that the lines belong to Stage I and that there is an appreciable amount of lead vapor in the arc at low currents. Two stages are found in the development of the lines of *mercury* and a possible third stage is suggested; furthermore, the lines develop more slowly than in the other metals. Two stages are found in the development of the lines of *thallium*. There are four stages in the development of the lines of *magnesium*, which correspond closely to what is known from other considerations as a resonance, first ionization, a subsequent resonance, and a second ionization. The changes are subtle, and the stages overlap. The different actions of nitrogen and hydrogen atmospheres are used in these experiments. *Enhancements* are carefully considered, and evidence is given which confirms the opinion, which has been frequently contested, that the enhanced lines correspond to hotter stages.

INTRODUCTION

In Parts I and II[1] the intensity of various spectral lines was generally computed from the formula

$$\log_{10} I = d, \tag{1}$$

where I is the intensity of illumination and d the density of the filter which reduces the transmitted light to the limit of visibility. This method is employed herein for the visible spectrum of mercury (see graphs, Fig. 2). In the present experiments the photographic method has been chiefly employed. If we attempt to use the density of a (negative) line as an index of intensity according to equation (1), we are subject to some restrictions. I am indebted to Mr. Lloyd Jones, of the Eastman Kodak Company, for an extensive discussion of this point. For the present purpose Figure 1

[1] *Astrophysical Journal*, 54, 191, 246, 1921.

will give us sufficient information. Here logarithms of intensity are represented as abscissae and the corresponding densities of the photographic image as ordinates. The two curves correspond to different wave-lengths. Uniform results are obtained only when lines are studied upon one branch (e.g., branch *b*). A method was available which permitted the study of spectral lines by an absorption method, and from the absorption a computation of the density of the lines. A less accurate method was judged sufficiently satisfactory for the purpose. Let us take, for example, three lines, *l*, *m*, *n*, whose intensities are 100, 10, and 1, respectively. Let us take three exposures of these lines, and let the time ratio of the exposures be 1, 10, and 100. Now we shall make the assumption that an increase in time of exposure is equivalent to an increase in intensity, which certainly is more accurate than the reading which we shall obtain. We find now that *m* in the second has the same density as *l* in the first photograph. Also, we find that *n* in the third photograph has the same intensity as *l* in the first and as *m* in the second. Thus by cross-comparison of differently "timed" exposures, one may make visual comparison of the intensities of lines. This procedure, of course, should be limited to the *b* branch of the curve, Figure 1, and should be subjected to a large correction for large change of wave-lengths. The latter correction has not been introduced, and lines of intensity 1, 2, lie upon the lower branch of the curve, Figure 1. Such lines are essentially brighter, therefore, than they appear.

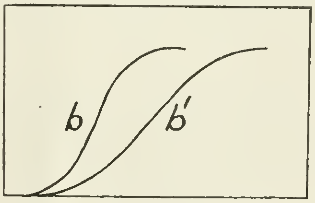


FIG. 1

One other feature should be noticed. If we are dealing with a photograph which has a number of lines whose intensities are distributed across the *b* section of the curve, Figure 1, and compare it with another photograph adequately timed to bring the strong lines down to the lower part of the branch *b*, then the lines which previously occupied the lower position upon the *b* curve will make practically no impression upon the lower branch of the curve. Hence the disappearance of these lines under such circumstances is

not an indication of the absence of the lines in the source of illumination. There are two methods which one may adopt to overcome this difficulty. First, one may establish some relationship between the intensities of certain lines and the current. In Parts I and II we have used

$$I = ki^x, \qquad (2)$$

where i is the current, and we found very serviceable values for x to be 2.5, 3 to 3.3. Then with x equal to 3 for two currents of ratio one to two, the times required for exposures which will give equal intensities are in the ratio of eight to one. In this way any inequality in the rate of development will appear. For the very weak lines one may resort to overexposure, the object of which is to push the line from the lower to the middle branch in Figure 1. This is essentially increasing x, and its application is limited by the fogging of the plate from low-intensity continuous radiation.

LEAD

The metal was placed in a small brass cup and was used as a lower negative pole. The upper positive pole was copper. The current was varied from 0.035 ampere to 0.1 ampere. With larger

TABLE I

Group I			Group II*		
λλ	I	Negative Pole—Middle	λλ	I	Negative Pole—Middle
4058	20	5	2073	5	10
3740	4	10	33	10	10
3680	20	5	23	3	10
3671	1	10	02	5	10
3640	20	5	2663	4	10
3573	3	10	28	8	10
			2577	1	
			11*c*	1	
			2476*a*	1	
			2446*b*	1	

* Groups according to Kayser and Runge, *Wied. Ann.*, 52, 93, 1894; "*a, b*," belong to Group III; "*c*" an unidentified line.

currents the metal volatilized rapidly. The arc was photographed in an atmosphere of nitrogen. In Table I is given a list of lines with an eye estimate of intensities for one current. With increased

exposure the continuous field grows, and, except for a small region about λλ 2400–2800, interferes with observations. In the latter region there was no trace of new lines at 100 times overexposure, although there are several recorded in Kayser's *Handbuch der Spectroscopie*, Volume VI, which might have been anticipated. Lead is characterized by a few very strong lines, a few of moderate intensity, and a greater number of very weak ones. Under the circumstances the very weak ones may have been absent because of inadequate exposure, but this cannot be said concerning the missing lines of moderate intensities. The inference is that the current had not reached sufficient intensity to excite these lines. Comparing the ratio of polar to middle intensities in Table I, it is evident that the slight variation does not give one adequate reason for thinking that the lines belong in different classes. They represent very easy excitations and probably fall in Stage I. They increase more rapidly than the cube of the current.

MERCURY

Table II gives a list of lines at a current of .08 ampere whose terms are listed in order of their relative intensities. A larger current was not used in the invisible spectrum, for boiling and spraying begins. Judgment concerning the middle of the arc could not be carried very far as an exposure forty times as long as at the pole begins to produce a fog which has its origin in continuous spectral radiation.

Table III gives a study of λ 5461 with the ray filters in the visible spectrum. To get the last readings for pole and middle it was necessary to use a large surface of mercury, and to have an assistant stir it and remove the oxidized portion. This line and a few others are shown upon the graph, Figure 2. Line λ 5769, negative pole, practically duplicates λ 5461, middle of the arc. Line λ 5790 is like λ 5769 but a little weaker. Line λ 4916, Type $2P–3S$, shows the same ratio to λ 5769 as λ 5769 shows to λ 5461. Lines λ 4339, Type $2P-4d'$ and λ 4347, Type $2P-4D$, are just becoming visible at the limits of the experiment. Line λ 4959 appears at the same time as λ 4916 and shows uniformly about one-third of the intensity of the latter. An unknown type is shown in λ 5365, a very important unknown type is found in Table II, line λ 4078.

TABLE II

MERCURY LINES

λλ	Intensities Negative Pole	Negative Pole—Middle	Type	Stage
2537	100	20	$1S-2p_2$	I
5461	....	....	$1.5s_1$	I
4358	100	20	$1.5s_2$	I
4047	60	20	$1.5s_3$	I
3663	10		$3d''_1$	II
55	12		$3d'_1$	II
50	20	30	$3d_1$	II
3131	....	....	....	....
26	15	30	$3d_2$	II
3028	10	....	$3d_3$	....
3028	....	....	$4d''_1$	....
23	....	....	$4d_1$	....
21	15	30	$4d_1$	II
2655, 2653, 2652 }*	15	....	$4d_2$	II
2635†	....	....	$4d_3$	II
3341	10	>40	$2.5s_1$	....
2893	7	....	$2.5s_2$	....
2753	4	....	$2.5s_3$	....
4078	10	....	?	....
2925	6	....	$3.5s_1$	....
2574	4	....	$3.5s_2$	....
2464	1	....	$3.5s_3$	....
2803	7	....	$5d_1$	....
2482	2	....	$5d_2$	....
2699	4	....	$6d_1$	....
2399	1	....	$6d_2$	....
5790	*g*	....	$2P-3D_1$	....
5769	*g*	....	$2P-3d$	....
4347	*g*	....	$2P-4D$	....
4339	*g*	....	$2P-4d_1$	....
4916	*g*	....	$2P-3.5S$	....
5335	*g*	....	....	....
4959	*g*	....	....	....

* Not separated. Broad. † Overlapped by λ 2537. *g* See graph.

TABLE III

λ 5461

Intensities	Negative Pole (amperes)	Middle (amperes)
10	....	.016 –.018
100	.0125	.0295–.0345
1000	.0215–.0245	.056 –.068
10000	.036 –.045	.095 –.125
100000	.11–.16*	....

* Probably reversal is taking place.

There is a marked degree of parallelism in the lines of the mercury graph, Figure 3. A rate of development proportional to the cube of the current is shown, and it is seen that the lines develop less rapidly than this power of the current. The same conclusion holds for the ultra-violet lines. Line λ 2537, Type $1.5s—2p_2$, and the triplet $2p_1—1.5s$ intercept the ordinate axis at zero current, i.e., they leap to well-defined magnitudes at the very lowest current at

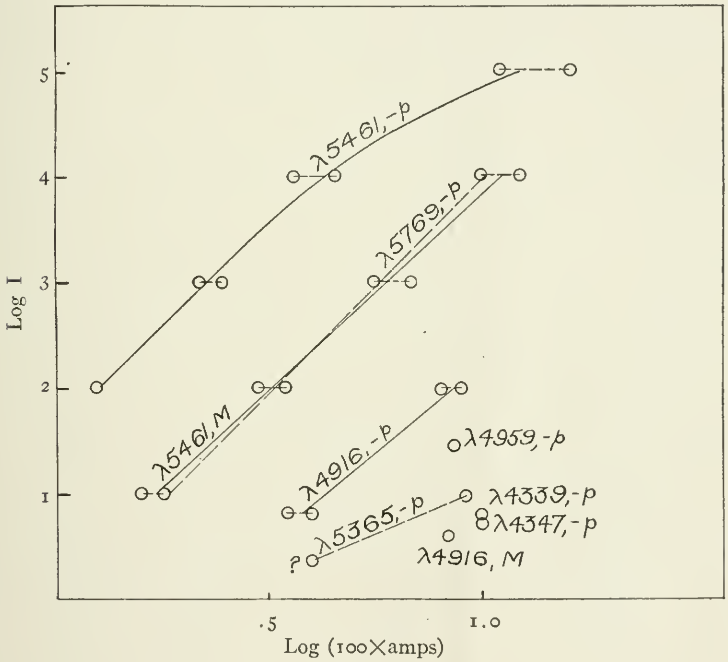


Fig. 2

which the self-sustaining arc will operate. They are therefore placed in Stage I. Then there follow others whose claim for Stage II are very distinct. Lastly there are weaker lines in the ultra-violet and some just becoming distinct in the visible spectrum for the largest current. The evidence that these lines belong in a third stage is not so well defined. But if they belong to an earlier stage, we must assume that the general slope of their development is very different from the other lines in Stage II. The lines are not assigned to any stage in Table II.

THALLIUM

Line λ 5350 is so very strong that it required too much superposition of filters to get a clear view of the field, and readings are too inaccurate to record. However, between 0.08 ampere and 0.32 ampere the rate of development seemed less than the 2.5 power of the current. An alloy of thallium and copper, containing 1.4 per cent of thallium, was tried. This was similar to the recorded line λ 5378 in cadmium but considerable brighter.[1] But at about .1 ampere the samples melted, rapidly oxidized, and the line fell off to about one-tenth its original intensity. With this scarcely complete analysis, it was concluded that the character of λ 5378 had been properly established in the investigation of cadmium, although the identity of the substance was wrong. With the alloy as negative pole, an exposure of the middle of the arc at .08 ampere, 100 times as long as the foregoing negative pole exposure brings out the lines $2s_2$, $3d'_1$, $3d_1$, $3d'_2$, $3d_2$, and the first-stage copper lines λλ 3247, 3274.

From this analysis, these lines are assigned to Stage I. They belong to the earliest ring types immediately following the resonance position and apparently cannot be separated from it in the self-sustaining arc. The remainder of the lines have the Stage II characteristics. They fall off rapidly in intensity as required, and increase more rapidly after being once established, than λ 3776. But the separation here, as with the photographic procedure in general, is not as sharp as observed in Parts I and II, and probably for reasons assigned in magnesium. No members of the principal series of doublets were observed. The first one to come into view would be $4p_2$.

Table IV gives a list of the lines observed in the spectrograms. The lines observed belong to the sharp series, s, and diffuse series, d. The quantum values of the lines are shown by n and the sequence in the members by a. There are 16 other lines or incipient bandheads, none of which identify with known arc or spark lines. If

[1] This thallium green line λ 5350 in the small spectroscope seemed to occupy about the same position as a line designated as cadmium λ 5378 Part II, page 249 ff. This line in cadmium had been taken for granted from the descriptions in spectroscopic tables. Measurements showed that the line in both cases was λ 5350 and that the line λ 3779 in cadmium was also an impurity of thallium and was the line λ 3776.

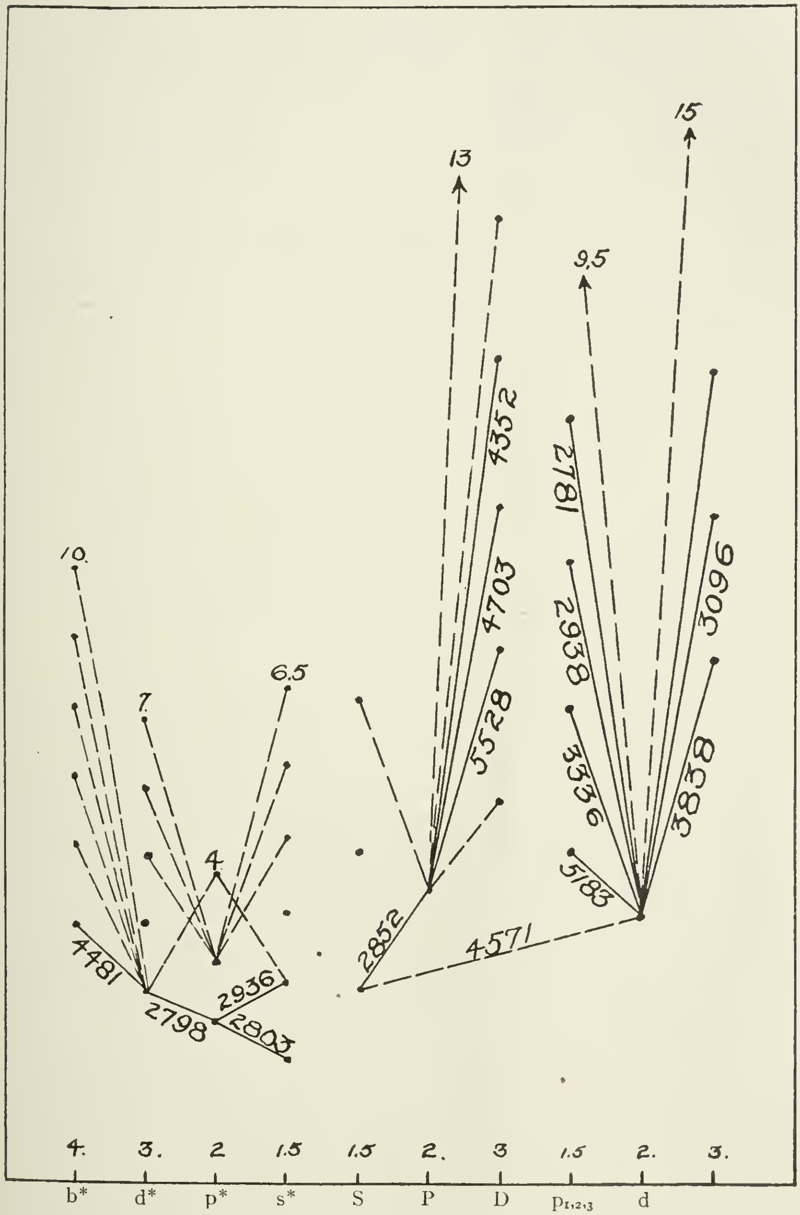

13
15
9.5
10.
7.
6.5
4.
4352
4703
5528
1817
2781
2938
2962
3336
3096
3838
5183
4481
2798
2936
2803
2852
4571
4.
3.
2
1.5
1.5
2.
3
1.5
2.
3.
b*
d*
p*
s*
S
P
D
p1,2,3
d

FIG. 3

the intensity of λ 5350 in the visible spectrum were comparable, these lines show an intensity difference of over three thousand fold. All these lines did not appear at the lowest current. For example, when λ 3776 is overexposed one thousand fold at .04 ampere the negative pole line 5.5*s* has not appeared. Let us assume that the rate of development is proportional to the 3.3 power of the current, then at .16 ampere an exposure ten times as long as the first exposure of λ 3776 is equivalent to the one thousand fold overexposure. When such an exposure is made at .16 ampere both 5.5*s* and 1.5*s*

TABLE IV

THALLIUM

Sharp Series, *s*. Doublets			Intensity	Diffuse Series, *d*			Intensity
λλ	*n*	*a*		λλ	*n*	*a*	
5350	1	1.....	....	3529	3	1'.....	400
3776	1	2.....	3000	3519	3	1.....	400
3230	2	1.....	600	2768	3	2.....	150
2580	2	2.....	50	2921	4	1'.....	20
2826	3	1.....	25	2918	4	1.....	20
2316	3	2.....	1	2380	4	2.....	1—
2666	4	1.....	15	2711	5	1'.....	3
....	4	2.....	....	2709	5	1.....	3
2585	5	1.....	2	....	5	2.....	....
....	5	2.....	....	2609	6	1'.....	
2538	6	1.....	1		6	1.....	1

appear. Hence, even if these lines were not absent at the lower current they have at least developed more rapidly than the 3.3 power of the current. The continuation of this line of analysis shows that λ 3776 is developing less rapidly than the third power of the current. If we compare the later with the earlier current we find that the line must be similar to λ 5461 of mercury, which exterpolated would give a large intercept upon the ordinate axis at zero current. The ratio of pole to middle arc is low for the lines $2.5s_2$, $3.5s_1$, $3d_1'$, $3d_1$, and $3d_2$. Unfortunately the record in the middle of the arc is not complete. For λ 5350 in the visible spectrum at .32 ampere the ratio of pole to middle is less than five.

MAGNESIUM

A brief discussion of this substance has already been given.[1] It was therein recorded that the resonance line λ 4571, Type $1.5S-2p_2$, is a result of "spraying and flaming" process, and that it does not belong in the arc process of the self-sustaining arc, while λ 2852, Type $1.5S-2P$ was found to be the leading line. It was not obtained entirely alone even in the middle of the arc. A group of companion lines seemed to accompany it upon the violet side. To make more than a guess at the identity of these lines, it was necessary to make use of a larger dispersion quartz spectrograph. The University of Minnesota kindly loaned an instrument which made possible the completion of the analyses herein recorded.

The previous study of magnesium was made with salts of magnesium upon carbon. The substitution of the metal in air gave nothing different, the substance rapidly oxidized and flaming was liable to ensue. In the present experiments the arc of the metal was studied in nitrogen and hydrogen under various pressures and in mixtures of these gases. This method did not make it possible to reduce the arc to the point where the leading lines, hereafter mentioned, did not appear along with the resonance line λ 2852. Whether even the middle of the arc was free from the last trace of these lines at the lowest current seemed doubtful.

Table V gives a record of these lines at .16 ampere. To bring out comparative densities, the exposures in the middle of the arc were made twenty and sixty times as long in nitrogen and hydrogen, respectively. Photographs were taken down to .03 ampere where the variations were larger, due to the irregularity in the position of the minute negative pole. Here, too, the continuous and band spectra are troublesome in the long exposures. In the middle of the arc the changes are more pronounced. Lastly there is given in Table V, for the purpose of comparison, a partial list of observations taken by Foote, Meggers, and Mohler for three characteristic potentials in the "separately excited" vacuum arc.[2] To be complete,

[1] *Astrophysical Journal*, 54, 215, 1921. In the eleventh line from the bottom of page 215, the expression "this line" refers to λ 4571 not to λ 5183.

[2] *Philosophical Magazine*, 42, 1006, 1921.

there should be a polar column for hydrogen also. The effect of the substitution of hydrogen for nitrogen is to increase the intensities of all lines at the poles, which is just the opposite of the effect of the substitution in the middle of the arc. Also, the lines which

TABLE V

λλ	Intensity			Intensity by F. M., and M.			Type
	Negative Pole	Middle ×20	Middle ×60	$20V$	$10V$	$7V$	
4571	0	....	....	20	15	5	$1S-2p_2$
2852	200	150	120	50	50	10	$2P$
5183	160	70	50	7	8	1−	$1.5s_1$
72, 67	80	50	30	10	10	1−	$1.5s_2$, $1.5s_3$
2803	60	30	8	10	8	....	$2p^*_2$
2795	50	25	6	10	9	....	$2p^*_1$
3838	30	25	12	12	10	....	$3d_1$
32, 29	25	20	10	12	10	....	$3d_2$, $3d_3$
2798	20	4	1	4	1	....	$3d^*_2$
90	30	5	1	6	1	....	$3d^*_1$
2936	25	4	1	6	5	....	$2.5s^*_2$
29	20	3	1	6	1	....	$2.5s^*_1$
4481	15	0	0	8	....	....	$4b^*$
3097	12	6	2	9	9	....	$4d_1$
92	....	....	....	6	7	....	$4d_2$
91	1	5	2	10	9	....	$4d_3$
3337	12	3	1	10	10	1	$2.5s_1$
32, 30	10	3	1	8	8	....	$2.5s_2$, $2.5s_3$
2779	10	4	2	4	4	....	....
83	8	3	1	5	5	....	....
81	6	3	1	4	4	....	$4.5s$
78	6	3	1	4	4	....	$4.5s$
77	4	2	1	5	5	....	$4.5s$
5528	4	4	2	1	0	....	$4D$
4703	2	2	2	15	10	....	$5D$
2942	2	1−3	0	10	9	....	$3.5s_1$
38, 37	2	2	0	4	5	....	$3.5s_2$, $3.5s_3$
4351	1	1	1	40	30	....	$6D$

decrease the most in the middle of the arc, are the ones which increase the most at the poles. For example, at the negative pole in hydrogen the intensities of the lines λ 2803, λ 2798, λ 2936, and λ 4481 which are first terms of four series, theoretically belonging to the excitation of the positive nucleus are about in the ratio 5, 4, 4, 3; whereas, as will be seen from Table V, their respective ratios in nitrogen are about 12, 4, 5, 3. This means an "enhancement"

of the three later lines in hydrogen with respect to λ 2802. Visually the "enhancement" of λ 4481 seems nearer tenfold than 2.4 fold, which would be no contradiction if there is an enhancement in λ 2802 of fourfold. However, it did not appear that the line λ 2802 was more than doubled for a common current upon passing from nitrogen to hydrogen. Although the polar spectrum in hydrogen is stronger than in the nitrogen, there are no extra lines brought out until a current of .22 ampere is reached, when the doublet λ 3104, type $5b^*$ appears. The same doublet has not yet appeared in a nitrogen atmosphere at .32 ampere. (Owing to the heat developed in hydrogen, the form of the apparatus would not permit the use of more than .22 ampere.)

In Table V the negative pole lines are given in the order of the relative intensities of the respective types. It is seen at a glance that the order is quite different in the vacuum arc data by Foote, Meggers, and Mohler. This is all brought about by reason of the fact that the intensities in the self-sustaining arc drop off very rapidly as one passes from the inner toward the outer Bohr rings, whereas by virtue of the freedom of motion in the vacuum arc there is less restraint upon the outer rings. This may be made clearer by the graph, Figure 3. The following brief explanation of this graph is adequate. Taking the equation for the limiting frequency in the Bohr-Sommerfeld theory,

$$\frac{N}{(1+k)^2}=K, \qquad \frac{N}{(1.5+s)^2}=A. \qquad \frac{N}{(2+p)^2}=a',$$

etc., the values

$$1.5+s, \qquad 2+p, \qquad 3+d,$$

etc., are computed. These values are then plotted as ordinates for arbitrary lateral displacements of s, p, d, etc. Then the value unity, or one quantum, is added in succession to represent the successive orbital positions. For lines which represent a recovery from the second ionization, we have for limiting frequency[1]

$$\frac{4N}{(1.5+s^*)^2}=\frac{N}{\left(\frac{1.5+s^*}{2}\right)^2}=A^*.$$

[1] Sommerfeld, *Atombau und Spectrallinien* (3d ed.), pp. 273–276, 457.

Here the value $\frac{1.5+s^*}{2}$ is computed. When one quantum is added to the numerator of this value it increases the graph step by one-half as much as in the case of the first ionization. The lines appear twice as frequently. The square of the distance from any position to the axis of abscissae is proportional to the radius of the electron orbit for that position (this is rigid only for the inner *circular* orbits). A conspicuous difference in the two types of arcs is the outer extension of their respective ring systems. The spectral lines are represented by diagonals running downward from an upper to a lower ring.[1] The types of the lines (see last column, Table V) are designated by the upper limit only, unless this designation would introduce confusion, as in case the line belongs to mixed type.[2] The full diagonal lines represent the types observed in these experiments. The dotted lines correspond to lines observed by Foote, Meggers, and Mohler. The numbers at the tops of the columns represent the quantum limits observed by the latter observers. The outer limits in the two types of arcs is very striking. The contrast is even larger when it is considered that the earlier types of lines in the self-sustaining arc are stronger than in the "separately excited vacuum arc." An effect of pressure or difference in the separation of the molecules of the gaseous medium suggests itself at once as a restraining force. But if this were a sufficient explanation, the ring limits in the second ionization lines would be just as extended as they are in the first ionization process. The other factor which enters in the "field state" is the temperature. The first duty of a self-sustaining arc light is to establish a working temperature; and very evidently, from what has proceeded, the temperature variations in the low current arc are very great. However, even the polar tip is only hot enough to excite the first rings of the second ionization type. It is seen that these rings are essentially no farther from the atom than the first rings of the first ionization type. It is also doubtful whether

[1] Sommerfeld, *ibid.*, p. 432.

[2] Absorption lines would be designated by the lines running in the inverse order. The upper position, a condition of ionization, is the field state before the beginning of radiation. The lower position is the state of the field such that the absorption spectra may be produced. It represents the neutral atom for the first ionization lines, and a simple ionization atom for the double ionization group.

the temperature is ever so low in the self-sustaining arc, that both electrons in this and other bivalent metals do not become excited at the polar tip. In the middle of the arc, particularly with hydrogen gas, we approach a condition which is free from the second ionization group of lines. We see this readily if we are free to assign a resonance to the second group lines λ 2803, λ 2796, and allow a small reflection in the middle of arc of polarized light coming from the poles. We also see that in the middle of the arc the first ionization lines are reduced to the limits of the first ring types, p, d, s. The $2p$ triplet is just as persistent as in zinc and cadmium, and the third type more so. In the middle only, then, is there a true separation of types, while at the poles there is no trace of evidence that we may build up a *one-ionization* spectrum, and then advance to a hotter state and form a *second ionization* spectrum, or that we might designate the former an "*arc spectrum*" and the latter a "*spark spectrum*."[1]

The real difference between polar and middle illumination is greater than a spectrogram will show, because the polar illumination wanders and does not stay upon the slit but part of the time. The envelope which surrounds the polar light only falls upon the slit, and this envelope has the same character as the middle of the arc. In the visible spectrum, where one may follow the lines with the eye, one may obtain a more faithful record by making observations only at the instant the desired polar point is upon the slit. Accordingly, attention was directed to two lines in the visible spectrum which have been the subject of much contention, and which have varying ratios in different types of stars. These lines are λ 4481, Type $3d^*-4b^*$, and line λ 4352, Type $2P$–$6D$. In some stars λ 4481 is the stronger, in others the weaker line. One theory has been that λ 4481 is due to a "hotter" star. This opinion is based upon the "enhancement" of the line upon passing from

[1] It is not advantageous to call things by a name, and tacitly admit that the name means something else. Sommerfeld, *op. cit.*, p. 457.

Evidence was given in Part II that polar illumination grows hotter as current increases, and that the later excitations therein were later ring formations of types already well defined. The "Zone" concept of type formation introduced by Lenard (*Annalen der Physik*, 11, 636, 1903), has some features in common with the present results but its discussion would lead too far.

the arc to the spark discharge. The following examination confirms this opinion. Because of the low visibility of λ 4352, the writer has used the next earlier line λ 4703, Type $2P-5D$. Since λ 4481 does not appear in the arc in air,[1] it was necessary to use a nitrogen atmosphere. For both lines the current was varied progressively from .04 ampere to .32 ampere. (The latter only could be used for very brief intervals due to the heating of the apparatus.) The intensities of the lines were compared through ray filters. As mentioned in Parts I and II, there is a large error in this procedure, but one can get a general perspective of the changes, at least. The growth of polar lines is indicated in equation 2. The exponent x lies between 3 and 3.3. Analysis shows that the value of k, which represents the intrinsic brightness coefficient, is at least eight times as large for λ 4481 as for λ 4703 at the negative pole. Next let us take the middle of the arc and compare the lines. Upon increasing the current from .04 ampere to .12 ampere, the line λ 4703 has about the same intensity in the middle of the arc as it had at .04 ampere at the pole; and three cubed is twenty-seven, which compares very favorably with the ratio of pole to middle, twenty-five, found in Table V. At .32 ampere the middle intensity of the line λ 4481 has not reached its polar intensity at .04 ampere. While at .24 ampere, line λ 4481 at the middle of the arc was traceable rather transiently and, even then, decidedly weaker than λ 4307 in the middle of the arc at .12 ampere. From this we see that, in the middle of the arc, the order of the intensities of the lines is just the reverse of that at the negative pole. Following Parts I and II we may characterize λ 4481 as a rigidly short line, and λ 4703 as an easily long one. This difference is explained by saying λ 4703 arises from a first, and λ 4481 from a second, ionization. The old theory that λ 4481 represents a *hotter* state is equally valid, for this is required, when energy magnitudes are considered in the ionization process. The observations make it clear also why one may obtain such anomalies as the experiments by Hartmann,[2] wherein it was found that the line λ 4481 increased in intensity as

[1] Some observers record this line in the air arc of the metals. The writer has only seen a trace of it with clean electrodes and never with an oxidized surface.

[2] Hartmann and Eberhart, *Astrophysical Journal*, **17**, 229, 1903.

the current decreased. We have noted above that the line increases at least with the cube of the current, and that it is *initially* absent at the middle of the arc. Hartmann operated with a low voltage arc which is necessarily a short one even with large currents, and which for decreasing currents becomes progressively shorter. For the lowest current Hartmann could not sustain the arc, and in order to secure a photogram he had to strike the arc continuously for a long period. In the light of the foregoing experiments, for the stronger currents Hartmann photographed the middle of the arc, then as the current and arc length decreased, he progressively received more polar illumination, and for his weakest current his spectrogram was entirely a polar view of the line.

It is plain, from the foregoing considerations, that lines develop at different rates with change in current, and that there are differences in polar and middle illuminations; and that the "enhancement" of some lines is quite marked in these changes. We see evidence for stages in the development represented, first by the line λ 2852, followed by the ionization types *s*, *p*, *d;* and the latter closely followed a second resonance type of lines $\lambda\lambda$ 2803, 2796, which are the basic lines of another ionization.[1] Finally, these are closely followed by lines which represent a complete second ionization. The whole is a condition of affairs which overlap at the poles of the arc but which is brought out by comparing the middle arc under different conditions. At the poles, in fact, when the current is carried down to the very limit at which the arc will operate, there are lines which theoretically have been assigned to both a first and a second stage of ionization.[2] The writer has called attention to the discontinuity in the current when the arc changes from the glow to the true arc discharge.[3] This discontinuity has its origin in the change of polar "drop" which occurs upon the establishment of an ionization at the electrodes. If there be a later stage or two stages in the ionization it is not observable. Since lines, which have been designated as representing two sequences in the loss of electrons for bivalent elements, are traceable to the very lowest limits in cur-

[1] Sommerfeld, *op. cit.*, p. 456.

[2] Sommerfeld, *ibid.*

[3] Chrisler, *Astrophysical Journal*, 54, 273, 1921.

rent for the self-sustaining arc, it is fair to conclude that the establishment of the arc always develops enough heat (or liberates enough energy) to free *both* valence electrons.

As pointed out above, long-sustained photographic exposure upon a shifting polar tip complicates the results. In the visible spectrum one can make observations at the instant the image of the pole is upon the slit of the spectroscope. To accomplish this, the writer took up again barium, whose P_1 line λ 5536, and $2p_2$*, λ 4554, and $2p_1$*, λ 4934 are in the visible spectrum. The arc was now operated in nitrogen and hydrogen under varying pressures. As the pressure falls, the current under which the true arc will operate increases, but at lower pressure the change in the current is larger when the form of the arc changes, and the changes are more frequent. These changes occur only when one is operating near the contact of the glow and true arc characteristics.[1] The observations confirm the earlier record that the last appearance of the barium line λ 5536 is in a broad band emitted by a glowing particle which was cooling below the limit of current for the true arc, and that $2p_2$* always appeared at the pole at the instant of the leap from glow to arc form of discharge. The conclusion that λ 5900 and λ 4525, Types 2.5*s**, are later in their formation was also reconfirmed. This latter type corresponds to λ 2936 and λ 2938 of magnesium and they are strictly short like λ 4481 in magnesium. Hence it appears that one may designate "extreme shortness" in lines as a characteristic of second ionization; and that the long features of the fundamental lines of the excited positive nucleus, represented in barium by λ 4554 and λ 4934, and in magnesium by λ 2803 and λ 2795, arise from a resonance of a positive nucleus accompanying the ionization spectrum of the atom.

I have called attention to the "enhancement" of the cadmium doublet λ 5379, λ 5339 and to the zinc doublet λ 4924, λ 4912, which do not appear in the air arc—a feature which, in so far as the present observations go, they have in common with the magnesium doublet λ 4481. Since the latter shows very conspicuous enhancement in various stars, and is the most conspicuously enhanced line in the magnesium arc, it was desirable to know whether the cadmium and

[1] Chrisler, *loc. cit.*

zinc doublets above designated would appear in a hydrogen or a nitrogen atmosphere. They were not found, and probably represent a "hotter" stage of excitation than λ 4481. Hartmann obtained all of these lines in an arc under water, which he called similar to the arc in hydrogen, and drew the conclusion that they are "cold" lines and under a subtle influence of hydrogen. This conclusion appealed to Kayser,[1] who says "dass hier von einer erheblichen Steigerung der Temperatur nicht gesprochen werden kann, dass alle Schlüsse auf Temperatur aus dem Auftreten der Linien verkehrt sind." But it is this conclusion which is as completely "twisted about" (*verkehrt*), as the arc is "*under*" not "*through*" water. The energy consumption of the arc is large, the pole drop is large and restricted, and therefore the polar illumination represents a *very* hot condition.

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March 1923

[1] Cf. *Handbuch der Spectroscopie*, 5, 710.

EXCITATION STAGES IN THE OPEN ARC-LIGHT SPECTRA

PART IV. HYDROGEN, AIR, WATER-VAPOR, PRESSURE EFFECT, MIXED ELECTRODES

By B. E. MOORE

ABSTRACT

The effect of the medium (hydrogen, air, water-vapor) upon the spectrum.—If hydrogen is substituted for air or nitrogen the middle of the arc is cooler, but it is hotter at the electrodes because of the increased polar drop of potential; and there is a corresponding effect upon the spectra of the electrode materials. The many-lined spectrum of hydrogen is more easily excited than the Balmer series. These lines have two features in common with air bands, nitrogen bands, certain chloride bands, and a large group of water-vapor lines, all of which were examined, viz., they are all easily excited, and they form only on the outside of the hot polar discharge. The inference is that the products which produce these types are dissociated by high heat.

The effect upon the spectrum of pressure and of mixed electrodes.—Decreased pressure is accompanied by a decrease in temperature and results in less contrast in the series lines. The spectral lines appear first at the negative pole, even when the substance in question is located at the positive pole. A low-volatilizing substance at the positive pole increases the intensity of the spectrum in the middle of the arc of a higher volatilizing substance used as a negative electrode. This is attributed to the vapor of the positive pole which conserves the heat in the middle of the arc and raises the temperature. As the vaporized electrode material increases in the arc it progressively suppresses the hydrogen spectrum. The effect begins at a lower current with an easily volatilized substance, e.g., cadmium, than it does with a substance which volatilizes at a higher temperature, such as copper.

General conclusion.—The changes due to pressure and temperature together with the variations in electron velocity are sufficient to explain all of the differences which have been noted in the spectra.

HYDROGEN

Attention was first directed to the effect of the substitution of hydrogen for heavier gases upon the spectra of the metallic electrodes. Naturally the spectra of hydrogen, in so far as they were present, could be noted also.

THE GENERAL EFFECT OF HYDROGEN

For all electrode materials tested, the substitution of hydrogen for air or nitrogen reduced the intensity of the spectra of the metals in the middle of the arc. At the poles, the effect was not always uniform. There was always some strengthening of the lines upon passing to a hydrogen atmosphere. The change ought to be considered in the light of the electrical behavior of the arc in hydrogen. The potential fall across the arc is greatly increased when hydrogen

is substituted for air. The writer will make use of some unpublished observations by Mr. V. L. Chrisler in this laboratory, who finds that the principal part of this important change takes place at the electrodes. This gives both greater electron velocity and greater heating at the electrodes. The middle of the arc upon the other hand is not so hot. This is not because less energy is produced there, but because hydrogen rapidly conducts the heat away from the arc to the inclosing vessel.[1] The greater velocity at the electrodes would bespeak a more extended pole effect as well as a more intense polar action. The former is not realized. The simple reason seems to be that the cooler middle encroaches upon the poles more than in the heavier gases. So that in many cases there is an actual shrinking in the length of the polar illumination, while, it, at the same time, becomes brighter. There is not much difference, for a given current, in the actual number of lines realized whether air, nitrogen, or hydrogen is used, i.e., the higher potential is counteracted by the natural disposition of hydrogen to carry the heat away from the arc. About the largest difference which can be realized has already been indicated under magnesium, Part III.

Most of the recent investigations upon spectral emissions have looked upon the lines as the result of electronic impact, wherein some definite potential is required to impart a velocity to the electrons. These investigations have been disposed to overlook the "heat factor" or "kinetic impacts" in the functioning of the arc. The action of the latter kind of motion is to sustain a field of vibration, while the special lines arise from a particular type of vibration in a particular form of unit in this field. As a definite energy impact is required to produce a definite radiation, one may resolve the energy into two components, viz.: the translational component due to the current flow, and the "heat factor" component. Naturally, in the self-sustaining arc, we may then regard the "heat factor" as the sum of all translational impacts in the current circuit which fall below the magnitude required to produce radiation excitations. Since most of this energy passes from the arc by convection and from the electrodes by conduction, and since this

[1] Stark, *Annalen der Physik*, 18, 213, 1915, finds that heavier gases conserve more heat and raise the temperature at constant pressure and constant current.

energy in the field must be kept at a certain definite value to prevent the arc from going out, it follows that a greater part of the energy of the arc is spent in maintaining this field energy. Hence the greatest field energy, or highest temperature, lies in close proximity to the electrodes, while the field energy in the middle of the arc depends upon the conserving power of the gases and vapors which may be present.

This conserving power is the least for hydrogen of all gases, and therefore the contrast between polar and middle illumination is the greatest for this gas. As the current increases the electrodes volatilize progressively, and for very large currents one approaches a condition where the middle of the arc is a vapor of the electrode material. These vapors are heavier than the surrounding medium, they conserve more heat,[1] the temperature rapidly rises, and the spectrum approaches its polar character.

We shall see that the results conform to the foregoing outline of the effect of the medium upon the spectra. There were two spectra noted, viz.: the Balmer series and the many-lined spectrum which is frequently designated as the "second spectrum." Three different electrodes were used, cadmium, zinc, and copper. The lines are weakest with cadmium and strongest with copper electrodes. This difference becomes distinct as the current increases. In harmony with the foregoing presentation, the explanation would be that cadmium volatilizes before copper, and begins to substitute its vapor in the middle of the arc for hydrogen at a lower current than copper. Four lines of the Balmer series were observed, but attention was directed almost entirely to the red line, $H\alpha$. The description will be given for the cadmium electrodes and comparisons given with the cadmium red line λ 6438, Type $2P-3d$.[2] We have, in the neighborhood of this line, the $H\alpha$ line and some important "second spectrum" lines. We can best understand the condition by trying to visualize them by means of Figure 1.

We will start with an extreme contrast case, about .1 ampere to .12 ampere, when the second spectrum lines have the appearance *a*,

[1] Stark, *ibid.*

[2] In Part II numerical values represented *sequences*, here they represent *quanta*, which is the customary notation.

the cadmium line the polar appearance b, and Hα the appearance c. Now introduce a 1 per cent transmission filter and the lines appear as shown by a', b', c', respectively. If the current is reduced, a takes the form a' and c the type c', but b disappears. The line a persists the longest, although c has the advantage of a polar formation. If one increases the current, b and c increase relatively and then absolutely as to a. Ultimately b passes c in brightness and becomes a long line. By this time the a line has nearly disappeared. When the cadmium line reaches this stage there begins a gradual absolute diminution of the Hα line. A similar weakening of band lines in the polar region was recorded in Part I, Figure 2, page 206. The explanation is the same. This type of radiation does not take place in the polar illumination, but only in the envelope surrounding it, whereas the Hα line is excited in the hot polar discharge. This indicates that hydrogen is completely dissociated into atoms within the polar illumination and that the a type of radiation is a molecular disturbance. We must conclude then that the "*second* hydrogen spectrum" is the *first* hydrogen excitation—a conclusion which may not be generally accepted. The line λ 6438 was found in Part II to belong to Stage III, which, from its Type $2P-3D$, is a rather tardy appearance. The line is an early polar type, but it is not of the rigid polar type represented by λ 4481 magnesium, Type $3d^*-4b^*$. The question then arises: Is it a later appearing type than Hα as appears at first glance from Figure 1? The following consideration makes this doubtful. The Hα radiation begins when the middle of the arc is almost pure hydrogen. The cadmium red line is not strong until very appreciable volatilization of the electrodes begins. When the heat is large enough to carry a considerable amount of the material into the arc the line surpasses Hα in brightness. This is further confirmed by the action of copper. Above .2 ampere with copper electrodes the Hα line is much brighter than when cadmium is used. With the chamber, which has been used, larger currents could be obtained only for brief intervals, but rater transient observations at .25 ampere point to a further

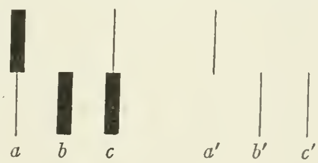

FIG. 1

increase in Hα with copper electrodes, and to an absolute diminution of the same line with cadmium electrodes. The writer is rather forced to the conclusion that if one is given temperatures high enough to insure the presence of both hydrogen and cadmium in the gaseous state, this third-stage cadmium line is essentially more easily generated than the Hα line.

AIR SPECTRA

This subject has already been noticed under band spectra, Part I, page 206; Figure 2 therein used doubtless refers to the character of a chloride band. The writer has not felt that the dispersion at hand afforded him a sufficiently accurate means of identifying these bands by their wave-lengths. Their common occurrence for different electrode materials leaves no doubt as to their air origin. Instead of approaching the polar behavior of these types as indicated by the foregoing figure, one may use the photographic method. It may be recalled that middle arc lines require from ten to a thousand times as long an exposure as the same lines at the negative pole. These bands cropped out more in the middle of the arc. The question arose whether they would be as strong at the poles as at the middle of the arc, if the exposure at the pole were continued as long as at the middle. A test of this kind showed that the lines are absolutely much weaker in the pole illumination, and all the effect actually observed might be attributed to a cooler envelope surrounding the polar glow. There are some bands which have their origin at the pole face; these are very strong in the glow arc of copper, and they nearly disappear upon change to the true arc.

Nitrogen contains nothing unnoticed in air.

WATER VAPOR

In Part II, page 217, two lines $\lambda\lambda$ 3069, 3064 are recorded which persisted under all conditions. Professor A. Fowler has kindly suggested that these lines had their origin in water-vapor. This conclusion seemed very acceptable, but it was desirable to make some test of the matter. An arc light of copper was tried in atmosphere of air, nitrogen, and hydrogen. The gases were passed over phosphorus pentoxide and a vessel of pentoxide was placed in the

arc-light chamber. The lines were still traceable. Thinking that this process had possibly not removed the last trace of moisture, a vessel of hot water was placed in the chamber filled with nitrogen; there was now a profuse supply of banded lines in this region.[1] The electrodes were magnesium and this water-vapor did not seem to interfere with the polar observations upon it. An overexposure for these lines at the pole in the saturated water-vapor was not made but the lines appear in the middle of the arc in the foregoing attempt to analyze the bands, and they did not come out there upon overexposure at the pole. Hence it must be that water-vapor dissociates in the position of the polar illumination. This feature it has in common with air bands, nitrogen bands, some of the chloride bands, some metallic oxides, and the hydrogen "second" spectrum.

The so-called cyanogen band head at λ 3883 was not very distinct except with carbon electrodes, and no definite results can be stated for it.

THE EFFECTS OF PRESSURE

It has been pointed out in Part II that the Bohr theory anticipates more rings, and therefore more lines, in an attenuated gas.[2] In Part III, Table V, one reason assigned for difference in the relative intensities of the writer's lines and those of Foote, Meggers, and Mohler was the difference in the attenuation of the gases.

In an attempt to reduce the pressure in the self-sustaining arc, one is confronted with the fact that the discharge passes from the arc to the glow at a larger current when the pressure is reduced. With a current of .1 ampere it was difficult to sustain the arc with a pressure of .1 atmosphere. Therefore, the pressures used for comparison were about $\frac{1}{8}$ and 1 atmosphere. The current was varied from .1 ampere to .32 ampere. Zinc and cadmium electrodes in nitrogen were used. An effort was made to extend the principal series triplet, p_i. This resulted in disappointment. The same number of lines appeared in both cases. But something else was apparently happening. The leading member, $2p_i$ seemed brighter under the higher pressure. Hence some attention was directed to $2p_i$, cadmium, and the arc subjected to pressures .1, 1, and 10

[1] Sommerfeld, *op. cit.*, p. 527.

[2] Part II, p. 258, *Philosophical Magazine*, 26, 9, 1913.

atmospheres. An assistant adjusted the current at values unknown to the observer, who examined the lines through filters. The readings were taken from .1 ampere to .25 ampere. The most uniform results were obtained at .19–.21 ampere. Here the ratio of intensity at 1 to that at $\frac{1}{10}$ atmosphere was $(6.5 \pm 2):1$. The ratio for 10 to 1 atmosphere was $(4 \pm 1.2):1$, where 1.2 is the average deviation from the mean. At lower pressure, then, there is less relative variation in the sequence of series terms. Evidently, also, one must reckon with the temperature differences when contemplating the differences in lines.[1] An increase in temperature and a decrease in pressure is the complete condition for obtaining a greater number of lines. In the arc a decreasing pressure gives an opportunity for the field to carry the heat away more rapidly and hence it may happen that a decrease in temperature becomes more important than a decrease in pressure. A great many factors would have to be known to settle such a point. In a strong gravitational field like the sun, one may have large fall (outward from the surface) in pressure and a comparatively small temperature gradient, such a condition might bring about such anomalies as a dominant "colder" inner line and a dominant "hotter" outer or high altitude line. Thus calcium λ 4227, resonance type, nucleus Ca, predominates at low solar levels, while calcium λ 3968 and λ 3933, second electron resonance type $2p_i$, nucleus Ca^+, predominates at higher altitudes, although at constant pressure it would require a higher temperature than the former line.[2]

MIXED ELECTRODES

The potential (and current) at which the change from glow to true arc occurs, increased roughly as the volatility of the electrode material decreases, and the spectral lines, in so far as noted, appear first at the negative electrode.[3] Where then, would the lines of a low boiling-point metal first appear if the substance were used as positive pole and some less volatile substance were used as negative pole? To answer this question, cadmium and zinc were tried as lower positive and copper as upper negative electrode.[4] A glow

[1] Saha, *Philosophical Magazine*, 40, 478, 809, 1920; 41, 267, 1921.

[2] Part II, p. 271.

[3] V. L. Chrisler.

[4] To reverse the electrodes does not change the result.

current with copper electrode can be maintained at much larger currents and for a longer time than with cadmium electrodes. With a copper-cadmium combination the copper will in due time become tarnished with the other metal. When the discharge passes more readily to the true arc form, such a tarnishing is sure to take place in time. But this is rather immaterial, as some of the lines of cadmium and zinc may be seen upon the first flare which accompanies the change in form of the arc. These lines appear at the copper instead of at the cadmium or zinc electrode. Next, copper was used as a positive lower electrode and platinum as a negative. Attention was directed to the copper line λ 5105. It appears first at the platinum electrode. Then a photograph of the spectrum was taken at the negative electrode. This reveals the copper-leading doublet and no line which could be attributed to platinum. Of course, tarnish might be effective here.

The atoms had crossed the electron field with no impact strong enough to cause an excitation until they came within the negative pole drop.

An attempt was made to find the copper doublet within the glow discharge. This pair is about the most favorable for this purpose. Copper was used for both electrodes and the chamber filled with nitrogen. This glow discharge may be described as a positive column which terminates in a minute Faraday dark space, near the negative electrode. Over the latter is a violet film, which yields bands. The positive column shows brightest adjacent to this dark space. A long exposure shows the copper doublet very weakly and the bands very brilliantly. The copper lines are lower, which shows that their position was in the head of the positive column and not in the band region. If there had been occasional minute true arc discharges the result would have been the same. So that there is no conclusive evidence that resonance excitation ever occurs in the glow arc.

Next, attention was directed to the effect of low volatile materials upon higher volatile ones. The introduction of materials in this way lowers the potential fall across the arc somewhat, but increases the intensities of lines instead of decreasing them. Let us take as an example the iron arc. At .1 ampere some lines are distinct but

not strong in the negative (upper) discharge. There are no lines beyond this polar region, which is suspended from the electrode like a drop. If we now substitute for the lower positive electrode lead, cadmium, magnesium, or mercury, a threadlike discharge comes from the tip of the positive electrode, and drives into the negative drop, making it heart-shaped upon the bottom. In this threadlike column there may now be found numerous iron lines, although the current is regulated to the same value as before.

This result would follow directly from an increase in temperature due to the conservation of heat upon the substitution of heavier vapors for nitrogen.

Brace Laboratory
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ON THE RADIATION AND TEMPERATURE OF THE EXTERNAL PHOTOSPHERIC LAYERS

By RAGNAR LUNDBLAD

ABSTRACT

Radiation, temperature, and optical properties of the photospheric layers of the sun.—Starting from the observations on the distribution of the energy over the sun's disk, the optical properties of the photospheric layers and the state of radiation within them are examined as closely as possible with a minimum of a priori assumptions. This is accomplished by a new method of inverting the integral equations of the problem. The solution develops two functions which can be said to express the amount of radiation converging toward and diverging from an element of any layer. From these can be computed the *intensity of radiation of each separate wave-length at any depth and in any direction*, and *the conditions for radiative equilibrium;* and in addition the temperature and extinction coefficients of each layer for each wave-length. From these theoretical results it is found that the outermost layers of the sun have an *absolute temperature* of 4500°, and that the temperature increases with the depth. Only a small fraction of the radiation reaches the limit of the atmosphere from the layer at which the temperature is 7500° K, and in this radiation there is a maximum at $\lambda=0.48\ \mu$ at which point 11.5 per cent of the radiation emitted straight outward reaches the outer atmosphere. The *extinction-coefficient* is very large in the extreme ultra-violet. It has a minimum at wave-length $\lambda=0.48\ \mu$ and a maximum at about $\lambda=0.95\ \mu$. A superior limit of the ratio of *scattering* to real absorption is assigned. The scattering is entirely negligible within the infra-red and visible regions.

By the well-known investigations of Schuster,[1] Schwarzschild,[2] and Emden[3] the theory of the transport of radiant energy through an atmosphere has obtained its solid foundation. Several authors have afterward dealt with the same subject, especially in its application on the solar envelope. Some [4] of them assume real absorption and radiative equilibrium, others[5] consider the scattering the most prominent and conclusive feature of the optics of the solar atmosphere. Common to all of them is the course of the investigation,

[1] A. Schuster, *Astrophysical Journal*, 21, 1, 1905.

[2] K. Schwarzschild, *Nachrichten v.d. Gesellschaft d. Wiss. zu Göttingen, Math.-phys. Kl.*, p. 41, 1906; *Sitz.-berichte d. Preuss. Akad. d. Wiss.*, p. 1183, 1914.

[3] R. Emden, *Sitz.-berichte d. Bayer Akad. d. Wiss., Math.-phys. Kl.*, p. 55, 1913.

[4] E. Öpik, *Astronomische Nachrichten*, 198, 48, 1914; B. Lindblad, Diss. Upsala, 1920; E. A. Milne, *Monthly Notices*, 81, 361, 375, 1921.

[5] A. Defant, *Sitz.-berichte d. Akad. d. Wiss. in Wien, Math.-phys. Kl.*, Abt. IIa, 125, 514, 1916; J. Spijkerboer, *Arch. Neerl.* (IIIA), 5, 1, 1918; P. H. van Cittert, *Proc. Amsterdam*, 22, 73, 1919; H. Groot, *Proc. Amsterdam*, 22, 89, 1919; *Physica*, 1, 7, 49, 1921.

starting with some assumption and then deducing the intensity distribution of the sun's disk; if the constants of the problem are disposed of in a convenient manner, the calculated distribution can be made to suit the observations rather well in several cases.

The disposition of this paper is the converse one. Making no assumptions as to the thermal equilibrium or the amount of scattering and real absorption we proceed from the observations on the intensity distribution over the sun's disk, computing two functions, which are characteristic of the radiative state of the various layers; then the intensity of radiation of each specified wave-length at each specified layer and in each specified direction, the temperature, and the coefficients of extinction of each layer are readily obtained; and it proves possible to assign limits of the coefficients of absorption and scattering separately. Ultimately, we shall see that the conditions of radiative equilibrium are satisfied.

I. THE MATHEMATICAL LAWS OF THE INTENSITY OF RADIATION TRAVELING THROUGH AN ABSORBING, SCATTERING, AND RADIATING ATMOSPHERE

In the sequel we shall assume that the sun is a great sphere, composed of concentric layers of radiating, absorbing, and scattering matter; the temperature of each layer and its coefficients of absorption and scattering for every wave-length λ being functions of the distance r from the center of the solar globe, and of this distance alone.

In the first place, we have to formulate the equation of the intensity change of a beam during its passage through an infinitesimal layer. We specify the direction of the beam by the cosine of the angle ζ between the beam and the outward normal to the layer, the quantity $\xi = \cos \zeta$ evidently being positive or negative according as the beam is running outward or inward. From reasons of symmetry we conclude that the intensity of the radiation of the wave-length λ must be a function of the two variables r and ξ only. Call this function $J_\lambda(r, \xi)$, and let $\alpha_\lambda(r)$ and $\beta_\lambda(r)$ denote the absorption and scattering coefficients of the layer, and $E_\lambda\ (r)$ the emissive power of an absolutely black body at the temperature prevailing in the layer.

According to well-known theoretical considerations the radiation round a scattering element would be partially polarized and more or less irregularly distributed. If the radiating layers of the sun were constituted like the earth's atmosphere we, of course, should have to pay regard to these results. The entire lack of polarization of the sunlight proves, however, that the analogy does not hold; the scattered radiation seems in fact more likely to be equally, or at least approximately equally, distributed over all directions, if it is of any account at all. Since every element of the photosphere is the convergency point of an infinitude of beams running from all directions and fairly equally distributed over at least one hemisphere, there is at all events no risk in assuming that the scattering works equally in all directions.

Then the energy balance of the beam is given by the equation

$$dJ_\lambda(r,\xi) = -[\alpha_\lambda(r)+\beta_\lambda(r)]J_\lambda(r,\xi)\frac{dr}{\xi}+[\alpha_\lambda(r)\cdot E_\lambda(r) + \beta_\lambda(r)\cdot G_\lambda(r)]\frac{dr}{\xi}, \quad (1)$$

where the "collustrivity" function

$$G_\lambda(r) = \tfrac{1}{2}\int_0^\pi J_\lambda(r,\cos\zeta)\sin\zeta\, d\zeta = \tfrac{1}{2}\int_{-1}^{+1} J_\lambda(r,\xi)d\xi \quad (2)$$

expresses the average intensity of the radiation converging toward an element of the layer from all directions. The left-hand side of the equation (1) signifies the total energy gain of the beam during its transit through the layer; the first term to the right stands for the attenuation by absorption and scattering, and the second term for the increase of intensity by emission and scattering. The equation holds for the radiation inward ($\xi<0$) as well as for the radiation outward ($\xi>0$).

Strictly speaking, the quantity ξ is a function of r, because the angle of incidence against the successive layers slowly varies as the beam traverses distances which cannot be neglected in comparison with the radius of the sun. But since we need not take such

enormous distances into consideration, we are allowed to treat ξ as a constant.

Then the equation (1) can be reduced to a linear differential equation with constant coefficients by means of the variable change

$$m_\lambda = \int_r^R [\alpha_\lambda(r)+\beta_\lambda(r)]dr; \qquad dm_\lambda = -[\alpha_\lambda(r)+\beta_\lambda(r)]dr; \tag{3}$$

where R is the radius of the sun. The new variable m_λ, which is a function of the wave-length, is called the optical mass with regard to the wave-length λ. It is zero at the external limit of the sun's atmosphere and increases inward. J, E, and G have now to be taken as functions of m. In order to avoid too complicated notation we retain the old symbols, e.g., understanding by $J\ (m, \xi)$ the value of $J\ (r, \xi)$ at the layer whose centri distance r corresponds to m by (3). The differential equation is found to be

$$\xi\frac{dJ}{dm} - J = -H, \tag{4}$$

where H denotes the function

$$H_\lambda(m) = \frac{\alpha E+\beta G}{\alpha+\beta} \tag{5}$$

which we shall call the "emissivity" function of the layer m. In (4) m is the independent and J the dependent variable; ξ is a constant parameter and H a function of m. H, J, and m are altogether functions of λ.

The two functions G and H now introduced are the very functions ruling the optical behavior of the medium.

On integrating (4) we obtain

$$J(m, \xi) = J(\mu, \xi)e^{\frac{m-\mu}{\xi}} - e^{\frac{m}{\xi}} \int_\mu^m \frac{H(m)}{\xi} e^{-\frac{m}{\xi}} dm\,, \tag{6}$$

where μ is an arbitrary constant.

According to the direction of the beam we dispose of μ in a different way. If $\xi > 0$ (outward radiation), we put $\mu = \infty$ and obtain the equation

$$\underset{\xi > 0}{J(m, \xi)} = e^{\frac{m}{\xi}} \int_m^\infty \frac{H(m)}{\xi} e^{-\frac{m}{\xi}} dm \, ; \tag{7}$$

if $\xi < 0$ (inward radiation), we put $\mu = 0$ and obtain (because $J[0, \xi] = 0$ for this radiation):

$$\underset{\xi < 0}{J(m, \xi)} = -e^{\frac{m}{\xi}} \int_0^m \frac{H(m)}{\xi} e^{-\frac{m}{\xi}} dm \, . \tag{8}$$

These are the fundamental laws of the radiation in the photospheric layers.

2. THE SOLUTION OF THE INTEGRAL EQUATIONS

The function $J(0, \xi)$ may be determined experimentally by measuring the intensity distribution over the sun's disk. The problem which we are now going to solve, to begin with from a purely theoretical point of view, is to determine the functions $H(m)$ and $J(m, \xi)$ for any values of m and ξ, the function $J(0, \xi)$ being known.

The solution is most easily achieved in an indirect way, starting from the converse problem to compute $J(0, \xi)$ when $H(m)$ is known. We assume the function $H(m)$ to be an empirical function, tabulated or graphically represented, but unknown as to its analytical expression. With a design to found the necessary computations on a simple algebraical expression, we begin with approximating $H(m)$ as a polynomial of the Nth degree

$$H(m) = \sum_{i=0}^{N} a_i m^i \, . \tag{9}$$

From the physical sense of $H(m)$ it follows that $H(m)$ is a continuous function of m with a restricted number of maxima and minima. The approximation will, therefore, probably prove very good even for a relatively small N and will naturally grow better the higher we

choose N. For the theoretical part we need not specify N at all.

Then, substituting (9) into (7) and (8) we obtain

$$\underset{\xi>0}{J(m,\xi)}=e^{\frac{m}{\xi}}\sum_{i=0}^{N}a_i\int_m^\infty\frac{m^ie^{-\frac{m}{\xi}}}{\xi}dm=\sum_{i=0}^{N}\frac{m^i}{i!}\sum_{k=0}^{N-i}(i+k)!\,a_{i+k}\xi^k\,; \tag{10}$$

$$\underset{\xi<0}{J(m,\xi)}=-e^{\frac{m}{\xi}}\sum_{i=0}^{N}a_i\int_0^m\frac{m^ie^{-\frac{m}{\xi}}}{\xi}\,dm=\sum_{i=0}^{N}\frac{m^i}{i!}\sum_{k=0}^{N-i}(i+k)!\,a_{i+k}\xi^k$$
$$-e^{\frac{m}{\xi}}\sum_{k=0}^{N}k!\,a_k\xi^k\,; \tag{11}$$

where $0!$ is to denote 1; the terms are arranged according to ascending powers of m.

From these equations the radiation at the external limit of the solar atmosphere is found by putting $m=0$:

$$\underset{\xi>0}{J(0,\xi)}=\sum_{k=0}^{N}k!\,a_k\xi^k\,; \tag{12}$$

$$\underset{\xi<0}{J(0,\xi)}=0. \tag{13}$$

The latter equation expresses only that there is no radiation toward the sun from the surrounding universe. The function (12) is the more interesting because of its peculiar analytical form.

In fact, if the functions $H(m)$ and $J(0,\xi)$ are expanded in series of m and ξ, respectively:

$$H(m)=\sum_{i=0}^{N}a_im^i\,; \tag{9}$$

$$J(0,\xi)=\sum_{i=0}^{N}b_i\xi^i\,; \tag{14}$$

the following simple relation holds between the coefficients a_i and b_i:

$$a_i = \frac{b_i}{i!}. \tag{15}$$

This connection is the essential basis of the method proposed in this paper. It obviously implies the solution of our original problem.

For, having observed the intensity distribution over the sun's disk, we know the function $J(0, \xi)$ empirically. The values observed can always be represented by means of an expansion like (14). According to a theorem due to Weierstrass we are able to reduce the difference between the function J and the polynomial (14) below any limit given in advance by choosing N great enough. Having found such a polynomial, we obtain by (15):

$$H(m) = \sum_{i=0}^{N} \frac{b_i}{i!} m^i ; \tag{16}$$

and according to (10) and (11)

$$\underset{\xi > 0}{J(m, \xi)} = \sum_{i=0}^{N} \frac{m^i}{i!} \sum_{k=0}^{N-i} b_{i+k} \xi^k ; \tag{17}$$

$$\underset{\xi < 0}{J(m, \xi)} = \sum_{i=0}^{N} \frac{m^i}{i!} \sum_{k=0}^{N-i} b_{i+k} \xi^k - e^{\frac{m}{\xi}} \sum_{k=0}^{N} b_k \xi^k . \tag{18}$$

Finally the function $G(m)$ can be calculated from (2), (17), and (18):

$$G(m) = \tfrac{1}{2} \int_{-1}^{+1} \sum_{i=0}^{N} \frac{m^i}{i!} \sum_{k=0}^{N-i} b_{i+k} \xi^k d\xi - \tfrac{1}{2} \int_{-1}^{0} e^{\frac{m}{\xi}} \sum_{k=0}^{N} b_k \xi^k d\xi . \tag{19}$$

The first set of terms can immediately be integrated; it gives

$$\tfrac{1}{2} \sum_{i=0}^{N} \frac{m^i}{i!} \sum_{k=0}^{N-i} \frac{1+(-1)^k}{k+1} b_{i+k} . \tag{20}$$

The second set contains the integrals

$$K(k)=\int_{-1}^{0} e^{\frac{m}{\xi}}\xi^k d\xi . \qquad (k=0, 1, \ldots . N).$$

Making the transformation

$$u=-\frac{m}{\xi} ; \qquad du=\frac{m}{\xi^2}d\xi ;$$

we find

$$K(k)=(-1)^k m^{k+1}\int_m^\infty \frac{e^{-u}}{u^{k+2}}du .$$

These integrals again can be reduced on the logarithm-integral by the general formula

$$\int_a^b \frac{e^{-u}}{u^n}du=\sum_{i=1}^{n-1}\frac{(-1)^i(n-1-i)!}{(n-1)!}\left[\frac{e^{-u}}{u^{n-i}}\right]_a^b+\frac{(-1)^{n-1}}{(n-1)!}\int_a^b\frac{e^{-u}}{u}du .$$

Consequently

$$K(k)=\frac{1}{(k+1)!}\left\{e^{-m}\sum_{i=0}^{k}(-1)^{k-i}(k-i)!\,m^i-m^{k+1}\int_m^\infty\frac{e^{-u}}{u}du\right\}. \qquad (21)$$

Introducing (20) and (21) into (19) we obtain in conclusion

$$G(m)=\tfrac{1}{2}\sum_{i=0}^{N}\frac{m^i}{i!}\sum_{k=0}^{N-i}\frac{1+(-1)^k}{k+1}\,b_{i+k}-\frac{e^{-m}}{2}\sum_{i=0}^{N}m^i\sum_{k=0}^{N-i}\frac{(-1)^k k!}{(i+k+1)!}\,b_{i+k}$$
$$+\tfrac{1}{2}\int_m^\infty\frac{e^{-u}}{u}du\cdot\left\{\sum_{k=0}^{N}\frac{b_k m^{k+1}}{(k+1)!}\right\}. \qquad (22)$$

We have thus shown how to determine the emissivity function $H(m)$, the collustrivity function $G(m)$, and the intensity $J(m, \xi)$ of the radiation in each specified direction and at each specified layer. It remains to show that the solution obtained is unique.

Otherwise there would exist at least two different functions H and H^* both satisfying the equation (7) in the special case giving the radiation outside the solar atmosphere:

$$\underset{\xi>0}{J(0, \xi)}=\int_0^\infty \frac{e^{-\frac{m}{\xi}}}{\xi} H(m)dm=\int_0^\infty \frac{e^{-\frac{m}{\xi}}}{\xi} H^*(m)dm\,.$$

But, as we shall see, these equations are inconsistent, unless $H=H^*$.

On subtracting the second equation from the first, we find

$$0=\int_0^\infty \frac{e^{-\frac{m}{\xi}}}{\xi}(H-H^*)dm. \tag{23}$$

But according to the theorem of Weierstrass quoted above it is always possible to find two polynomials

$$P(m)=\sum_{i=0}^{N} a_i m^i$$

and

$$P^*(m)=\sum_{i=0}^{N} a_i^* m^i$$

such as to render each of the differences

$$|H(m)-P(m)|$$

and

$$|H^*(m)-P^*(m)|$$

less than $\epsilon/4$ for every m within a finite interval $0 \leqq m \leqq M$, ϵ being any positive quantity given in advance. Hence

$$\left|\int_0^M \frac{e^{-\frac{m}{\xi}}}{\xi}(P-P^*)dm\right|+\int_M^\infty \frac{e^{-\frac{m}{\xi}}}{\xi}(H-H^*)dm<\frac{\epsilon}{2}\,.$$

Moreover, by (23)

$$\left|\int_M^\infty \frac{e^{-\frac{m}{\xi}}}{\xi}(H-H^*)dm\right|<\frac{\epsilon}{4}\,,$$

provided M is taken sufficiently great. Consequently

$$\left| \int_0^M \frac{e^{-\frac{m}{\xi}}}{\xi} (P-P^*)dm \right| < \frac{3\epsilon}{4} ;$$

or on integration

$$\left| \sum_{i=0}^{N} i!\,(a_i - a_i^*)\xi^i \left\{ 1 - e^{-\frac{M}{\xi}} \sum_{k=0}^{i} \frac{1}{k!} \left(\frac{M}{\xi}\right)^k \right\} \right| < \frac{3\epsilon}{4} .$$

Now in virtue of a theorem due to Abel the module of the series

$$\sum_{i=0}^{N} a_i u_i$$

never exceeds the quantity

$$a_0 \left| \sum_{i=0}^{N} u_i \right| ,$$

provided the quantities a_i are positive and decreasing. The left-hand side of the inequality above is therefore less than

$$(1 - e^{-\frac{M}{\xi}}) \left| \sum_{i=0}^{N} i!\,(a_i - a_i^*)\xi^i \right| ;$$

but M is a large number and therefore $e^{-\frac{M}{\xi}}$ is certainly less than $\frac{1}{4}$; and in conclusion

$$\left| \sum_{i=0}^{N} i!\,(a_i - a_i^*)\xi^i \right| < \epsilon . \tag{24}$$

A well-known theorem now states that if a power series is identically zero within a finite interval, each separate coefficient must be zero; hence:

$$\lim_{\epsilon=0} | a_i - a_i^* | = 0 \qquad \therefore \quad \lim_{N=\infty} P^*(m) = \lim_{N=\infty} P(m)$$

or finally

$$H^*(m) = H(m)$$

which establishes the result stated above.

The function $H(m)$ and in consequence the functions $G(m)$ and $J(m, \xi)$ can thus be determined without ambiguity by the aid of the method explained in this paragraph. Before pursuing the discussion of these functions in their bearings on the optics of the photosphere we are, however, obliged to work out the computations numerically.

3. NUMERICAL COMPUTATION OF THE EMISSIVITY AND COLLUSTRIVITY FUNCTIONS AND THE INTENSITY OF THE RADIATION

The most complete set of observations concerning the intensity distribution along the diameter of the sun's disk is furnished by the careful and laborious investigations of Abbot, Fowle, and Aldrich[1] of the Smithsonian Institution at Mount Wilson. Their researches will be the base of the following computations.

From the point of view of this paper the principal desiderata as to the observations are the following:

1. The accuracy must be sufficiently high to enable us to evaluate a satisfactory number of coefficients in the expansion (14). The observations quoted admit of fixing the four first coefficients b_0 to b_3.

2. The observations must be continued to the immediate vicinity of the sun's limb. The Smithsonian material only extends to 0.95 of the radius of the disk, corresponding to the interval $1 \geqq \xi \geqq 0.3123$; this fact necessitates some disadvantageous extrapolations.

3. The material must be homogeneous. There are indications of a slight variability of the intensity distribution from day to day and from year to year. Therefore each day of observation ought to be represented in the mean for each wave-length and not only in a few of the means. This condition is not fulfilled, the number of

[1] C. G. Abbot, F. E. Fowle, and L. B. Aldrich, *Annals of the Astrophysical Observatory of the Smithsonian Institution*, 3, 157, 1913.

observations being, e.g., for $\lambda = 0\overset{\mu}{.}386$ only three, but for $\lambda = 0\overset{\mu}{.}501$ more than a hundred.

4. The wave-length to be observed must be selected in a suitable manner so that the Fraunhofer lines intervene as little as possible: for even if the disturbances of the intensity distribution by the lines perhaps are negligible, owing to the remarkable constancy of the contrast between the lines and their spectral surroundings all over the sun's disk, the absolute amount of the radiation is naturally affected by them.

In a later publication of the Smithsonian Institution[1] there are to be found some results of more recent investigations; these seem to be more adequate to our requirements. Although the main features of our results most likely should be unaltered, I regret that I cannot find time to renew the computations utilizing these observations, since they illustrate another part of the sun-spot period.

As stated above, the calculations are to be commenced by finding the polynomial approximations. For this purpose I have used the method of developing intermediately in series of Legendrian polynomials.[2] The merit of this method lies in the fact that every following approximation is obtained simply by adding one term without altering the coefficients of the preceding ones.

The coefficients b obtained in this manner are included in Table I. They are determined so as to suit the graphical representation of the function $J(0, \xi)$ as well as possible within the interval $0.3 \leqq \xi \leqq 1$. (Evidently we have $\xi = \sqrt{1-\rho^2}$, where ρ is the distance of an element from the center of the disk as a fraction of the radius.)

The first line in the table refers to observations made by Schwarzschild and Villiger.[3] The units are the same as used in the original papers, the intensity at the center being equated to one for each wave-length apart. The remaining differences between the observed data and the polynomial (14) hardly present any systematic features.

[1] C. G. Abbot, F. E. Fowle, and L. B. Aldrich, *Smithsonian Miscellaneous Collections*, 66, No. 5, 1916.

[2] H. v. Sanden, *Praktische Analysis*, p. 113, Leipzig, 1914.

[3] K. Schwarzschild und W. Villiger, *Astrophysical Journal*, 23, 284, 1906.

From the data given in Table I the emissivity function $H(m)$ and the collustrivity function $G(m)$ can immediately be found in virtue of the relations (16) and (22). The results are collected in

TABLE I

THE COEFFICIENTS OF THE POLYNOMIAL $J(0, \xi)$

λ	b_0	b_1	b_2	b_3
0.323 μ	+0.1269	+0.7625	+0.2109	−0.1283
0.386	+0.1699	+0.7553	+0.1244	−0.0525
0.433	+0.2126	+0.7002	+0.1903	−0.1108
0.456	+0.1786	+0.9939	−0.2026	+0.0292
0.481	+0.2192	+0.9638	−0.2200	+0.0350
0.501	+0.2444	+0.9507	−0.2610	+0.0641
0.534	+0.3119	+0.7694	−0.0312	−0.0525
0.604	+0.3288	+0.9794	−0.4819	+0.1749
0.670	+0.3566	+1.0359	−0.6133	+0.2216
0.699	+0.3716	+1.0215	−0.6146	+0.2216
0.866	+0.4563	+0.9499	−0.6450	+0.2391
1.031	+0.5163	+0.8220	−0.4984	+0.1633
1.225	+0.5572	+0.7756	−0.4972	+0.1633
1.655	+0.6491	+0.6726	−0.5019	+0.1808
2.097	+0.6851	+0.6322	−0.5139	+0.1983

the subsequent tables. The second line of the head of the tables contains the corresponding values of a new variable p defined by the relation

$$p=\frac{m}{m+1};$$

this variable is preferable to m for graphical representation, because it runs through the finite interval 0 to 1 at the same time that m runs from 0 to ∞. The last column $m=4$ is naturally not very exact; only the e^{-4}th part, i.e., 0.018, of a beam originating in that layer and going straight outward reaches the face.

Finally the radiation at various layers and in various directions can be calculated from (17) and (18). To give a survey I have indicated the results graphically in the figures below. The intensity in a specified direction is proportional to the radius vector in that same direction. The inmost curve of each figure illustrates the radiation at the external limit of the absorbing layers ($m=0$); then there follow consecutively the curves $m=0.429$, $m=1$, and

$m=2.333$. Of course it must be remembered that m is a function of the wave-length. In section 5 we shall examine the relation between the optical masses with respect to different wave-lengths.

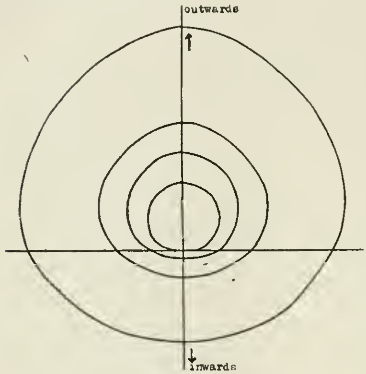

FIG. 1

$\lambda=0^{\mu}.386$

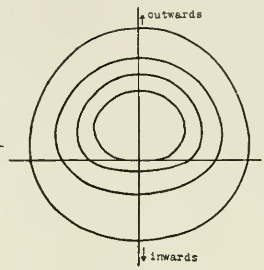

FIG. 2

$\lambda=0^{\mu}.670$

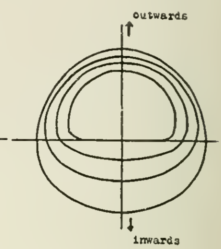

FIG. 3

$\lambda=1^{\mu}.655$

The intensity as a function of the direction of the beams

4. SCATTERING, OR REAL ABSORPTION?

As mentioned in the introduction there are very different opinions about the importance of scattering in solar optics. By the aid of the results obtained in the preceding paragraph we are now going to examine the various possibilities.

We begin with the extreme assumption that there is a vast atmosphere, scattering, but not absorbing, surrounding the seat of emission in such powerful layers as to cut off all direct radiation from within practically completely.

The coefficient of real absorption then must be zero throughout the atmosphere, and accordingly, by (5)

$$H(m)=G(m). \tag{25}$$

Moreover, by (2), (7), and (8):

$$G(m)=\tfrac{1}{2}\int_0^{+1} e^{\frac{m}{\xi}}d\xi\int_m^{\infty}\frac{H(m)}{\xi}e^{-\frac{m}{\xi}}dm-\tfrac{1}{2}\int_{-1}^{0} e^{\frac{m}{\xi}}d\xi\int_0^{m}\frac{H(m)}{\xi}e^{-\frac{m}{\xi}}dm\ ;$$

or applying a simple transformation due to King and Schwarzschild:

$$G(m) = \tfrac{1}{2} \int_0^\infty H(\mu) d\mu \int_{|m-\mu|}^\infty \frac{e^{-u}}{u} du \; ;$$

or finally by (25)

$$G(m) = \tfrac{1}{2} \int_0^\infty G(\mu) d\mu \int_{|m-\mu|}^\infty \frac{e^{-u}}{u} du \; ; \qquad (26)$$

where μ and u are integration variables. Since the equation (26) is an integral equation of Fredholm's second type, it admits of a unique solution; and since the wave-length does not occur in (26), this solution must be the same function of the optical mass for all

TABLE II

THE EMISSIVITY FUNCTION, $H(m)$

λ	$m=0$	0.111	0.25	0.429	0.667	1	1.5	2.333	4
	$p=0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$0^{\mu}.323$...	0.127	0.213	0.324	0.471	0.676	0.974	1.436	2.209	3.495
0.386...	0.170	0.255	0.363	0.504	0.698	0.979	1.413	2.165	3.623
0.433...	0.213	0.292	0.393	0.529	0.716	0.990	1.415	2.129	3.352
0.456...	0.179	0.288	0.421	0.586	0.798	1.076	1.458	2.008	2.847
0.481...	0.219	0.325	0.453	0.613	0.815	1.079	1.437	1.943	2.686
0.501...	0.244	0.348	0.474	0.629	0.823	1.076	1.413	1.888	2.644
0.534...	0.312	0.397	0.503	0.638	0.815	1.057	1.401	1.910	2.577
0.604...	0.329	0.435	0.559	0.707	0.883	1.096	1.354	1.673	2.259
0.670...	0.357	0.468	0.597	0.747	0.922	1.123	1.345	1.573	1.955
0.699...	0.372	0.481	0.608	0.756	0.927	1.123	1.337	1.551	1.902
0.866...	0.456	0.558	0.674	0.807	0.958	1.124	1.290	1.424	1.650
1.031...	0.516	0.605	0.707	0.825	0.962	1.116	1.280	1.423	1.558
1.225...	0.557	0.640	0.736	0.846	0.972	1.111	1.253	1.359	1.423
1.655...	0.649	0.721	0.802	0.894	0.995	1.101	1.195	1.234	1.250
2.097...	0.685	0.752	0.828	0.911	1.002	1.093	1.167	1.182	1.220

wave-lengths. Hence, summing up the contents of (25) and (26): *If the extinction is due to pure scattering all the way through the atmosphere, the emissivity and collustrivity functions must be equal and independent of the wave-length.*

Tables II and III demonstrate that this is not the case. The assumption is thereby proved to be erroneous.

Next, suppose that the interior layers are absorbing as well as scattering, but the external layers (say $0<m<m^*$) exclusively scattering. Then the equation (25) obviously subsists within the same interval $0<m<m^*$; but (26) naturally not. Hence *if the sun is enveloped by an exclusively scattering layer, the emissivity and collustrivity functions must coincide within the corresponding interval of m.*

If the scattering layer were equal to the earth's atmosphere as to its optical properties, the interval in question would be around 0.15 in the visible spectrum and 1.25 in the extreme ultra-violet.

TABLE III

THE COLLUSTRIVITY FUNCTION $G(m)$

λ	$m=0$	0.111	0.25	0.429	0.667	1	1.5	2.333	4
	$p=0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$0^{\mu}.323$...	0.273	0.346	0.438	0.566	0.748	1.020	1.452	2.183	3.396
0.386...	0.288	0.365	0.459	0.585	0.762	1.024	1.440	2.165	3.598
0.433...	0.299	0.380	0.474	0.598	0.771	1.027	1.428	2.109	3.270
0.456...	0.308	0.393	0.497	0.633	0.816	1.069	1.432	1.973	2.818
0.481...	0.318	0.408	0.512	0.645	0.822	1.064	1.406	1.906	2.662
0.501...	0.324	0.416	0.520	0.651	0.824	1.057	1.383	1.860	2.643
0.534...	0.337	0.431	0.532	0.657	0.821	1.048	1.375	1.864	2.500
0.604...	0.351	0.452	0.559	0.688	0.847	1.050	1.311	1.659	2.331
0.670...	0.363	0.469	0.578	0.708	0.865	1.057	1.286	1.554	2.050
0.699...	0.367	0.474	0.583	0.712	0.867	1.055	1.277	1.531	1.997
0.866...	0.388	0.502	0.612	0.735	0.878	1.043	1.224	1.405	1.752
1.031...	0.401	0.519	0.627	0.745	0.881	1.039	1.215	1.392	1.612
1.225...	0.410	0.531	0.638	0.753	0.883	1.028	1.185	1.327	1.477
1.655...	0.432	0.560	0.666	0.774	0.889	1.011	1.129	1.213	1.329
2.097...	0.440	0.571	0.676	0.781	0.890	1.001	1.104	1.169	1.315

Once more referring to Tables II and III we find a marked discrepancy between the stated conclusion and the experimental data, especially in the extreme ultra-violet region. The underlying assumption must therefore be false again.

We now proceed to apply our results to the hypothesis of pure absorption, assuming that the whole of the dissipating energy is converted into heat and that the radiation is a pure temperature radiation. On this hypothesis we have by (5)

$$H(m)=E(m); \tag{27}$$

i.e., *if the scattering is negligible, the emissivity function is equal to the emissive power of an absolutely black body of the temperature of the layer.*

In virtue of Planck's law the emissive power of an absolutely black body is a known function of the temperature; we have in a frequent notation

$$E=\frac{2c^2h}{\lambda^5}\cdot\frac{1}{e^{\frac{ch}{\kappa T\lambda}}-1},$$

or, using the C.G.S. units:

$$E=\frac{1.177\cdot 10^{-5}}{\lambda^5\left(e^{\frac{1.43}{\lambda T}}-1\right)}. \tag{28}$$

By means of this equation the absolute temperature of each layer can be computed as a function of the overlying optical mass.

If our assumption concerning the nature of the extinction is true, the temperature of the outermost layer $m=0$ must be one and the same, irrespective of the wave-length used in the computations. We shall utilize this circumstance to test the assumption.

To begin with, we have to reduce $H(0)$ to C.G.S. units. The units employed above were defined as the radiation issued within the unit solid angle from a body which radiates like the central parts of the sun's disk. Now the energy-curve of the solar radiation outside the earth's atmosphere is determined at Mount Wilson[1] in some arbitrary unit; on interpolation we have obtained the figures given in the second column of Table IV. As the solar constant is accurately measured (1.932 cal. per cm² and min. $=1.348\cdot 10^6$ erg per cm² and sec), we can readily put them into absolute units. The numbers thus obtained would give the absolute intensity of the radiation from the entire solar disk at the wave-lengths specified in the table. Since the solid angle occupied by the sun is $6.796\cdot 10^{-5}$ (assuming the sun's semidiameter to be 959ʺ.7), the radiation from a body which emits like the sun but occupies the unit solid angle is found by dividing by that quantity. In order to account for the fact that the measurements bear upon the joint radiation of the

[1] *Annals of the Smithsonian Astrophysical Observatory*, 3, 197, 1913.

entire solar disk but the units of the preceding paragraph upon the radiation at the center, we have further to divide by the quantity

$$\frac{2\pi\int_0^1 J \cdot \rho d\rho}{\pi \cdot 1^2} = 2\int_0^1 J(0, \xi).\ \xi d\xi = 2\sum_{i=0}^{N} \frac{b_i}{i+2}\ ;$$

$\rho = \sqrt{1-\xi^2}$ denoting the distance of a zonal element from the center of the disk. By this procedure the units will be expressed in the C.G.S. system; they are given in the third column of Table IV and are there denoted U_λ. Finally, multiplying the value $H(0)$ obtained above with the corresponding U_λ, we get $H(0)$ expressed in ergs per cm² and sec, as tabulated in the fourth column of Table IV.

TABLE IV

REDUCTION OF $H(0)$ TO C.G.S. UNITS

λ	C	$U_\lambda \cdot 10^{-14}$	$H(0) \cdot 10^{-13}$ (abs.)	λ	C	$U_\lambda \cdot 10^{-14}$	$H(0) \cdot 10^{-13}$ (abs.)
0.μ323....	1180	1.014	1.29	0.μ670....	4060	2.887	10.29
0.386....	3590	2.962	5.03	0.699....	3660	2.589	9.62
0.433....	5490	4.437	9.43	0.866....	2270	1.556	7.10
0.456....	6140	4.824	8.62	1.031....	1540	1.011	5.34
0.481....	6200	4.780	10.48	1.225....	1030	0.686	3.82
0.501....	6060	4.624	11.30	1.655....	480	0.311	2.02
0.534....	5780	4.326	13.50	2.097....	190	0.124	0.85
0.604....	4990	3.632	11.94				

The temperature of the layer $m=0$ is now readily found by (28). The results are given in the tabular summary below. The departures from the mean cannot be said to be small, but I think they might be entirely accounted for by the inaccuracy of the experimental material. The precision of the observations is not sufficiently great to enable us to evaluate higher terms than the fourth in the expansion of J (0, ξ). Therefore, if the outermost layers of the sun only give rise to a short interval of m with feeble radiation, they are not revealed by the computations; so it is to be expected that the apparent limiting temperatures will come out higher, the lower the absorption coefficient is, and vice versa. We shall now determine the values of the absorption coefficients in the sequel;

utilizing these values in anticipation we give below the graphical run of the absorption coefficients of the outermost layer together with the difference between the mean and the separate values of Table V.

TABLE V

THE ABSOLUTE TEMPERATURE OF THE LAYER $m=0$

λ	T	λ	T
$0^{\mu}.323$	4360°	$0^{\mu}.670$	4800°
0.386	4680	0.699	4750
0.433	4920	0.866	4640
0.456	4790	1.031	4640
0.481	4890	1.225	4670
0.501	4920	1.655	4960
0.534	5040	2.097	4580
0.604	4910		

Mean .... 4770° abs.

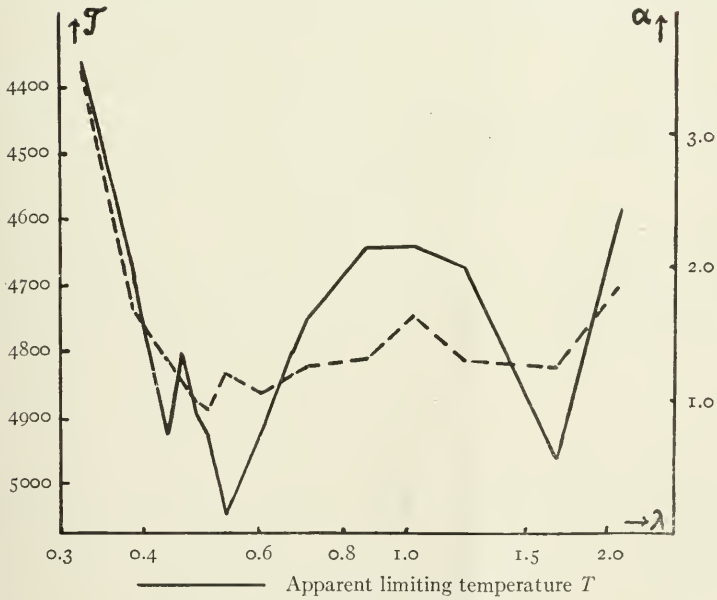

FIG. 4.—Apparent limiting temperature and absorption coefficients

The real temperature of the utmost layer which can make itself perceptible by its absorption and radiation is most correctly indicated by the lowest number of the table. Because of the Fraun-

hofer lines that number should be raised a little.[1] The most likely value of the limiting temperature therefore appears to be around 4500° abs.

It is evident that no real objection against the postulated conception of the extinction can be made from the discussion above. But there remains yet an assumption to be examined, namely, the one that both the coefficients of absorption and scattering are different from zero.

This being the case, we should have by (5)

$$E=H\left(1-\frac{\beta}{\alpha}\frac{G-H}{H}\right). \tag{29}$$

The scattering must not render the expression within the parenthesis negative, which enables us to find a superior limit of β:

$$\beta\leqq\frac{\alpha H}{G-H}$$

or, in particular, for the wave-length $0\overset{\mu}{.}323$:

$$\beta\leqq 0.867\,\alpha\,. \tag{30}$$

Having thus found a superior limit for the shortest wave-length, we can readily assign a superior limit for each of the longer wave-lengths too. According to Lord Rayleigh's law

$$\beta=\frac{32}{3}\pi^3\frac{(n-1)^2}{N\lambda^4}, \tag{31}$$

the selective properties of scattering are in the main due to the factor λ^4 of the denominator. It may be that the formula is not exactly valid; the factor λ^4 is safe in any case, since it can be deduced from pure dimensional considerations.[1] The quantity

$$\beta_\lambda=\beta_{0.323}\cdot\frac{0.323^4}{\lambda^4} \tag{32}$$

[1] According to Rowlands' spectral atlas, the absorption by the lines in the region about $0\overset{\mu}{.}323$ does not seem to be very much more extensive than in the visible region.

[1] Lord Rayleigh (J. W. Strutt), *Philosophical Magazine* (5), 41, 110, 1871.

is therefore a superior limit of the scattering coefficient of the wave-length λ. If the whole of (31) were exact, the limit would be still smaller.

Thus, the extinction in the sun can be caused either by pure absorption, or by absorption together with scattering within the indicated limits. In any case the scattering is negligible within the red and infra-red parts; the question whether it is also negligible within the remaining parts of the visible spectrum and the ultra-violet must be left unsettled if we wish to avoid entering into somewhat hypothetical considerations as to the state and optical behavior of the layers.

To get, nevertheless, at least an idea of the probable value of the scattering coefficients, notice in the first place that the limit above is very wide, so wide in fact that it even admits the temperature of the outmost layer to be zero on the absolute scale; I need not point out that this temperature and any low temperature is absurd in the overwhelming ardor of the radiation from within;[1] in particular, venturing the assumption that the layer is in radiative equilibrium, we should be able to reduce the maximum of the scattering to a small fraction of the foregoing amount; most likely the scattering is negligible all through the spectrum.

In one of his celebrated memoirs on solar physics Schwarzschild[2] appears to be disposed in favor of the scattering theory of the solar atmosphere; his opinion is based on a study of the Fraunhofer lines. There is clearly no contradiction between his result and mine; for the layers with which we have been concerned above are those giving rise to the continuous spectrum, i.e., the photospheric layers, while Schwarzschild's result bears upon the optics of the reversing layer with its discontinuous absorption and radiation; according to Julius' measurements during the annular eclipse of 1912 the total amount of radiation from the entire solar atmosphere outside the photosphere is 0.001 at most of the sun's total emission; such small amounts are not discovered by our computations.

[1] From a spectroscopic determination the temperature of the reversing layer is found to be 4300° abs. R. T. Birge, *Astrophysical Journal*, 54, 273, 1922.

[2] K. Schwarzschild, *op. cit.*, p. 1200, 1914.

5. THE TEMPERATURE OF THE PHOTOSPHERIC LAYERS

The temperature of any layer can be computed from the equation

$$E=H\left(1-\frac{\beta}{\alpha}\frac{G-H}{H}\right), \tag{29}$$

provided either β/α is known or $\frac{G-H}{H}$ is a small quantity. From Tables II and III we find that the latter is the case for the deep layers, at least as a good approximation, so we have simply

$$E=H. \tag{27}$$

Inversely, having computed E for a specified temperature by means of Planck's formula, we know by (27) the function $H(m)$ for the layer where that temperature exists, and can then read off

TABLE VI

Optical Mass and Temperature

λ	T						
	5100°	5500°	5900°	6300°	6700°	7100°	7500°
0.μ323	0.53	1.09	1.90	3.20	....	....	....
0.386	0.21	0.48	0.87	1.36	1.97	2.69	3.54
0.433	0.09	0.30	0.59	0.92	1.32	1.79	2.36
0.456	0.08	0.25	0.46	0.74	1.11	1.59	2.19
0.481	0.07	0.23	0.44	0.72	1.09	1.56	2.16
0.501	0.08	0.23	0.45	0.74	1.11	1.59	2.19
0.534	0.03	0.22	0.48	0.78	1.17	1.61	2.15
0.604	0.07	0.24	0.47	0.78	1.22	1.66	2.69
0.670	0.11	0.30	0.54	0.87	1.38	2.36	3.74
0.699	0.13	0.33	0.59	0.95	1.55	2.66	....
0.866	0.22	0.43	0.76	1.24	2.45	....	....
1.031	0.23	0.49	0.80	1.27	2.28	....	....
1.225	0.22	0.43	0.72	1.13	1.99	....	....
1.655	0.05	0.25	0.49	0.80	1.26	....	....
2.097	0.27	0.54	0.93	....	....	....	....

the corresponding values of m, on a set of diagrams representing Table II. In this way Table VI has been deduced. From this table we can obtain the correspondence between the optical masses with regard to different wave-lengths.

By the aid of the table we can further trace a kind of isothermals, which are very instructive and make the optical conditions easy to

survey. In Figure 5 λ and m are co-ordinates, and the curves connect points with equal temperature. The scale of wave-lengths is a logarithmic one. From the figure it is obvious that an appreciable

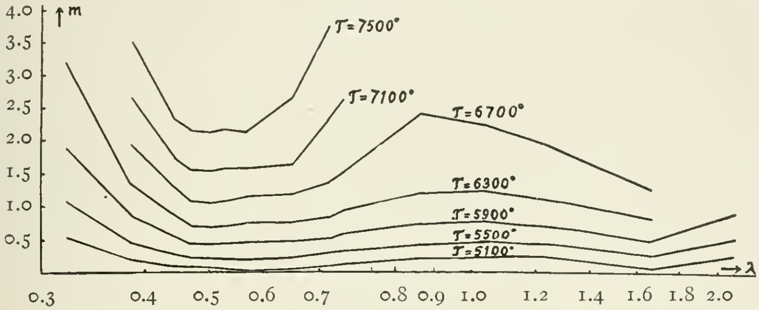

FIG. 5.—Isothermals: $T=5100$, 5500, 5900, 6300, 6700, 7100, and 7500

part of the green and the blue light comes from deep and hot layers, while the radiation of the ultra-violet and part of the infra-red spectrum in the main is due to cooler external layers.

6. THE EXTINCTION COEFFICIENTS

The optical mass was defined above by the differential relation

$$dm_\lambda = -(\alpha_\lambda + \beta_\lambda)dr = -\gamma_\lambda dr\,, \tag{3}$$

where the coefficient γ—the sum of the coefficients of absorption and scattering—might be called the coefficient of extinction. The first differences of Table VI measure by (3) the integral

$$m(T_2) - m(T_1) = \int_{T_2}^{T_1} \gamma dr\,,$$

taken between the layers indicated by their temperatures T_1 and T_2.

Let l_{12} be the distance between the layers T_1 and T_2. We have then approximately

$$\int_{T_2}^{T_1} \gamma dr = l_{12} \cdot \gamma \left\{ \frac{T_1 + T_2}{2} \right\}\,,$$

or

$$m(T_2)-m(T_1)=l_{12}\cdot\gamma\left\{\frac{T_1+T_2}{2}\right\},$$

where $\gamma\left\{\frac{T_1+T_2}{2}\right\}$ denotes the extinction coefficient at the layer $\frac{T_1+T_2}{2}$. From this equation γ may be computed for each of the layers $T=5300$, 5700, 6100, 6500, 6900, and 7300°; however, as we do not know l_{12}, only in relative numbers. Table VII contains the ratio of the extinction coefficient of the wave-length λ to that of the wave-length 0.481 μ.

TABLE VII

EXTINCTION-COEFFICIENTS

λ	T					
	5300°	5700°	6100°	6500°	6900°	7300°
0.323 μ	3.50	3.86	4.64	....	....	....
0.386	1.69	1.86	1.75	1.65	1.53	1.42
0.433	1.31	1.38	1.18	1.08	1.00	0.95
0.456	1.06	1.00	1.00	1.00	1.02	1.00
0.481	1.00	1.00	1.00	1.00	1.00	1.00
0.501	0.94	1.05	1.04	1.00	1.02	1.00
0.534	1.19	1.24	1.07	1.05	0.94	0.90
0.604	1.06	1.10	1.11	1.19	0.94	1.72
0.670	1.19	1.14	1.18	1.38	2.09	2.30
0.699	1.25	1.24	1.29	1.62	2.36	....
0.866	1.31	1.57	1.71	3.27	....	....
1.031	1.63	1.48	1.68	2.73	....	....
1.225	1.31	1.38	1.46	2.32	....	....
1.655	1.25	1.14	1.11	1.24	....	....
2.097	1.86	....	....	....	....	....

If the extinction is entirely due to real absorption, the data of Table VII naturally denote absorption coefficients. Otherwise, they are maximal values of the absorption coefficients. Assuming that the quotients of the extinction coefficients of the outmost layer are equal to those of the layer $T=5300°$ we can also give minimal values for that layer and at the same time maximal values of its scattering coefficients. From (30) and (32) we obtain the results contained in Table VIII. The extinction coefficient of $\lambda=0.481\,\mu$ is still retained as unit. I repeat that the actual values of β/α must

be much lower than those of the table; for if these were correct, we should obtain quite incongruous values of the limiting temperature.

TABLE VIII

Minimum of Absorption and Maximum of Scattering

λ	α min.	β max.	λ	α min.	β max.
$0^{\mu}.323$	1.87	1.63	$0^{\mu}.670$	1.10	0.09
0.386	0.89	0.80	0.699	1.18	0.07
0.433	0.81	0.50	0.866	1.28	0.03
0.456	0.65	0.41	1.031	1.62	0.01
0.481	0.67	0.33	1.225	1.30	0.01
0.501	0.66	0.28	1.655	1.25	0.00
0.534	0.97	0.22	2.097	1.86	0.00
0.604	0.93	0.13			

7. THE HEAT ECONOMY OF THE PHOTOSPHERIC LAYERS

Several of the papers quoted at the outset start with the assumption that the external layers of the photosphere are in radiative equilibrium, i.e., the energy gain by absorption is equivalent to the energy loss by emission:

$$\int_0^\infty \alpha G d\lambda = \int_0^\infty \alpha E d\lambda \,.$$

We are now able to test this assumption. Provided the extinction is a real absorption all through, we have $\alpha=\gamma$, so we can compute the integrals from Tables III and VII. The ratio

$$q=\left\{\int_0^\infty \alpha(G-E)d\lambda\right\}:\left\{\int_0^\infty \alpha E d\lambda\right\}$$

is evidently an index of the departures from the radiative equilibrium. By graphical integration I have found the following values:

T	q
5300°	−0.019
5700°	−0.025
6100°	−0.037
6500°	−0.034

The differences are most likely entirely caused by inevitable inaccuracies; and so we obtain, as our last conclusion, the verification of the very starting-point of most of the previous investigations.

Luleå, Sweden
September 1922

Minor Contributions and Notes

NOTES ON TWO STARS HAVING VARIABLE BRIGHT LINES

ABSTRACT

Two spectroscopic binaries of class B, 47 ω Orionis and 43 θ^2 Orionis, previously known to have bright lines at Hβ, were observed at the Yerkes Observatory in 1922 and 1923. The *intensity* of the bright lines is found to vary in the course of a few days.

47 ω Orionis ($\alpha = 5^h34^m$, $\delta = 4°4'$, 1900; mag., 4.5; spectral type, B3) has exceedingly wide and diffuse absorption lines, except the K line, which is sharp and narrow. Hγ is fairly good for measurement. This star was found to be a spectroscopic binary by Professor E. B. Frost.[1]

Paul W. Merrill[2] noticed in 1912 on two plates, taken on September 23 and October 14, that Hα was a strong bright line, while Hβ was a well-defined double, bright superposed on absorption.

In a paper presented at the twenty-second meeting of the American Astronomical Society in 1919, C. D. Perrine[3] classifies ω Orionis among the stars that have lost bright Hβ. His data are based on objective-prism photographs taken with the astrographic refractor and a 20° prism (in Córdoba).

In 1921 the star was observed by F. Henroteau[4] at Ottawa. Two plates taken on January 25 show a rather weak emission line, which is certainly single, in the middle of the absorption line at Hβ.

In the *Henry Draper Catalogue* the following remark refers to ω Orionis (No. 37490): "No bright lines are seen on a photograph taken with the 11-inch telescope on January 31, 1906."

Forty-five spectrograms of ω Orionis were obtained with the Bruce Spectrograph of the Yerkes Observatory between October 30, 1922, and March 31, 1923. The first plate on October 30 showed a

[1] *Lick Observatory Bulletin*, **181**, 22, 1910.

[2] *Ibid.*, **237**, 168, 1913.

[3] *Popular Astronomy*, **27**, 90, 1919.

[4] *Publications Dominion Observatory*, Ottawa, **5**, 337.

single bright line at Hβ, while the next plate on November 6 had two bright components of unequal intensity. The later plates confirmed the changes of the intensity of the two components, which occur in rather short intervals of a few days. Some of the plates entirely fail to show the emission. Preliminary measures indicate that the changes in the emission lines are not due to orbital motion of the binary system, for the separation of the two components apparently does not vary, averaging about 240 km/sec. It seems probable that the two components really vary in intensity. Hγ does not show any trace of emission on our plates, but the velocity derived from it differs by about 30–40 km/sec, from that of the K line and *He* 4472 and agrees very well with the velocity determined from the mean of the two bright components of Hβ. This seems to indicate that Hγ is also affected by emission which does not stand out on the background of the continuous spectrum.

The presence of bright Hβ had previously been found by Professor Frost from some of the early plates, numbering eleven and taken between 1903 and 1912. But on none of these plates does the bright component appear so strong as on most of the plates of 1922–1923. Per contra, most of the early plates show a wide absorption line at Hβ without any trace of emission, while the great majority of plates taken this season show two bright components. This indicates that in addition to the rapid changes in the spectrum there has been a slow change in the appearance of Hβ during the past twenty years. Observations of this star will be resumed in the autumn.

43 θ^2 Orionis ($\alpha = 5^h 30^m$, $\delta = -5^\circ 29'$, 1900; mag., 4.9; spectral type, B1). The lines of this star are rather poor for measurement, except K, which is sharp. It was announced as a spectroscopic binary by Frost and Adams[1] in 1904. No bright lines were visible on the plates taken at that time.

In 1919 F. Henroteau[2] found that the broad hydrogen absorption lines were divided by sharp and well-defined emission lines.

In February, 1922, three plates of this star were taken by Professor S. B. Barrett with the Bruce Spectrograph. No emission visible.

[1] *Astrophysical Journal*, **19**, 153, 1904.

[2] *Publications of the Dominion Observatory*, Ottawa, **5**, 19.

In 1923 six plates were taken by Mr. Barrett and myself. Some of them show a fairly strong, single emission line at Hβ, while others, though of good quality, fail to show any bright lines. No emission is visible on any of the other absorption lines. The variation in intensity of bright Hβ and the absence of emission at Hγ and of the nebular lines prove that the observed emission line is not due to the Orion Nebula.

It should be remembered that 43 θ^2 Orionis is not one of the trapezium stars. It is Bond 685 and follows the trapezium about 6^s, south $100''$.

The star will be observed again next season.

OTTO STRUVE

YERKES OBSERVATORY
April 14, 1923

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ON THE DOUBLE STAR 9 ARGUS

By OTTO STRUVE

ABSTRACT

Observations of the visual double star 9 Argus, made with the Bruce spectrograph of the Yerkes Observatory, were used for finding the *sign of the inclination* and the actual *dimensions of the orbit.* The period of revolution of this system is 23.34 years. A comparison of the spectrographic results with the visual orbit (by Aitken) gives $\pi m_1/m = 0''.14$. The most probable value of the *absolute parallax*, obtained by the trigonometric and other methods, is $\pi = 0''.078$. The *mass-ratio* is approximately $m/m_1 = 0.6$. The *total mass of the system* is found to be near 1.0–1.1 ☉. This is in good agreement with the value 1.2 ☉ obtained by Miller and Pitman using the law of gravitation.

The star 9 Argus, known as β 101 ($\alpha = 7^h 47^m.1$, $\delta = -13°38'$ (1900); mag., 5.6 and 6.7; spectral type, F8)[1] was placed by Professor Edwin B. Frost on the observing program for radial velocity at the Yerkes Observatory in 1905. The intention was to accumulate data for determining the sign of the inclination and the absolute parallax (where possible) for a number of visual binaries having well-established orbits. Since that time eleven spectrograms of this star, of good quality, have been obtained. All the plates have been measured and reduced by the usual Hartmann formula. Most of the plates were measured independently by different observers and no plate was measured less than twice. The spectrum of the fainter component does not appear on our plates and the measures

[1] In the *Henry Draper Catalogue*, for the brighter component, the spectral type is given as G0 and the combined visual or "photometric" magnitude as 5.34.

refer only to the brighter component. Table I contains the results of the measurements. The means of the various values for each plate were adopted as final and are given in the last column.

The visual orbit of the double star 9 Argus is known with great accuracy. A number of orbits have been published for this star. The best is undoubtedly the one by R. G. Aitken,[1] though his elements differ only slightly from the elements computed by Burnham[2]

TABLE I

Plate	Date	G.M.T.	Taken By	Velocity	Meas. By	Mean
				km/sec		km/sec
IB 493....	1905, Jan. 27	19^h06^m	F, B, S	−17.6, −15.8, −13.3	F, P, L	−15.6
IIB 89....	1906, Nov. 23	21 42	B, S	−16.5, −14.6	L, σ	−15.5
IB 933....	1906, Dec. 17	20 14	B, S	−17.5, −17.9, −20.6	F, P, L	−18.7
IB 939....	1907, Jan. 4	19 38	F, B, S	−22.9, −22.6, −18.2, −23.4	F, P, L, σ	−21.8
IB 1322....	1908, Jan. 13	19 30	L, B, S	−26.0, −22.2, −20.2	P, M, L	−23.8
IB 4789....	1917, Jan. 29	15 40	B	−19.5, −19.5	M, σ	−19.5
IB 6376....	1922, Jan. 27	18 37	σ, S	−13.1, −16.0	σ, σ	−14.6
IB 6384....	1922, Jan. 30	19 00	σ, S	−14.4, −18.3	σ, σ,	−16.4
IB 6465....	1922, April 7	14 48	σ, S	−14.7, −18.9	σ, σ	−16.8
IB 6653....	1922, Nov. 6	22 09	σ, S	−12.8, − 8.0	σ, σ,	−10.4
B 966....	1922, Nov. 20	20 31	σ, S	−17.3, −16.7	σ, σ,	−17.0

F=E. B. Frost, B=S. B. Barrett, P=J. A. Parkhurst, L=O. J. Lee, M=C. A. Maney, S=F. R. Sullivan, σ=O. Struve.

TABLE II

Ephemeris by Aitken			Observations			
	Pos. Angle	Distance		Pos. Angle	Distance	
1917.25...	280°.3	0″.40				
1918.25...	284.3	0.53	1918.18..	289°.1	0″.48	Van Biesbroeck
1919.25...	287.0	0.59	1919.11..	292.6	0.54	Van Biesbroeck
1920.25...	289.3	0.62				
1921.25...	291.5	0.62	1921.14..	299.9	0.52	Van Biesbroeck
1922.25...	293.8	0.62	1922.19..	293.4	0.56	Paraskévopoulos
			1923.27..	304.5	0.62	Van Biesbroeck

and later by Lohse.[3] Unpublished observations made by Professor G. Van Biesbroeck with the 40-inch telescope of the Yerkes Observatory and by Dr. J. Paraskévopoulos with the 16-inch telescope at Athens, as compared with the computed ephemeris, prove that the orbit, though derived in 1912, still represents the observations with sufficient accuracy.

[1] *Publications of the Lick Observatory*, 12, 51.

[2] Burnham's *General Catalogue*, No. 4310, p. 504.

[3] *Publicationen des Astrophys. Observatoriums*, Potsdam, 20, 93.

It seemed therefore unnecessary to undertake a new determination of the visual orbit, and Aitken's elements were adopted without change.

ELEMENTS

$P=23.34$ years	$a=\ 0''.69$
	$w=\ 74^\circ.65$
$T=1892.60$	$i=\pm 79^\circ.8$
$e=\ 0.75$	$\Omega=\ 99^\circ.7$
	Angles increasing

The inclination i is positive since the companion is being carried away from the observer at the nodal point.

In order to combine the spectrographic observations with the visual orbit, for each spectrographic velocity an equation of the following form was computed:

$$\frac{d\zeta}{dt}=V+\frac{dz}{dt}$$

$$\frac{dz}{dt}=\frac{\mu a_1 \sin i}{\sqrt{1-e^2}}[e \cos \omega+\cos(v+\omega)]$$

The notation is the same as in Aitken's *The Binary Stars,* pages 72 and 135–136.

It should be remembered that the spectrographic velocities are referred to the center of gravity of the system, while the visual orbit refers to the brighter component as the center of the coordinates.

Therefore we shall call a the semimajor axis of the visual orbit, while a_1 is the semimajor axis of the spectrographic orbit.

These equations were solved by the method of least squares and gave the following normal equations:

$$1.\ 3779x+\ 2.3185V+36.5127=0$$

$$2.\ 3185x+11.0000V+188.1000=0$$

where

$$x=\frac{\mu a_1 \sin i}{\sqrt{1-e^2}}$$

and V is the velocity of the center of mass of the system.

Solving these equations, we obtain:

$$V=-17.8 \text{ km/sec} \qquad x=3.5241.$$

No probable errors were computed for these quantities, for they would be entirely illusory, since their accuracy is based mainly on the accuracy of the visual orbit.

Remembering that

$$\mu=\frac{2\pi}{P_{\text{seconds}}}$$

we find $a_1=300{,}000{,}000$ km, while from the visual orbit $a=0\overset{''}{.}69$.

If the masses of the two components were equal, we would have

$$a=2a_1$$

If the mean distance of the earth from the sun is 150,000,000 km, then we would have:

$$\pi=\frac{0\overset{''}{.}69\times 150}{600}=0\overset{''}{.}17.$$

In the case of 9 Argus the mass-ratio is not known with sufficient accuracy and may differ considerably from unity. Therefore we find only:

$$\pi\left(\frac{m_1}{m}\right)=0\overset{''}{.}17.$$

We could solve our problem in a somewhat different way. If we draw a smooth curve through the spectrographic observations, retaining only the period of the visual orbit, we would find a range of 16 km/sec between maximum and minimum velocities.

Using the equation

$$a_1 \sin i=\text{const.}\ \frac{\kappa\sqrt{1-e^2}}{\mu}$$

we find

$$a=2a_1=900{,}000{,}000 \text{ km}.$$

This gives

$$\pi\left(\frac{m_1}{m}\right)=0\overset{''}{.}11.$$

Taking the mean between the two values $0\overset{''}{.}17$ and $0\overset{''}{.}11$ we have finally

$$\pi\left(\frac{m_1}{m}\right)=0\overset{''}{.}14.$$

Below is given a list of other determinations of the parallax of 9 Argus:

(1) Flint	$+0\overset{''}{.}035 \pm 0\overset{''}{.}026$	Meridian circle
(2) Sproul	$+0.121 \pm 0.009$	Photography
(3) McCormick	$+0.036 \pm 0.008$	Photography
(4) Adams and Joy	$+0.079$	Spectroscopic method
(5) Jackson and Furner	$+0.067$	Hypothetical value

Using only the best determination (2–5) and adding $0\overset{''}{.}005$ to the values obtained by the trigonometric method, we adopt for the absolute parallax

$$\pi = 0\overset{''}{.}078.$$

Combining this value with the one found from the spectrographic observations, we obtain for the mass-ratio approximately:

$$\frac{m}{m_1} = 0.6.$$

Considering the different values for the parallax of this star, as determined by the trigonometric method, we must admit that the discrepancy between the results obtained by J. A. Miller at the Sproul and by S. A. Mitchell at the McCormick observatories is rather unusual. The epochs of the parallax plates were the same at the two observatories, i.e., 1915–1916–1917. It seems not impossible that the photographic images of the parallax star have been affected by the light of the companion, which was in periastron in 1915.9 and which was moving very rapidly in position angle between the years 1915 and 1917 (99°–280°). Dr. O. J. Lee suggests that the large difference between the two values of π may be due to the circumstance that the light of the companion was not entirely eliminated by cutting down the brightness of the parallax star by means of a rotating sector, and that probably the effect of the sector was different at the two observatories. Therefore the light of the companion might have affected the image of the parallax star more in the one case than in the other.

In spite of this discordance, there is no doubt as to the reality of the parallax of 9 Argus, and it seems highly probable that the adopted value $0\overset{''}{.}078$ is very close to the truth.

S. A. Mitchell[1] finds from a comparison of proper motions as given by Boss and by his own determinations that the masses of the two components are in the ratio 0.73 to 0.27, or approximately

$$\frac{m}{m_1}=0.4.$$

This value agrees fairly well with the mass-ratio found from the spectrographic observations,

$$\frac{m}{m_1}=0.6.$$

In addition it may be noted that the mean density of this system was computed by E. Öpick,[2] and later, on the basis of better observational material by E. Bernewitz.[3] The two values are

$$\delta=2.3 \qquad \text{and} \qquad \delta=1.01,$$

the unit being the density of the sun.

Though differing very considerably from each other they still indicate that the density of this system is rather high.

If we adopt with Burnham the apparent magnitudes of the two components as, respectively, 5.6 and 6.7, then the absolute magnitude of the brighter component would be 5.1 (for $\pi=0''078$). H. N. Russell's tables[4] give for the diameter of such a star

$$D=0.9,$$

expressed in diameters of the sun. This would correspond to about 0.7 of the volume of the sun.

Applying Bernewitz's value for the mean density of the star, i.e., 1.01, we find that the mass of the brighter component is 0.7 that of the sun.

If our value for the mass-ratio (0.6) is correct, then the mass of the fainter component will be 0.4 that of the sun and

$$m+m_1=1.1 \odot .$$

[1] *Publications of the Leander McCormick Observatory*, 3, 239.

[2] *Astrophysical Journal*, 44, 298, 1916.

[3] *Astronomische Nachrichten*, 213, 1, 1921.

[4] *Publications of the Astronomical Society of the Pacific*, 32, 307, 1920.

This result agrees remarkably well with the value of the total mass of the system, derived from Kepler's law by Miller and Pitman[1] and independently by B. Meyermann.[2] Their values are identical, being based on the same parallax ($\pi=0''.079$) and give

$$m+m_1=1.2.$$

Knowing the mass-ratio, we could compute the density of the brighter component from Bernewitz' formula (1):

$$\log \delta_1=\log\left(\frac{a^3}{u^2}\right)+0.6(m_1-i_1)-\log\left(1+\frac{m}{m_1}\right)+0.089.$$

The computation gives

$$\delta_1=0.8.$$

Hence approximately

$$m_1=0.6\odot\,,\qquad m=0.4\odot$$
$$m+m_1=1.0\odot.$$

It must be remembered that the value for $m+m_1$, derived from the law of gravitation, depends on the third power of π. Therefore, a small error in the adopted parallax produces a great uncertainty in the mass. The good agreement between our hypothetical values of $m+m_1$ and the one derived by Miller and Pitman is probably the best proof that the adopted parallax $0''.078$ or $0''.079$ is not far from its true value. On the other hand, it must be admitted that the hypothetical value for the diameter, being true in average, may be subject to considerable uncertainty if applied to a particular star.

At the present time, the radial velocity of 9 Argus is slowly decreasing, the brighter component being carried away by its orbital motion with decreasing speed. Since the velocity of the system as a whole is -17.8 km /sec, the resulting velocity is always negative. A very rapid change from minimum to maximum velocity will occur in the years 1938–1941, and it will be of great value to secure a number of good spectrograms at that time.

[1] *Astronomical Journal*, **34**, 127, 1922.

[2] *Astronomische Nachrichten*, **216**, 302, 1922.

Since the completion of the preceding work in April, 1923, special experiments have been made by the writer with the Bruce spectrograph to prove that the observed spectrum of 9 Argus was affected by the light of the brighter component only.

It is now proved that, in a composite spectrum, the lines of the fainter component are entirely invisible if the difference in brightness of the two components equals or exceeds 1.0 magnitude. For a difference in brightness of 0.5 magnitude, the intensity of the lines of the fainter component is only about 15 per cent of the intensity of the lines of the brighter star. If the difference in magnitude of the two components of a close double star is less than 1.0 magnitude, and the resolving power of the spectrograph not sufficient for separating the two lines, it would be necessary to apply corrections, to the measured velocities, for the effect of blending with the lines of the fainter component. In the case of 9 Argus the difference in brightness exceeds 1.0 magnitude and the correction is therefore negligible.

Yerkes Observatory
June 30, 1923

A DETERMINATION OF e/m FROM MEASUREMENTS OF THE ZEEMAN EFFECT[1]

By HAROLD D. BABCOCK

ABSTRACT

The methods used in the determination of the ratio e/m.—Forty-nine separate values were derived from spectroscopic observations of the Zeeman effect, with absolute measurements of the magnetic field-intensity. About 40 lines in the blue region of the chromium spectrum were most frequently used. These were supplemented by 76 other lines of chromium, titanium, zinc, and barium. At *field-strengths averaging about 30,000 gausses* the separations of these lines were measured for the derivation of the normal separation, a. A wide variety of types of separation was used, but most dependence was placed on lines known to have small denominators in their fractional relation to a. Field measurements were made with ballistic galvanometer, test coil, and mutual inductance, in the primary circuit of which a known current could be reversed. *The uncertainty* affecting each factor in the working equation is discussed and the combined effect of all upon the result is derived. The chief difficulty in this method is the determination of a, the only other source of appreciable error being the magnetic area of the test coil.

Weighted mean e/m.—The final result from all the observations is $e/m = 1.761 \times 10^7$, with a probable error of one part in 1800 and an estimated actual uncertainty of 2 or 3 parts in 1800.

INTRODUCTION

Among the numerous determinations which have been made of the ratio e/m there is found a considerable discrepancy. Many of the older observations are so obviously inaccurate as to be of little value, while, even among those results which are apparently the best, the differences are objectionably large. Webster and Page,[2] using Paschen's data for hydrogen and helium series, find $e/m = 1.7686 \times 10^7$, which they say "is probably more accurate than any other determination." They do not give the probable error of this result. Birge,[3] on the other hand, using the most recent experimental results for hydrogen and helium, finds $e/m = 1.758 \times 10^7$, with a probable error of fully 0.5 per cent. He points out the desirability of new measurements based on deflection methods or on the Zeeman effect.

[1] *Contributions from the Mount Wilson Observatory*, No. 263.

[2] "A General Survey of the Present Status of the Atomic Structure Problems," *Bulletin of the National Research Council*, 2, 368 (No. 14), 1921.

[3] *Nature*, 111, 287, 1923.

The observations to be described were made during the progress of an investigation of the Zeeman effect for iron, chromium, and vanadium which is not yet published. The large amount of observational material thus collected has now been summarized, and this has made feasible the derivation of the results given here.

METHODS AND INSTRUMENTS

From the fundamental equation for the Zeeman effect we have:

$$\frac{e}{m}=\frac{4\pi V}{H}\frac{\Delta\lambda}{\lambda^2}$$

where H = intensity of magnetic field

$\Delta\lambda$ = separation of either n-component of a normal triplet of wave-length λ, from the undisturbed position of the line.

V = velocity of light in the medium in which $\Delta\lambda$ and λ are measured.

The quantity $\Delta\lambda/\lambda^2$ will be replaced throughout this paper by the symbol a.

The procedure which has been followed is to measure a for a selected group of lines of different elements, making absolute determinations of H, and to assume for V the value 2.9978×10^{10} cm/sec. In their important paper on the ratio of the electromagnetic to the electrostatic unit of the electricity, Rosa and Dorsey[1] conclude that the most probable value of the velocity of light *in vacuo* is 2.9986×10^{10} cm/sec, with an uncertainty of 1 part in 10,000. The index of refraction of air under normal conditions for λ 4500 is given by Meggers and Peters[2] as 1.00028. Since this number represents the ratio of the velocity of light *in vacuo* to its velocity in air with sufficient accuracy over the range of working conditions encountered in the present work, it is used for deriving the single value of V which enters into the equation for evaluating e/m.

The values of a were determined from photographs made with the vertical Littrow spectrograph in the Pasadena Laboratory. Second- and third-order spectra were used, sometimes at the 13-foot (4 m) focus, but nearly always at the 30-foot (9.2 cm) focus. A majority of the observations was made with a grating ruled by

[1] *Bulletin of the Bureau of Standards*, 3, 541, 1907.

[2] *Ibid.*, 14, 731, 1919.

Anderson at the Johns Hopkins University. This grating has 42,386 lines and was found to give its theoretical resolving power in the second and third orders. Two other gratings, one ruled by Rowland and one by Michelson, were used occasionally.

The slit width was calculated by dividing the mean wave-length for a given photograph by the angular width of the ruled surface as seen from the slit. This gives four times the normal slit width as defined by Schuster,[1] and proves a satisfactory compromise between the opposing claims of intensity and resolving power.

The light was produced by the discharge of a condenser supplied with power from an alternating current transformer. The energy flowing into the transformer on the low-tension side, as measured with an indicating wattmeter, was never less than one kilowatt nor more than one and one quarter kilowatts. Inductance was always added to the discharge circuit to improve the sharpness of the spectral lines and to reduce the intensity of the air spectrum. The spark terminals consisted of lumps or rods of metal held in a non-magnetic clamp. Chromium, titanium, brass, and cadmium were used. In some cases the two terminals were different but more often they were of the same metal. Occasionally a solution of barium bromide was dropped upon the metal terminals, thus giving the strong barium line λ 4554 in addition to the spectrum of the terminals themselves.

Table I gives the lines which were used in this work, the elements to which they belong, and the fractions which indicate the part of a associated with each component. For the spectral lines under consideration the components occur in pairs symmetrical about the normal position of the line, except that in the case of lines having an odd number of components the middle p-component is undisplaced. This is indicated by 0 in the numerator of the fraction. Parentheses inclose each group of p-components, and in the case of lines having normal separation the denominator 2 is arbitrarily assigned. The fraction would be written (0)2/2 for lines of this kind. The sign $\pm$ is always understood to precede the fractions as written.

[1] *Astrophysical Journal*, 21, 207, 1905

TABLE I

Spectral Lines Used for Determination of *a*

λ	Element	Fractional Separation	No. of Plates	λ	Element	Fractional Separation	No. of Plates
3841.32	Cr	(?), 6/7	1	4280.43	Cr	(0), 20/21	1
3852.23	Cr	(0), 3/2	1	4289.73	Cr	(2, 1, 0), 8, 7, 6, 5, 4/3	6
3857.66	Cr	(26), 53, 27/21	1	4295.79	Cr	(0), 5/3	4
3865.57	Cr	(0), 11/10	1	4297.78	Cr	(0), 12, 0?/13	4
3883.34	Cr	(0), 3/2	3	4300.53	Cr	(0), 2/2	1
3885.25	Cr	(0), 3/2	3	4301.21	Cr	(0), 13/12	1
3886.83	Cr	(0), 3/2	3	4325.16	Cr	(0), 15/17	2
3894.07	Cr	(0), 3/2	3	4337.59	Cr	(1, 0), 3, 2, 1/2	22
3902.95	Cr	(0), 3/2	3	4339.44	Cr	(2, 1, 0), 7, 6, 5, 4, 3/4	3
3908.77	Cr	(0), 3/2	3	4351.06	Cr	(3), 3, 0/2	1
3916.27	Cr	(0), 3/2	3	4359.64	Cr	(2, 1), 4. 3, 2, 1/2	22
3919.22	Cr	(0), 3/2	3	4363.13	Cr	(0), 5/6	3
3921.06	Cr	(0), 3/2	3	4371.30	Cr	(3, 2, 1, 0), 8, 7, 6, 5, 4, 3/4	2
3928.66	Cr	(0), 3/2	3	4375.36	Cr	(0). 12/13	2
3941.52	Cr	(0), 3/2	3	4376.81	Cr	(0), 13/11	2
3963.74	Cr	(0), 7/6	1	4391.77	Cr	(2, 1, 0), 5, 4, 3, 2, 1/2	8
3983.88	Cr	(0), 9/11	1	4403.47	Cr	(0), 2/2	4
3990.00	Cr	(0), 8/7	1	4432.20	Cr	(0), 3/2	4
3991.15	Cr	(0), 5/8	1	4458.53	Cr	(0), 12/11	1
3992.87	Cr	(0), 28/17	1	4482.89	Cr	(1), 4, 3/2	4
4001.46	Cr	(0), 22/21	1	4488.04	Cr	(0), 13/10	4
4003.33	Cr	(0), 20/21	1	4492.32	Cr	(1, 0), 4, 3, 2/2	1
4012.52	Cr	(0), 20/21	1	4496.86	Cr	(2, 1, 0), 7, 6, 5, 4, 3/3	23
4039.10	Cr	(0), 9/8	1	4498.73	Cr	(0), 31/21	12
4048.78	Cr	(0), 22/21	2	4501.28	Ti	(0), 9/10	1
4058.81	Cr	(0), 5/6	1	4506.80	Cr	(0), 20/21	3
4065.73	Cr	(0), 13/12	1	4511.92	Cr	(2), 7, 2/4	19
4077.69	Cr	(1), 5, 3, 1/3	1	4512.74	Ti	(0), 3/2	1
4121.29	Cr	(0), 2/2	1	4518.03	Ti	(0), 3/2	1
4121.85	Cr	(0), 2/2	1	4526.48	Cr	(0), 21/16	3
4123.40	Cr	(0), 7/6	1	4533.25	Ti	(0), 4/3	1
4126.54	Cr	(0), 4/3	1	4542.62	Cr	(0), 11/10	1
4142.15	Cr	(0), 5/4	1	4549.64	Ti	(0), 22/21	1
4161.45	Cr	(0), 13/12	3	4552.46	Ti	(0), 3/2	1
4163.65	Cr	(0), 7/6	1	4554.04	Ba	(1), 5, 3/3	10
4165.55	Cr	(0), 2/2	2	4555.49	Ti	(0), 3/2	1
4172.80	Cr	(0), 13/12	1	4556.16	Cr	(3), ?/5	2
4179.28	Cr	(0), 15/16	2	4563.77	Ti	(0), 4/5	2
4191.29	Cr	(0), 13/14	1	4564.18	Cr	(0), 2/2	2
4206.90	Cr	(0), 2/2	2	4569.62	Cr	(11), 26/15	1
4240.69	Cr	(0), 8/7	2	4571.98	Ti	(0), 20/21	1

TABLE I—*Continued*

4580.07	Cr	(1, 0), 5, 4, 3/2	36	4722.16	Zn	(1), 4, 3/2	11
4591.42	Cr	(2, 0), 13, 11, 9/6	4	4727.16	Cr	(0), 7/6	2
4595.58	Cr	(0), 2/2	3	4729.74	Cr	(0), 9/8	1
4613.33	Cr	(0), 5/2	23	4737.34	Cr	(0), 8/7	2
4616.11	Cr	(4, 2), 13, 11, 9, 7/6	22	4752.12	Cr	(0), 2/2	5
4619.54	Cr	(0), 31/21	2	4756.13	Cr	(0). 13/12	2
4626.16	Cr	(2), 5, 3/2	37	4758.13	Ti	(0), 2/2	1
4651.32	Cr	(2, 0), 5, 3, 1/2	35	4759.25	Ti	(0), 8/7	1
4652.21	Cr	(12, 6, 0), 38, 32, 26, 20, 14/17	12	4764.32	Cr	(0), 22/21	1
4654.74	Cr	(0), 6/2	4	4792.52	Cr	(6, 3, 0), 16, 13, 10/10	1
4656.47	Ti	(0), 9/11	1	4799.90	Cd	(1), 4, 3/2	1
4667.59	Ti	(0), 2/2	1	4801.03	Cr	(0), 4/3	1
4678.17	Cd	(0), 4/2	1	4810.55	Zn	(1, 0), 4, 3, 2/2	11
4680.17	Zn	(0), 4/2	11	4824.11	Cr	(0), 15/11	2
4681.91	Ti	(0), 16/15	1	4848.25	Cr	(0), 5/4	2
4589.40	Cr	(2, 1, 0), 5, 4, 3, 2, 1/2	5	4856.02	Ti	(0), 22/21	1
4693.96	Cr	(0), 2/2	6	4870.81	Cr	(0), 5/7	1
4700.62	Cr	(1, 0), 3/2	2				

To illustrate the meaning of the fractions the chromium line λ 4580.07 is drawn to scale in Figure 1. Lines at the top and bottom of the diagram indicate the position of the line in the absence of the magnetic field. The p-components are shown separated from the n-components just as they appear upon the photographs described below.

Table I also shows the number of plates upon which each line was measured. It will be seen that, in the main, chromium lines having small denominators were used between the wave-lengths λ 3883 and λ 4752. The complex types having six, twelve, or more components, when suitable for measurement, are of great usefulness on account of the checks they afford on the value of a for a given plate, and also because they increase the number of separate values

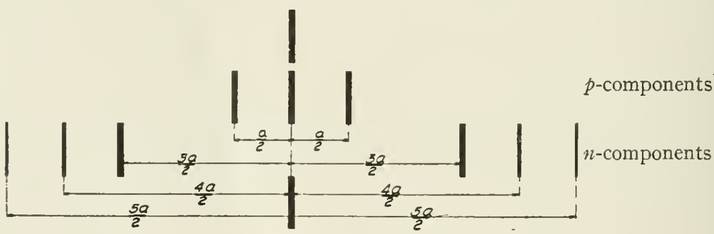

FIG. 1.—Zeeman effect for Cr λ 4580

obtainable. Triple lines whose separation is well known, but different from the normal interval, are just as useful for the determination of a as those which are normal. An example of this is λ 4613 of chromium, which is an excellent triple line whose separation is $5a/2$. Such lines are of more value than those having the normal separation because the percentage error in the value of a derived from them is less.

With regard to the lines in Table I to which are assigned fractions having large denominators, e.g., 13, 17, 21, it may be said that the extended investigation of the Zeeman effect for iron, chromium, and vanadium referred to above has furnished clear evidence for the existence of these denominators, which will be published as soon as possible. Upon the basis of 1177 lines of iron, 1123 of chromium, and 1462 of vanadium, for which the Zeeman effect has been measured, it becomes comparatively easy to ascertain the reality of even

such large denominators as 21. It will be observed, however, from an inspection of Table I, that these large denominators play a minor part in the present investigation.

In selecting lines for measurements of a the following considerations were kept in view: definiteness of the fractional system pertaining to the line, sharpness of the components, and absolute amount of their separation and number of measurable lines occurring on a given photograph. It is not feasible to use the fainter lines on account of circumstances relating to exposure time, heating of spark terminals, and the desirability of keeping the field strength as high as possible. The blue and violet region of the chromium spectrum fulfils the conditions satisfactorily.

In combining the separate values of a for a given photograph, weights were assigned depending on the ease of measurement and the absolute separation of the components. The probable error of the weighted means thus derived shows that when 40 or 50 separate measurements of a are combined, the result for the plate has an uncertainty of about 1 part in 600.

The measured displacements of the magnetically separated components were reduced to angstroms by means of a series of curves obtained from measurements on iron lines of known wavelength. The same spectrographic adjustments were used for these lines as in the general study of the Zeeman effect, the source being an iron arc.

The magnetic field was supplied by a large oil- and water-cooled magnet designed by Weiss, which has already been described.[1] Examination of the performance of the magnet under actual working conditions showed that for the portion of the magnetization curve employed in this work no appreciable error is introduced by breaking the magnetizing current during an exposure, or between the exposure and the measurement of the magnetic field-strength. The control of the magnetizing current was accomplished by means of a fine adjustment of the voltage of the generator which supplied the power, the current being measured and kept constant throughout each exposure by means of a sensitive ammeter. For different

[1] Annual Report, Mt. Wilson Observatory, *Year Book Carnegie Institution of Washington*, No. 11, 1912, p. 202.

observations the current ranged between 35 and 70 amperes, while the air gap in the magnetic circuit was from 7 to 12 mm. The lowest field-strength used was 24,290 gausses, the highest 31,880 gausses. The magnet is so mounted that the beam of light may traverse the field either at right angles or parallel to the lines of force. Both arrangements were used, but the former more frequently, since it has the advantage of giving all the components into which a line may be divided. It has, however, the disadvantage of reducing the *n*-components to one-half the intensity which they have when viewed along the lines of magnetic force. It was customary, when working with light perpendicular to the field, to separate the *p*- and *n*-components along the length of the slit by placing a nicol prism before the slit with an orientation such as to transmit one group of components and suppress the other. One-half the nicol and a corresponding segment of the slit were then covered by a half-wave plate of mica, with its axis inclined 45° to the slit. The combination of half-wave plate and nicol transmitted the group of components extinguished by the nicol alone, so that both groups could be simultaneously photographed. In the case of light parallel to the field no analyzer was required, since the *n*-components alone were visible and there was no occasion for distinguishing between the two states of circular polarization presented.

Measurements of the field-intensity were made by removing from the air gap a very small test coil which was connected in series with a long-period ballistic galvanometer, a manganin resistance of 20,000 ohms, and the secondary circuit of a mutual inductance. Galvanometer deflections of opposite sign were obtained by inverting the test coil between successive withdrawals. This method was found preferable to that of inverting the coil in the field, on account of the smallness of the air gap in which the coil had to be placed. The series of deflections thus secured was intermingled with another series obtained by reversing known currents in the primary of the mutual inductance while the test coil was stationary outside of the field of the magnet. The current which would give the same deflection as that produced by the test coil could then be found by interpolation.

Let I represent this current expressed in c.g.s. units, M, the mutual inductance in centimeters, and f, the magnetic area of the

test coil in square centimeters; the field-strength, H, is then given by the expression

$$H = \frac{2MI}{f} \text{ gausses.}$$

The factor 2 is required by the fact that I is reversed in direction, while the test coil is not inverted but merely withdrawn from the field. Since nearly all the resistance in the ballistic galvanometer circuit was of manganin, no correction was needed for the small changes of temperature which occurred during a series of measurements. This high resistance also made unnecessary any correction for damping for such a range of deflections as was used.

The mutual inductance was designed for these measurements. It was sent to the Bureau of Standards for certification, where its value was found to be 3.918 millihenrys. Its construction is such as to insure a high degree of permanence in this value. The two circuits were wound simultaneously with their respective wires side by side, so that the mutual inductance is nearly equal to the self-inductance of each circuit. The spool on which the wire is wound is made of seasoned mahogany. The terminals are small and as far from the region of concentrated magnetic field as possible. When in use the coil was supported upon dry wood well away from metallic objects and far beyond the influence of the large magnet whose field-strength was being measured.

A second mutual inductance whose value was obtained by comparison with that described above, and by independent measurements in terms of a known capacity and known resistance according to the Carey-Foster method, was used for part of the observations. It is of similar construction and has a mutual inductance of 17.25 millihenrys.

Two test coils were used in the direct observations while five other auxiliary coils were employed in determining the magnetic areas. The two principal coils have magnetic areas of 14.112 sq. cm and 7.434 sq. cm, respectively. They are wound on ivory spools and have several layers of No. 40 enameled copper wire. The dimensions of the spools and of each layer of wire were measured during their construction by mounting them under the microscope of a measuring machine. From the number of turns in each layer and the mean diameters the effective areas could be calculated.

The screw of the measuring machine was standardized by means of a precision scale ruled upon glass, calibrated and certified by the Bureau of Standards. Independent determinations of the areas were then made by intercomparing these coils with the five auxiliary coils, which were all single layer coils whose areas could be calculated with higher accuracy than is possible with multiple layer coils. In order to make the areas of the single-layer auxiliary coils nearly equal to those of the multiple-layer coils, which is desirable for reducing the errors of comparison, their diameters were increased. On account of their large volume the single-layer coils were not suited to direct measures on concentrated fields.

For comparing the areas of different types of coils the following plan was adopted. The two coils to be compared were temporarily fastened together and connected in series with each other and with the ballistic galvanometer and a suitable high resistance. They were then placed in a uniform magnetic field of sufficient intensity, obtained from the Weiss magnet by using an air gap of 30 mm between circular pole faces 50 mm in diameter. The coils were placed in position near the center of the field and, after quieting the galvanometer, were quickly withdrawn and the ballistic deflection was read. They were then inverted and a corresponding deflection of opposite sign was observed. After recording a series of such deflections, the terminals of one of the two test coils were reversed and the series was repeated. The amplitude of the deflections depends on the direction of the respective windings of the two coils, and will be proportional to either the sum or the difference of their magnetic areas. Simple combinations of the observations, therefore, give the ratio of the areas of the two coils.

This method was extended by connecting three test coils in series instead of two, each one of the coils in turn being kept outside of the magnetic field while the other two were used as described above. The ratio of a given pair of areas was in this way measured directly as well as indirectly, thus affording a useful check on the accuracy. The calculated and observed areas agree within the range indicated by their probable errors. It is thought that the mean area of the two coils used for the final measurements may be relied upon to 1 part in 1500.

The constants M and f being evaluated, consider now the measurement of the variable quantity, I, the current which is reversed in the primary circuit of the mutual inductance. At the beginning of the investigation this current was measured simultaneously by two separate instruments, a high-grade direct-reading ammeter, and a galvanometer with low-resistance shunt. These instruments were standardized by comparison with a silver voltameter, observing the usual precautions required in this operation.

A more satisfactory method was soon adopted, however, whereby the galvanometer was used only as a null reading indicator, thus avoiding the necessity for frequent standardization and at the same time securing higher accuracy. The current to be measured was passed through a standard 10-ohm resistance coil shunted by an adjustable resistance whose value was accurately known. The fall of potential across this combination was measured by opposing it to that of a Weston Standard cell with galvanometer in series, as in the ordinary potentiometer method. For each chosen value of the adjustable resistance the current flowing through the primary of the mutual inductance was varied until the galvanometer in series with the standard cell showed no deflection, whereupon the direction of the current through the mutual inductance was reversed by a quick-acting switch and the ballistic deflection was read. From the known values of the resistance and of the voltage of the standard cell the corresponding values of the current were derived by Ohm's law.

Two standard cells of the Weston type were used. One has for its electrolyte a solution of cadmium sulphate which is saturated at $+4°$ C., while the other has an excess of cadmium sulphate crystals. The former cell was standardized at the Physikalisch-Technische Reichsanstalt and later both cells were compared with a new Weston cell having a manufacturer's certificate. The mean e.m.f. of the two cells was 1.0183 international volts. The temperature coefficients are negligible over the range of temperatures at which the observations were made.

The ballistic galvanometer has a resistance of about 2100 ohms and a period of 16 seconds. It is of the moving-coil type and was used at a scale distance of 4 meters with an observing telescope

giving a magnification of 36 diameters. Under these conditions the ballistic sensitivity of the galvanometer was 6×10^{-10} coulombs per millimeter of deflection. The galvanometer used in the potentiometer circuit was sufficiently sensitive to show variations amounting to a few parts in 100,000 in the current passing through the standard resistance.

The standard 10-ohm coil used in this work was constructed by Otto Wolff according to the specifications developed by the Bureau of Standards.[1] No correction was required for temperature. The shunt in parallel with the standard coil was taken from a box of high-quality manganin resistances which was calibrated in terms of the 10-ohm standard.

An examination of the combined effect of the uncertainties in the e.m.f. of the standard cell and in the resistance shows that the probable error of the current can hardly exceed 1 part in 5000.

It is well known that with magnetic fields such as those used in this work the intensity varies considerably from point to point within the air gap. This effect may easily be shown either by a suitable exploring device such as a small bismuth spiral, or by means of the Zeeman effect itself. In order to avoid as far as possible the effects of this non-uniformity of the field, the spectrograph slit was placed at right angles to the direction of the magnetic field and was kept rather short. As a result the spectral lines appeared straight on the photographs. When the field-strength is measured with a test coil, however, the volume occupied by the coil makes a certain amount of integration inevitable. In order to correct for the difference between the field which was effective for the photographs and that which was measured, it was necessary to map the intensity from pole to pole across the air gap over an area equivalent to the cross-section of the test coil. The bismuth spiral, although not well adapted to precise absolute determinations of magnetic intensity, is nearly ideal for such relative measurements as are required for this purpose. It was mounted with its plane parallel to the pole faces of the magnet and was moved by successive small steps across the air gap, its resistance being read at each point.

[1] *Bulletin of the Bureau of Standards*, 5, 413, 1908–1909.

The resulting curves of field-strength as a function of position in the air gap were then plotted and mechanically integrated over the volume occupied by the test coil in order to find the desired correction. This quantity was generally 175 or 200 gausses.

Since, as stated above,

$$\frac{e}{m}=\frac{4\pi aV}{H},$$

and

$$H=\frac{2MI}{f},$$

the working equation is

$$\frac{e}{m}=\frac{2\pi aVf}{MI}.$$

To determine the combined effect of the uncertainties of the various factors on the resulting value of e/m, the following values, which closely represent those actually occurring in the observations, were assigned arbitrarily. The appended quantities are probable errors, for a single photograph in the case of a, and for a single complete determination of the field-strength in the case of I.

$a=1.230\pm0.002$	cm^{-1}
$v=(2.9978\pm0.0003)10^{10}$	cm/sec
$f=14.112\pm0.0093$	cm^2
$M=(3.918\pm0.0005)10^6$	cm
$I=(0.4750\pm0.0001)10^{-1}$	c.g.s.

Let R = probable error of e/m. Then, by the usual equation for the propagation of errors,

$$R^2=\left\{\frac{2\pi vf}{MI}\right\}^2 4\times10^{-6}+\left\{\frac{2\pi af}{MI}\right\}^2 9\times10^{12}+\left\{\frac{2\pi av}{MI}\right\}^2 8.6\times10^{-5}$$
$$+\left\{\frac{2\pi avf}{IM^2}\right\}^2 25\times10^4+\left\{\frac{2\pi avf}{MI^2}\right\}^2 10^{-10}.$$

Evaluating the successive terms in this expression, we find

$$R^2=(8.16+0.03+1.33+0.05+0.14)\times10^8=9.71\times10^8$$

and

$$R=\pm3\times10^4=\pm0.003\times10^7$$

Since the value of e/m is approximately 1.76×10^7, the percentage probable error of the result derived from a single photograph is 1 part in 600, which is the same as that for the corresponding determination of a. The uncertainties of the other factors in the equation are negligible except in the case of the magnetic area of the test coil, which contributes, however, only one-seventh of R^2.

Table II shows the results of all the measurements made. The weight assigned to each value of e/m depends on the sum of the weights for the individual observations of a on the corresponding photograph. Among the first 12 plates, the weights were in some cases reduced from 2 to 1 because of the less accurate method used

TABLE II

Plate No.	a	H	e/m	Wt.	Plate No.	a	H	e/m	Wt.
1......	1.352	29030	1.7543	1	26.....	1.486	31810	1.7598	4
2......	1.346	29030	1.7465	1	27.....	1.243	26500	1.7670	4
3......	1.360	29030	1.7647	1	28.....	1.234	26500	1.7542	4
4......	1.274	27670	1.7344	1	29.....	1.228	26500	1.7457	2
5......	1.287	27670	1.7521	1	30.....	1.229	26500	1.7471	3
6......	1.284	26950	1.7946	1	31.....	1.233	26500	1.7528	4
7......	1.281	26950	1.7904	1	32.....	1.233	26500	1.7528	3
8......	1.385	29360	1.7769	1	33.....	1.236	26500	1.7571	2
9......	1.423	30300	1.7690	1	34.....	1.241	26500	1.7642	2
10......	1.411	30300	1.7542	1	35.....	1.241	26500	1.7642	2
11......	1.306	27660	1.7786	1	36.....	1.143	24290	1.7727	4
12......	1.341	28320	1.7837	1	37.....	1.145	24290	1.7758	4
13......	1.485	31810	1.7586	5	38.....	1.138	24290	1.7649	4
14......	1.494	31810	1.7693	4	39... .	1.243	26500	1.7670	4
15......	1.487	31810	1.7610	4	40.....	1.236	26500	1.7571	3
16......	1.477	31810	1.7572	4	41.....	1.239	26500	1.7613	2
17......	1.490	31810	1.7646	2	42.....	1.238	26500	1.7599	4
18......	1.475	31810	1.7468	5	43.....	1.242	26500	1.7656	4
19......	1.498	31810	1.7740	5	44.....	1.242	26500	1.7656	2
20......	1.464	31810	1.7338	4	45.....	1.235	26500	1.7556	3
21......	1.485	31810	1.7586	5	46.....	1.236	26500	1.7571	3
22......	1.483	31810	1.7563	5	47.....	1.232	26500	1.7514	3
23......	1.490	31810	1.7646	5	48.....	1.242	26500	1.7656	2
24......	1.484	31810	1.7575	5	49.....	1.248	26500	1.7741	2
25......	1.490	31810	1.7646	2					

for measuring the current in the mutual inductance. The most probable value of e/m is the weighted mean of all the determinations. This is found to be

$$\frac{e}{m}=1.761\times 10^7$$

with a probable error of 1 part in 1800.

It would be manifestly unfair to give each observation the same weight, on account of the variation in the amount of data involved, but it is interesting to note that, if they are combined in this manner,

the mean becomes 1.762×10^7. This indicates that within fairly wide limits no great importance attaches to the weight assigned.

As noted above, the probable error of the weighted mean is 1 part in 1800. From this and the preceding analysis of the uncertainty of e/m as dependent on the various factors involved, considered in connection with the amount of observational material, it appears reasonable to believe that the actual uncertainty in the final value of e/m is not more than 2 or 3 parts in 1800.

This conclusion is supported by the recent work of Birge[1] on the value of the Planck constant h. Using the value

$$e/m = 1.761 \times 10^7,$$

he finds

$$h = 6.556 \times 10^{-27}$$

while the most probable value of h, which he derives by seven different methods, using revised data, is

$$h = 6.557 \times 10^{-27}$$

Although it is thought that the value of e/m found here may properly claim considerable accuracy, further improvements now seem quite possible. Inasmuch as the chief difficulty lies in the determination of a, it is evident that the use of much higher resolving power would yield a marked gain. In addition, a more extended list of spectral lines is now available for which fractional separations are sufficiently well known to make them useful. Since the completion of the observations described here, a powerful solenoid[2] capable of furnishing fields of 35,000 gausses has been constructed in the instrument shop of the Observatory. With this instrument and the other improvements referred to, it is hoped that in the future our knowledge of e/m may be still further increased.

My grateful acknowledgments are due to Miss Lois M. Keener and several former members of the Computing Division for much assistance in the measurements and reductions involved in this work.

[1] *Nature*, **111**, 811, 1923; *Physical Review*, **14**, 361, 1919.

[2] Annual Report, Mt. Wilson Observatory, *Year Book Carnegie Institution of Washington*, No. 18, 1919, pp. 220, 251; *ibid.*, No. 19, 1920, p. 256; *ibid.*, No. 20, 1921, p. 285.

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PHOTOMETRIC STUDIES OF SZ HERCULIS AND RS VULPECULAE

By RAYMOND S. DUGAN

ABSTRACT

Studies of the eclipsing variables SZ Herculis and RS Vulpeculae with the polarizing photometer have been carried out and the *mean light curves* of these stars have been drawn and discussed. The probable error of a single observation with the comparison star $+33°2925$ is $\pm 0^M.026$. *New light elements* are given, also *tables* of normals and plotted light curves. For both stars the uniform curve represents the observations better than the darkened. The spectroscopic observations aided in the selection of the correct interpretation of the light curve of RS Vulpeculae. Orbital elements and other data are summarized.

The variability of star $+33°2931$, one of the comparison stars of SZ Herculis, is apparent from the distribution of the residuals.

SZ Herculis (1900, $\alpha = 17^h 36^M.0$, $\delta = +33°1'$, $10^M.0$).—Observation of this star was begun in 1915. At that time sufficient observations had not been made during primary minimum to define this part of the light curve satisfactorily. Very few observations had been made when the star was free from eclipse and nothing was known concerning the secondary minimum. The star $A = +33°2931$ proved to be variable over a small range. The observations are not sufficient to determine the nature of the variation. The other comparison star, $B = +33°2925$, showed no variation. The distribution of the residuals of the B–V observations is quite normal, while the non-accidental occurrence of the A–V observations is apparent. These conclusions are represented also by the size of the probable errors,

$\pm 0^M.026$ for a B–V observation,
$\pm 0^M.042$ for an A–V observation.

The B–V observations are not sufficiently numerous to independently determine all parts of the curve. In order to make use of the A–V observations, a preliminary solution was made of the B–V observations and values for ellipticity and reflection assumed on the basis of the values of the relative dimensions thus secured. The secondary is well defined by the B–V observations. Residuals of the A–V observations from this provisional theoretical curve

FIG. 1.—Distribution of residuals of observations of SZ Herculis. The V-B residuals are apparently accidental, while the distribution of the A-V residuals indicates that comparison star A is variable.

were read off and their average value on each night applied to the A–V observations of that night. The final values of the ellipticity and reflection coefficients are somewhat affected by this procedure and should not be used in any general discussion of these effects.

The newly determined heliocentric elements are

$$\text{Min.} = \text{J.D. } 2418495.406 + 0^{d}.8180972\, E.$$

The letter following the current number in the table of normals indicates the comparison star used. The sum of the weights of the

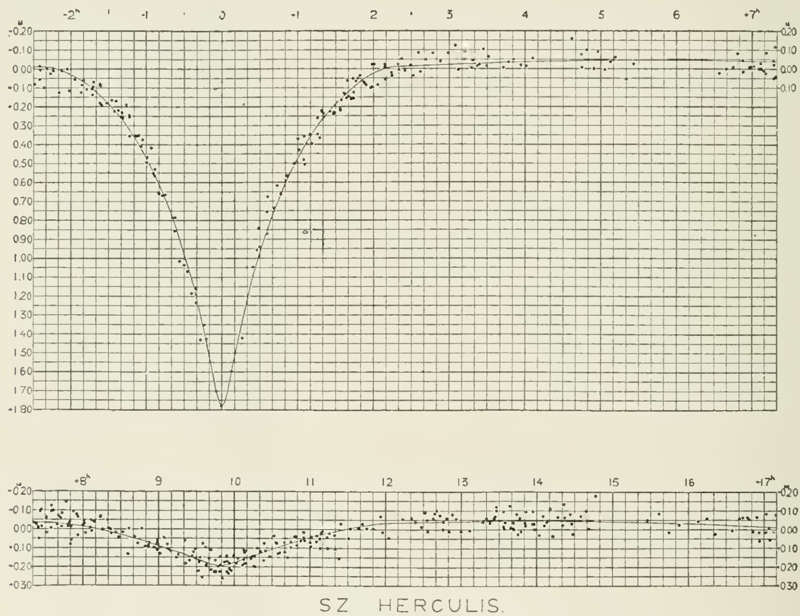

FIG. 2.—Mean light curve of SZ Herculis. Single observations plotted. Theoretical "uniform" curve drawn out.

observations in a normal is reduced by a factor which is unity when no two observations in the normal were made on the same night and is 0.5 when all the observations were made on one night. O–C is the deviation of the observed normal from the final computed curve.

In Figure 2 the individual observations are plotted and the accepted theoretical curve drawn out.

The solution assumes stars presenting uniformly illuminated disks. No satisfactory "totally darkened" solution was found.

RS Vulpeculae ($\alpha = 19^h 13^m.4$, $\delta = +22°16'$, $7^m.0$, B8, B9).—This star was observed on 63 nights between October, 1919 and December, 1922. It was necessary to cover thoroughly the primary and the region of a possible secondary minimum and a few points between eclipses.

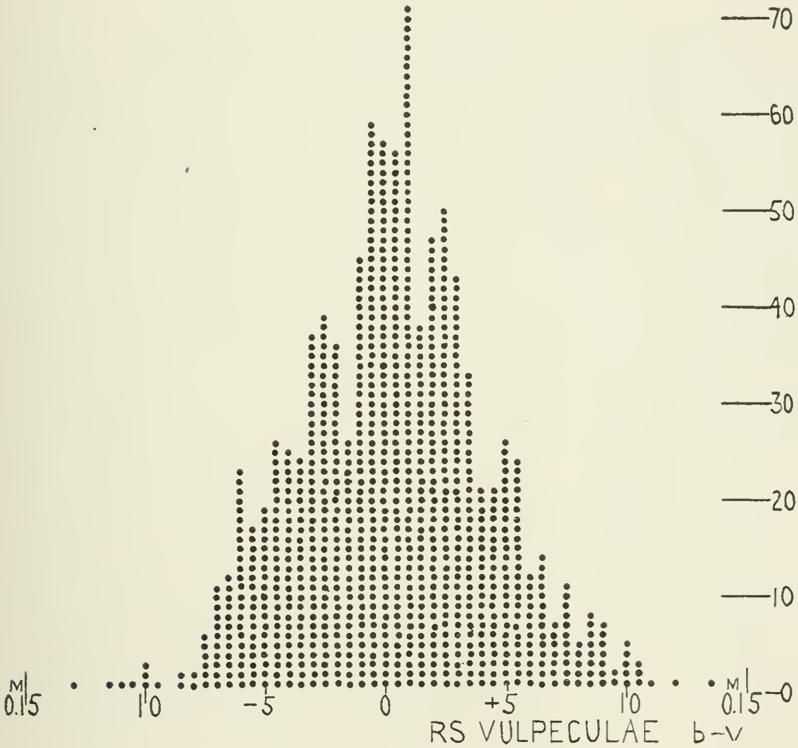

FIG. 3.—Distribution of residuals of observations of RS Vulpeculae. An apparently normal distribution giving no evidence of any variation of the comparison star.

New light elements were adopted as follows:

$$\text{Min.} = \text{J.D. } 2420606.6226 + 4^d.477666\, E.$$

Comparison star $b = +22°3644$ was used throughout the series and gave no indication of being itself a variable. The probable error of a single observation is $\pm 0^M.026$.

The curve drawn through the normals is the final curve of the uniform solution of partial eclipse. The variation in brightness when the system is free of eclipse and during secondary minimum, in the lower part of the figure, has been exaggerated by increasing the magnitude scale and compressing the time scale. The two minima are drawn the same length.

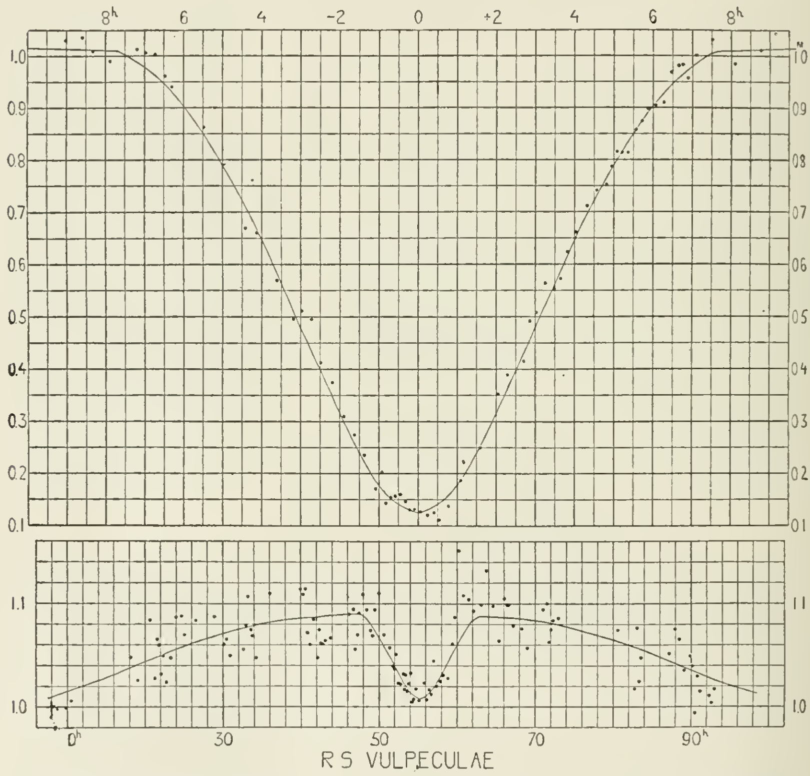

Fig. 4.—Mean light curve of RS Vulpeculae. Normals plotted. Theoretical "uniform" curve drawn out.

The ellipticity and reflection coefficients are well determined by the observations and the secondary minimum is clearly outlined.

The change in brightness at bottom of primary minimum is very gradual. One would hardly quarrel with the assertion that the light is constant for an hour or so. The interpretation of primary minimum as a total eclipse is not consistent with the value of the

ratio of diameters and the depth of secondary minimum. Plaskett's statement that the spectral lines of the faint component are visible and about one-fourth as strong as those of the bright component makes it impossible to accept the interpretation of primary minimum as an annular eclipse. If it were not for the spectrographic evidence one might be almost content with the representation of the

SUMMARY OF RESULTS

	SZ Herculis	RS Vulpeculae
Nature of each eclipse	Partial	Partial
Duration of each eclipse	$4^{h}30^{m}$	$15^{h}19^{m}$
Magnitude at maximum	10.0	7.0
Magnitude at primary minimum	11.8	8.0
Magnitude at secondary minimum	10.2	7.1
Loss of light at primary, due to eclipse	$1^{\mathrm{M}}64$	$0^{\mathrm{M}}97$
Loss of light at secondary, due to eclipse	$0^{\mathrm{M}}22$	$0^{\mathrm{M}}08$
Displacement of secondary minimum	0	$+1\frac{1}{2}^{h}$
Inclination of orbit plane	$88^{\circ}4$	$79^{\circ}0$
Ratio of radii of two stars	0.960	0.765
Ratio of axes of each star	0.965	0.984
Fraction of surface of brighter star covered at primary minimum	0.977	0.636
Difference in intensity of sides of fainter star	0.030	0.078
Light of brighter star	0.800	0.804
Light of bright side of faint star	0.200	0.196
Ratio of surface brightness	4.0	4.1
Max. radius of brighter star	0.3295* 1.5⊙†	0.2045* 4.27⊙
Max. radius of fainter star	0.3432* 1.6⊙†	0.2674* 5.58⊙
Distance of centers	4.6⊙†	20.86⊙
Mass of brighter star	1.4‡	4.59⊙
Mass of fainter star	0.6‡	1.44⊙
Density of brighter star	0.38	0.060
Density of fainter star	0.16	0.008

* Distance of centers is unity.
† Mass of each component assumed equal to mass of sun.
‡ Assumed.

observations on this hypothesis. There remains the case of partial eclipse. The theoretical curve fits the observations very satisfactorily. Again there is no evidence of darkening at the limb.

The secondary is displaced toward the following primary about $1\frac{1}{2}$ hours. Plaskett's spectroscopic orbit, of which the eccentricity is but 0.05, would require a shift of about two hours in the opposite direction. Otherwise the light curve is sensibly symmetrical.

TABLE OF NORMALS. SZ HERCULIS

Norm. No.	Phase	Wt.	V–B	O–C
1B..........	$-2^h 20^m.63$	2.5	$0^M.000$	$+0^M.012$
1A..........	20.30	3.5	+0.077	+ 89
2A..........	2 1.20	4	+0.047	+ 24
3A..........	1 48.90	4	+0.097	+ 24
4A..........	38.90	4	+0.131	+ 2
2B..........	33.50	1.5	+0.122	− 39
3B..........	20.30	3.5	+0.248	− 11
5A..........	19.88	3	+0.205	− 58
4B..........	6.50	2.5	+0.379	− 20
6A..........	1 3.50	2.5	+0.374	− 59
5B..........	0 53.75	2.5	+0.563	+ 4
7A..........	49.33	2	+0.618	+ 1
8A..........	35.38	1.5	+0.892	+ 32
6B..........	30.50	1	+1.043	+ 78
9A..........	22.00	1.5	+1.168	+ 13
7B..........	17.25	1	+1.293	0
10A..........	14.00	1.5	+1.426	+ 36
11A..........	− 0 3.13	1.5	+1.726	− 14
12A..........	+ 0 11.75	.5	+1.508	+ 48
8B..........	29.75	3	+0.875	− 102
13A..........	35.50	1.5	+0.775	− 79
9B..........	0 43.63	2	+0.708	− 6
14A..........	1 1.50	3	+0.479	+ 23
10B..........	7.63	2	+0.355	− 29
15A..........	13.38	3.5	+0.341	+ 20
16A..........	24.20	4	+0.233	+ 6
11B..........	26.83	1.5	+0.208	− 1
17A..........	36.40	4.5	+0.172	+ 28
18A..........	44.60	4.5	+0.096	+ 1
12B..........	49.10	2.5	+0.080	+ 12
19A..........	1 56.40	4	+0.039	+ 3
20A..........	2 11.33	3.5	+0.001	+ 5
13B..........	17.70	3	+0.016	+ 28
14B..........	35.00	3.5	−0.021	− 5
15B..........	2 59.40	2.5	−0.085	− 63
21A..........	3 19.33	3	−0.039	− 12
16B..........	32.30	3.5	−0.025	+ 5
17B..........	3 58.63	2	−0.035	0
22A..........	4 47.05	4	−0.070	− 26
23A..........	5 14.28	1.5	−0.009	+ 37
24A..........	6 43.90	3	−0.022	+ 21
25A..........	6 59.60	4	−0.004	+ 36
26A..........	7 10.80	4.5	−0.027	+ 12
18B..........	19.50	3.5	+0.015	+ 53
27A..........	+ 7 21.10	4.5	−0.029	+0.007

TABLE OF NORMALS. SZ HERCULIS—*Continued*

Norm. No.	Phase	Wt.	V–B	O–C
28A..........	+ $7^h29^m.80$	4	$-0^M.065$	$-0^M.031$
29A..........	39.20	4.5	−0.065	− 33
19B..........	43.39	3.5	−0.073	− 45
30A..........	49.50	5	−0.023	+ 3
20B..........	7 54.80	4.5	−0.020	0
21B..........	8 3.50	4	−0.023	− 15
22B..........	11.80	4.5	−0.033	− 33
23B..........	22.00	4.5	−0.004	− 18
24B..........	38.40	4	+0.042	− 3
31A..........	42.21	2	+0.060	+ 8
32A..........	57.93	2	+0.075	− 7
25B..........	8 59.70	4	+0.090	+ 3
26B..........	9 20.00	4	+0.126	− 7
33A..........	21.00	2	+0.137	+ 2
27B..........	31.70	5	+0.184	+ 23
28B..........	37.70	5	+0.142	− 35
29B..........	43.50	5	+0.188	0
30B..........	50.33	4.5	+0.194	− 1
31B..........	55.50	5	+0.195	+ 9
34A..........	9 56.83	1.5	+0.198	+ 14
32B..........	10 3.50	4.5	+0.165	− 4
35A..........	10.80	4.5	+0.154	+ 4
33B..........	14.20	4	+0.132	− 9
36A..........	21.10	4	+0.131	+ 4
34B..........	32.80	4	+0.113	+ 13
37A..........	34.72	3	+0.077	− 17
35B..........	45.80	4	+0.087	+ 14
36B..........	10 55.60	4.5	+0.050	0
37B..........	11 4.90	4	+0.061	+ 28
38A..........	7.28	3.5	+0.038	+ 8
39A..........	21.17	4	+0.053	+ 45
38B..........	33.00	2.5	+0.015	+ 23
39B..........	50.83	2	+0.002	+ 29
40A..........	11 59.25	1	−0.013	+ 20
40B..........	12 26.88	2	−0.066	− 28
41A..........	12 31.00	3	−0.039	− 1
42A..........	44.50	4	−0.052	− 12
43A..........	12 57.30	3.5	−0.022	+ 19
41B..........	13 24.10	4	−0.065	− 21
44A..........	28.50	3	−0.048	− 3
42B..........	13 35.70	4.5	−0.050	− 4
43B..........	43.60	4.5	−0.031	+ 15
44B..........	52.60	4	−0.049	− 3
45A..........	13 59.60	3.5	−0.028	+ 18
46A..........	14 16.70	3.5	−0.068	− 22
45B..........	22.33	4	−0.023	+ 22
47A..........	14 35.13	2.5	−0.084	− 39
48A..........	15 40.25	2	−0.037	− 2
46B..........	+16 54.00	5	−0.002	+0.014

TABLE OF NORMALS. RS VULPECULAE

Norm. No.	Phase	Wt.	b−v	O−C
1	$-9^h\ 1^m.3$	2.5	$1^M.030$	$+0^M.017$
2	8 34.7	4.0	1.036	+ 24
3	8 19.7	3.5	1.009	− 3
4	7 53.7	2.5	0.991	− 19
5	7 11.8	3.5	1.013	+ 24
6	6 58.5	3.5	1.006	+ 28
7	6 43.7	3.0	1.003	+ 41
8	6 28.9	3.5	0.962	+ 17
9	6 18.1	3.5	0.940	+ 9
10	5 29.8	3.0	0.863	+ 14
11	4 25.2	3.5	0.669	− 41
12	4 8.0	3.0	0.661	− 8
13	3 36.9	2.5	0.570	− 13
14	3 11.5	4.0	0.497	− 14
15	2 58.6	4.0	0.512	+ 38
16	2 44.0	4.0	0.496	+ 59
17	2 29.1	4.0	0.413	+ 16
18	2 11.9	3.5	0.375	+ 25
19	1 53.9	3.5	0.310	+ 10
20	1 37.8	3.5	0.275	+ 16
21	1 22.1	3.0	0.236	+ 13
22	1 4.5	4.0	0.172	− 13
23	0 55.7	4.5	0.203	+ 33
24	0 49.5	4.5	0.143	− 19
25	0 42.1	4.5	0.154	+ 2
26	0 35.3	4.0	0.157	+ 10
27	0 27.8	5.0	0.161	+ 22
28	0 19.9	4.5	0.147	+ 12
29	0 13.3	5.0	0.130	− 1
30	− 0 5.9	4.0	0.132	+ 5
31	+ 0 3.9	4.0	0.127	0
32	0 14.7	4.5	0.121	− 11
33	0 24.4	4.0	0.126	− 12
34	0 32.8	4.5	0.112	− 33
35	0 46.4	3.5	0.138	− 20
36	1 5.3	3.0	0.187	− 1
37	1 35.6	2.5	0.251	− 2
38	2 2.7	3.5	0.353	+ 28
39	2 17.1	3.0	0.390	+ 27
40	2 41.9	4.0	0.416	− 14
41	2 51.4	4.0	0.492	+ 35
42	3 1.6	4.0	0.508	+ 24
43	3 15.0	3.5	0.564	+ 42
44	3 28.5	4.0	0.553	− 5
45	+ 3 38.3	4.0	0.573	−0.014

TABLE OF NORMALS. RS VULPECULAE—*Continued*

Norm. No.	Phase	Wt.	b–v	O–C
46	$+3^h48^m.7$	4.0	$0^M.624$	$+0^M.008$
47	4 1.3	3.5	0.662	+ 10
48	4 18.2	3.5	0.712	+ 16
49	4 33.0	4.0	0.742	+ 13
50	4 48.6	4.5	0.753	− 12
51	4 56.5	4.5	0.788	+ 6
52	5 5.1	4.5	0.816	+ 15
53	5 13.0	4.5	0.814	− 3
54	5 21.2	4.0	0.814	− 20
55	5 33.4	4.0	0.858	+ 1
56	5 42.2	4.0	0.874	0
57	5 51.4	4.5	0.898	+ 8
58	6 3.3	4.0	0.905	− 4
59	6 16.8	4.0	0.910	− 19
60	6 27.9	4.5	0.968	+ 24
61	6 38.2	4.5	0.982	+ 25
62	6 45.3	4.5	0.983	+ 18
63	6 53.1	4.0	0.957	− 16
64	7 5.5	4.0	1.002	+ 17
65	7 30.2	2.5	1.032	+ 26
66	8 5.4	3.0	0.985	− 26
67	8 45.3	3.5	1.012	− 1
68	9 8.4	4.0	1.043	+ 30
69	9 48.6	3.0	0.999	− 16
70	10 28.7	3.0	1.006	− 11
71	18 6.3	3.0	1.048	+ 10
72	19 1.1	3.0	1.026	− 15
73	20 37.5	3.0	1.084	+ 38
74	21 12.2	3.5	1.028	− 20
75	21 30.8	4.0	1.066	+ 18
76	21 45.1	4.5	1.060	+ 11
77	22 1.8	4.5	1.032	− 18
78	22 18.5	4.0	1.050	− 2
79	22 44.8	3.5	1.024	− 29
80	23 18.6	2.5	1.048	− 7
81	23 59.9	2.5	1.087	+ 31
82	24 41.7	4.0	1.088	+ 30
83	25 4.2	4.0	1.070	+ 11
84	26 30.4	3.0	1.084	+ 21
85	28 57.3	3.0	1.088	+ 19
86	30 7.0	3.0	1.061	− 10
87	30 25.7	3.5	1.066	− 7
88	30 55.4	3.0	1.050	− 24
89	32 39.9	3.5	1.056	− 21
90	+32 57.3	3.5	1.079	+0.002

TABLE OF NORMALS. RS VULPECULAE—*Continued*

Norm. No.	Phase	Wt.	b–v	O–C
91	$+33^h15^m.5$	3.5	$1^M.107$	$+0^M.030$
92	33 31.3	3.5	1.075	− 4
93	33 47.2	3.5	1.069	− 10
94	34 13.6	2.5	1.048	− 32
95	36 3.5	0.5	1.110	+ 28
96	39 55.6	3.5	1.114	+ 28
97	40 14.6	3.5	1.109	+ 23
98	40 33.5	3.5	1.114	+ 28
99	40 51.1	3.5	1.072	− 15
100	41 9.6	3.5	1.061	− 26
101	41 32.1	3.5	1.085	− 2
102	41 46.4	3.5	1.085	− 2
103	42 6.8	4.0	1.048	− 39
104	42 18.7	4.0	1.075	− 13
105	42 29.9	4.0	1.062	− 26
106	42 41.9	4.0	1.061	− 27
107	43 7.8	5.0	1.064	− 24
108	43 45.3	2.5	1.067	− 22
109	46 9.0	3.0	1.090	0
110	46 40.1	3.0	1.095	+ 5
111	46 56.9	3.5	1.056	− 34
112	47 11.8	3.5	1.070	− 20
113	47 26.1	3.0	1.091	+ 1
114	47 53.9	2.5	1.109	+ 20
115	48 23.5	2.5	1.094	+ 9
116	48 51.7	3.0	1.074	− 7
117	49 10.0	3.5	1.069	− 9
118	49 27.0	3.5	1.094	+ 20
119	49 59.1	2.5	1.110	+ 43
120	50 35.4	2.5	1.070	+ 12
121	51 21.4	4.0	1.057	+ 8
122	51 45.0	4.0	1.039	− 4
123	51 57.0	4.0	1.037	− 3
124	52 9.9	3.5	1.051	+ 14
125	52 23.9	3.5	1.023	− 10
126	52 43.5	3.0	1.022	− 6
127	53 1.4	4.0	1.032	+ 8
128	53 12.2	4.0	1.017	− 5
129	53 22.5	4.0	1.031	+ 11
130	53 33.6	4.0	1.015	− 3
131	53 47.1	4.0	1.022	+ 7
132	54 0.1	4.0	1.033	+ 20
133	54 13.1	3.5	1.005	− 7
134	54 24.6	3.5	1.007	− 4
135	+54 41.4	3.0	1.017	+0.008

TABLE OF NORMALS. RS VULPECULAE—*Continued*

Norm. No.	Phase	Wt.	b–v	O–C
136..........	$+55^h\ 7^m.6$	2.0	$1^M.006$	$-0^M.002$
137..........	55 47.0	2.5	1.023	+ 13
138..........	56 6.7	3.5	1.007	− 4
139..........	56 23.7	3.5	1.017	+ 4
140..........	56 39.3	3.5	1.012	− 3
141..........	56 57.5	3.5	1.045	+ 27
142..........	57 18.8	3.5	1.020	− 3
143..........	57 49.1	3.0	1.025	− 4
144..........	58 13.6	2.5	1.031	− 4
145..........	58 49.0	3.0	1.028	− 16
146..........	59 22.2	4.0	1.061	+ 10
147..........	59 40.0	4.0	1.087	+ 32
148..........	60 15.6	3.0	1.151	+ 87
149..........	60 50.2	3.0	1.108	+ 37
150..........	61 27.3	3.0	1.104	+ 25
151..........	62 5.3	3.5	1.093	+ 9
152..........	63 6.7	2.0	1.099	+ 11
153..........	63 47.2	1.5	1.132	+ 44
154..........	64 35.0	1.5	1.098	+ 11
155..........	65 20.9	3.0	1.064	− 24
156..........	66 2.8	3.5	1.105	+ 18
157..........	66 20.9	3.5	1.099	+ 12
158..........	66 36.5	3.5	1.099	+ 13
159..........	67 11.1	3.0	1.079	− 7
160..........	68 17.2	3.0	1.076	− 10
161..........	68 57.8	3.0	1.058	− 28
162..........	70 59.3	3.5	1.095	+ 12
163..........	71 28.9	3.5	1.100	+ 18
164..........	71 43.8	3.5	1.063	− 19
165..........	71 59.9	3.5	1.075	− 6
166..........	72 15.5	4.0	1.084	+ 3
167..........	72 52.9	3.5	1.086	+ 6
168..........	80 31.8	2.0	1.075	+ 11
169..........	82 42.0	3.5	1.018	− 41
170..........	83 3.9	3.5	1.077	+ 19
171..........	83 18.2	3.5	1.025	− 32
172..........	83 35.0	3.5	1.048	− 8
173..........	87 7.4	3.5	1.052	+ 7
174..........	87 54.3	3.0	1.076	+ 34
175..........	88 28.6	2.5	1.066	+ 25
176..........	89 3.0	2.5	1.036	− 3
177..........	89 48.6	3.0	1.050	+ 14
178..........	90 9.2	3.5	1.040	+ 5
179..........	90 25.9	3.5	0.994	− 40
180..........	90 45.1	3.5	1.030	− 4
181..........	91 8.7	4.0	1.015	− 18
182..........	92 17.1	3.0	1.011	− 18
183..........	92 35.6	3.0	1.004	− 24
184..........	+92 56.2	3.5	1.018	−0.009

PRINCETON UNIVERSITY OBSERVATORY
May 1923

THE RELATION BETWEEN THE SPECTRA AND THE SIZES OF THE ALKALI METAL ATOMS.

By LOUIS A. TURNER

ABSTRACT

Spectra and relative sizes of the positive ions.—A *theory* is developed which gives the relation between the energies and quantum numbers of orbits of valence electrons in atoms differing in size but geometrically similar. It is shown that the *wave numbers* for some of the outer orbits of the four alkali metal atoms Na, K, Rb, and Cs are in fair accord with the assumption that the structure beneath the valence electron, or kernel, is geometrically similar but different in size for the four atoms. The relative sizes of these kernels obtained by this method agree with results obtained by other methods as well as those results agree among themselves. One important consequence of the theory is that for orbits with the same quantum numbers the wave number must be larger for the atom which has the larger kernel. The *quantum numbers* for the *p*, *d*, and *b* terms as given by Bohr in his new theory of atomic structure give the best agreement with this theory.

The radii of the orbits and the field of force around the kernel.—An expression for the maximum radius of a valence electron orbit is derived in terms of the wave number, azimuthal quantum number, strength of the field of the kernel at the outermost point of the orbit, and known constants. For circular orbits, such as the $3d$- and $4b$-orbits, both the field-strengths and radii are computed. These results combined with the size ratios of the kernels give curves for the variation of the field-strength as a function of the radius, for the four atoms. Using the equation and these curves, the radii of the atoms in the normal condition are found. They are: Li, 2.38×10^{-8} cm; Na, 2.72; K, 3.45; Rb, 3.61; Cs, 3.94. The application of this method to the $4b$-terms of the alkaline earth metal spectra shows that the kernel of each must be much smaller than that of the corresponding alkali.

Professor A. Fowler in his *Report on Series in Line Spectra* at the conclusion of the chapter on "Spectra and Atomic Constants" remarks:

These results are in a sense disappointing. It would seem that the spectra must for the present be regarded as constants of the elements which show no simpler relation to other constants than is shown by some of the constants among themselves. Thus, in the alkali group, the curve connecting atomic weights with melting-points, or that connecting atomic weights and atomic volumes is closely similar to that relating the limits of the subordinate series to atomic weights, and a similar discrepancy is shown by potassium in each case. Again there is no simple relation between the atomic weights and densities in this group of elements, just as there is no simple law connecting the limits of the principal series. It can only be concluded that although the spectra change progressively with atomic weights, atomic volumes, or atomic numbers, the

laws governing the changes are not clearly indicated by any of the foregoing investigations.

Apparently there is need for new work in this field, based on new points of view. In Part I of this paper after assuming atomic models in accord with modern theories of atomic structure, I have shown that the differences between the spectra of the different alkali metals can be related to the differences of the sizes of the structures immediately underlying the valence electron. In Part II I have calculated the radii of some of the electron orbits and the approximate mean field of force in the outer parts of the valence electron's orbits.

I. SPECTRA AND RELATIVE SIZES OF THE POSITIVE IONS

According to Bohr's new theory of atomic structure, and also to Bury's[1] modification of Langmuir's theory, the alkali metal atoms are built of an inner structure of nucleus and electrons surrounded by an outer shell or group of eight, similar in structure for all, and a still more loosely attached single valence electron. It is reasonable to suppose that the great similarity between the spectra of these elements is due to this similarity in structure. Departure from the simple hydrogen spectrum and differences between the spectra of the different elements will arise chiefly because of differences in the outer group of eight. This must be accurately true for lines due to orbits far outside the group of eight, and at least approximately true for all orbits which do not come into the region of strong forces from the second inner group. In what follows I assume, therefore, that for these outer orbits the atom can be considered as a nucleus of charge $+9e$, surrounded by a configuration of eight electrons and a valence electron, and that the configuration of eight has a mean field of force which is geometrically similar but differs in size for the different atoms. This latter assumption can be no more than a good approximation, but it seems reasonable enough and is strongly suggested by the similarity of the spectra of these elements. The change of the outer orbits, and therefore of the spectra, from one element to another of higher atomic number will be due, then, primarily to the change of the size of the struc-

[1] C. R. Bury, *Journal of the American Chemical Society*, 43, 1602, 1921.

ture of eight, and only indirectly due to the increasing inner complexity. I shall refer to the structure beneath the valence electron as the kernel.

It is necessary to consider the possibility of, and conditions for, geometrically similar atoms of different sizes. Imagine two such atoms geometrically similar but different in size, oriented the same way, and consider the similar orbits of the outer electron. Let atom A' be k times as large as atom A. Since the forces acting upon the outer electron are assumed to be electrical they vary as the inverse square of the distance and since in atom A' all distances are k times as great, all forces and proportional accelerations will be $1/k^2$ times as great. With acceleration $1/k^2$ times as great and distances k times as great, it follows immediately from the theory of dimensions that corresponding velocities in A' will be $1/\sqrt{k}$ times as great as those in A. Further application of the theory of dimensions shows at once that the energies, kinetic and potential, at each point of A' will be $1/k$ of the energies at the corresponding point of A, and the angular momentum will be $\sqrt{k}$ times as great. If orbit A is an actual one with constant energy and angular momentum, orbit A' is a possible one, since it, too, has constant energy and constant angular momentum. The azimuthal quantum numbers for A' will be $\sqrt{k}$ times those for A, since they equal the angular momenta divided by $1/2\pi$ times Planck's constant h. Other quantum numbers will be increased in the same way since they are obtained by dividing quantities of the dimensions of angular momentum by h alone, or by h and a numerical factor. The wave numbers ν of the spectral terms of A' will be $1/k$ times the corresponding wave numbers for A, since these are proportional to the energies.

Consider a sodium atom with the outer electron in some particular orbit and assume for the sake of illustration that a potassium atom has a kernel which is 1.5 times as large. What will be the energy and quantum number of the geometrically similar orbit in the potassium atom? As shown above, the quantum numbers will be $\sqrt{1.5}$ or 1.227 times as great, and the energy, $1/1.5$ or 0.667 times as great. Thus if we take any series of terms, such as the p-terms, of sodium and plot the wave numbers ν as ordinates and

the corresponding total quantum numbers as abscissas we obtain a curve characteristic of sodium which may be transformed into a curve characteristic of potassium by multiplying these ordinates by 0.667 and the abscissas by 1.227. All *p*-orbits of sodium have azimuthal quantum numbers 2. The orbits corresponding to points on the new curve for potassium have azimuthal quantum numbers 1.227 times 2, or 2.454. These new orbits are geometrically similar to the corresponding orbits in sodium not only as regards the parameters of the orbits themselves, but also as regards the relation of the orbits to the kernel. They are dynamically possible orbits, but will not be actual orbits, permitted by the quantum theory, since their azimuthal quantum numbers are not integers. We may designate these "in between," or fractional quantum number orbits as "hypothetical" orbits. The true ratio between the sizes of the sodium and potassium kernels should be that ratio which makes all the actual orbits of sodium transform into such a scheme of hypothetical potassium orbits as will be consistent with its actual orbits.

The difficulty in the immediate application and testing of this idea is that we do not yet know enough about the field of force surrounding the kernel to say just what the curve for orbits with these hypothetical azimuthal quantum numbers should be, i.e., how the hypothetical orbits of fractional azimuthal quantum numbers should be related to the actual orbits, and therefore we have no basis for the acceptance or rejection of various assumed ratios of the sizes.

There is a method of procedure, however, if we restrict our consideration to circular orbits. According to both Bohr and Sommerfeld the $3d$ and $4b$ terms of the four alkali metals Na, K, Rb, and Cs correspond to circular orbits, with azimuthal quantum numbers 3 and 4 respectively, and radial quantum numbers 0.[1] For the Cs atom, for instance, there will be some curve with ν for circular orbits for ordinates and azimuthal quantum numbers for

[1] Different numbers for some of these orbits are given in the table in the appendix to Foote and Mohler's book, *The Origin of Spectra.* Dr. Foote has told me that the book is in error, that the quantum numbers given here are the ones which Bohr believes to be correct.

abscissas. The $3d$- and $4b$-terms give two points on this curve. We have no other orbits of this class actually known for Cs, but these two are but a special pair of the infinite number possible as far as the mechanics of the structure is concerned. These other mechanically possible circular orbits are the "hypothetical" orbits with fractional quantum numbers, some of which can be obtained by transformation of the $3d$- and $4b$-orbits of Na, K, and Rb according to their sizes. If we now imagine the Na, K, and Rb atoms to be expanded so that their kernels will have the size of the Cs kernel, their six circular orbits will become hypothetical circular orbits of the Cs atom. The problem is to find the size factors which will bring all these points on a smooth curve. That is what I have done, essentially, in the following, but by an indirect method which is easier and more certain than it would be to try to do the thing directly.

If we take the equation $\nu = N/n^2$, where N is a constant and plot ν as a function of n, this curve will transform into itself by the foregoing transformation. That is, if we take the values for n and ν for any point on the curve, multiply n by any factor $\sqrt{k}$ and divide ν by k, the new values of ν and n will satisfy the equation and give a new point on the same curve. If we let N be the Rydberg constant, the values of ν corresponding to integral values of n are the wave numbers of the terms of the hydrogen spectrum. I shall refer to values of ν on this curve as ν_H. Take ν for some term of the Na spectrum corresponding to a total quantum number n and find the ratio of this ν to ν_H for the same total quantum number. The geometrically similar orbit in an atom which is k times as large will have a value of ν $1/k$ times as great and a value of $n\sqrt{k}$ times as great. The value of ν_H corresponding to this new n will also be $1/k$ times as great as the old one, so the ratio ν/ν_H will be the same for both of these similar orbits. If we plot these ratios ν/ν_H for Na as a function of n, we get the curve for an atom k times as large, simply by multiplying the values of n by $\sqrt{k}$, leaving the values of ν/ν_H the same. It is more convenient to plot $\log \nu/\nu_H$ because one uses logarithms in the computation of the ratios, and to plot $\log n$ because the multiplication of the values of n by $\sqrt{k}$ is replaced by the addition of $\frac{1}{2} \log k$ to $\log n$, a simple lateral shift in the direction of the abscissas, $\log n$.

The following table gives the available data. The mean between the two d-terms is taken to be the true value for ν, the separation apparently being due to some magnetic effect not considered here.

The range of values of log ν/ν_H is too great for satisfactory plotting on one curve, so I have multiplied them by 10^5 and again taken the log. If there is to be a smooth curve representing log ν/ν_H as a function of log n, there will also be a smooth curve for

TABLE I

	ν	Mean ν	log ν	log ν/ν_H	log(10^5·log ν/ν_H)
			$3d$		
Na..........	12276	12276	4.08906	0.00317	2.501
K...........	$\begin{Bmatrix}13467.5\\13470.3\end{Bmatrix}$	13469	4.12934	0.04345	3.638
Rb..........	14334.3 -missing	14334	4.15637	0.07048	3.848
Cs..........	$\begin{Bmatrix}16905\\16807\end{Bmatrix}$	16856	4.22676	0.14087	4.148
H...........	12187	12187	4.08589	....	....
			$4b$		
Na..........	....	6860.4	3.83635	0.00034	1.531
K...........	....	6878.5	3.83749	0.00148	2.170
Rb..........	....	6897.6	3.83870	0.00269	2.430
Cs..........	....	6932.6	3.84090	0.00489	2.689
H...........	....	6855.0	3.83601	....	....

log 3=0.477; log 4=0.602

log (10^5·log ν/ν_H) as a function of log n. I have actually plotted the data in the last column of Table I.

By a shift of 0.024 for Rb, 0.044 for K, and 0.142 for Na, seven of the eight points are brought on to a smooth curve, which is almost a straight line, and which gives the relation between wave number and quantum number for the actual and hypothetical circular orbits of Cs. Similar curves, parallel to this, could be drawn for the circular orbits of the other elements. One of the points for Na

alone falls off the smooth curve. No shifts deviating markedly from these would give any reasonable sort of a curve through all of these points, for it is highly improbable that the curve should have many inflections. These shifts give $\frac{1}{2}$ log k where k is the ratio of the size of the Cs kernel to that for the other atom concerned.

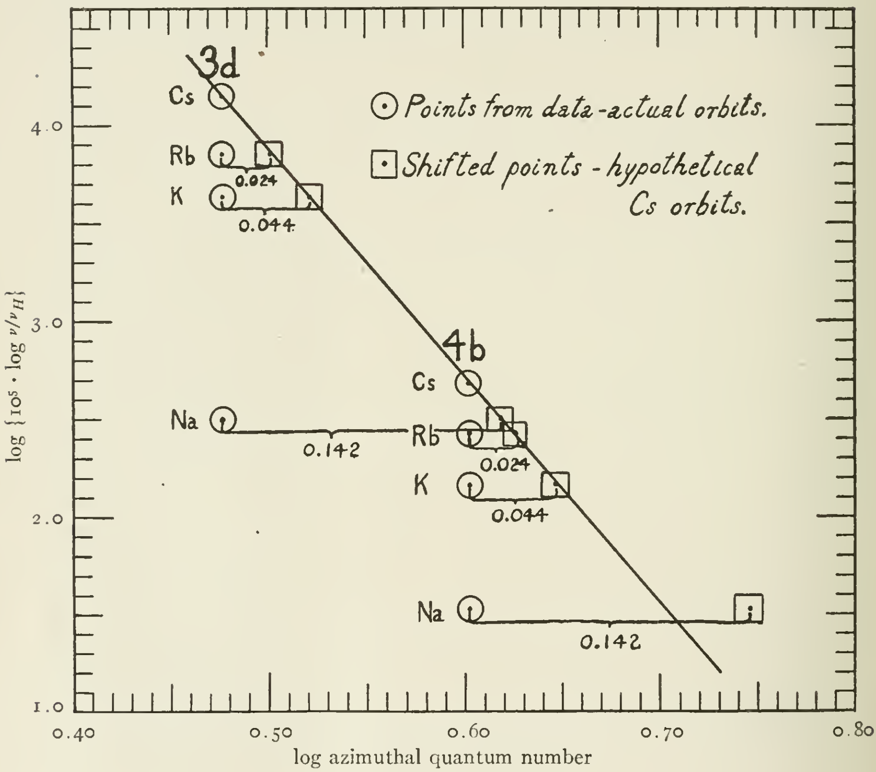

FIG. 1

The values of k are as in Table II. I shall reserve discussion of them until I have found another set of values in a similar manner.

One important result of this theory, even if it should be only a rough approximation, is that the values of ν must be *larger* for the same kind of an orbit in an atom of larger kernel. Otherwise, a shift of this sort would be in the opposite direction from that above and would indicate that the size of the atom increased

with decreasing atomic number. Thus, contrary to Sommerfeld's assignment of quantum numbers, the $2p$-terms of the alkalis cannot all have the same total quantum numbers, for they *decrease* in value as one goes to atoms of higher atomic number. The same can be said for the $3d$-terms in the enhanced doublet spectra of the alkaline-earth elements. The total quantum number for the first p-term will have to be at least one greater for each atom as one goes up the scale of atomic numbers. That is precisely the way that Bohr has assigned the numbers in the new theory, according to which the azimuthal quantum number for all the p-terms is 2, and the total quantum numbers for the first terms for each atom are as follows: Na, 3; K, 4; Rb, 5; Cs, 6. These are indicated in his notation, which I shall use, as 3_2, 4_2, 5_2, and 6_2 orbits.

TABLE II

	$\frac{1}{2}\log k$	$k=R_{Cs}/R_x$
Rb	0.024	1.12
K	0.044	1.23
Na	0.142	1.92

It is not possible to test this size theory in just the same way with the p-orbits as with the $3d$- and $4b$-orbits, because the orbits, not being circular, lie in regions of variable force, and we do not know enough about the nature of this variation to properly interpret the relations between the "hypothetical" orbits. Failing this, we can still consider some particular cases if we make the plausible assumption that there should be a relation between ν/ν_H and total quantum number for such orbits as have the *same ratio of azimuthal to total quantum number.* In a hydrogen-like atom, this would mean geometrical similarity of orbits, and there would be no new assumptions involved in treating the case as we treated the case of circular orbits. In the cases of the alkali metals, the additional assumption involved is that for orbits with the same ratio of azimuthal to total quantum number the variations of wave number depend in a continuous manner on the azimuthal quantum number for each atom. The curves representing this relation must shift into one another by the size transformation used above, since the

change of size does not alter the *ratio* between the two quantum numbers.

As an example, we can take the 6_2 and the 9_3 terms for all the elements. This will be the first p-term for Cs, the second for Rb, the third for K, and the fourth for Na, and the seventh d-term for all. For all of these, the ratio of the total to the azimuthal quantum number is 3. The same argument applies to these orbits as to the

TABLE III

	ν	Mean ν	$\log \nu$	$\log \nu/\nu_H$	$\log(10^3 \cdot \log \nu/\nu_H)$
			6_2		
Na..........	{4151.3, 4152.8}	4152.0	3.618	0.134	2.127
K...........	{6001.2, 6009.3}	6005.2	3.778	0.294	2.468
Rb..........	{9896.6, 9974.1}	9935.3	3.996	0.512	2.709
Cs..........	{19672.3, 20226.3}	19949.3	4.300	0.816	2.912
H...........	....	3047	3.484	....	....
			9_3		
Na..........	....	1357.2	3.132	0.001	0
K...........	....	1433.6	3.156	0.025	1.397
Rb..........	....	1464.1	3.165	0.034	1.544
Cs..........	....	1506.1	3.177	0.046	1.662
H...........	....	1354	3.131	....	....

log 2 = 0.301; log 3 = 0.477

circular ones before, and the same process of finding the shifts to bring all the points on one curve can be applied. It should be emphasized that this does not necessarily imply that the 6_2 and 9_3 and other such orbits in any one atom are necessarily geometrically similar, but simply that, in the nature of the case, we would expect to find some curve giving the value of ν as a function of azimuthal quantum number for all such orbits.

The data are given in Table III. Figure 2 shows the curve obtained.

The shift for Rb is 0.023, and that for K is 0.052. The upper Na point comes on to the curve by a shift of 0.098, but the lower could not be made to lie on the curve. As in Figure 1, these shifts bring all but one point on to a smooth curve, but the agreement

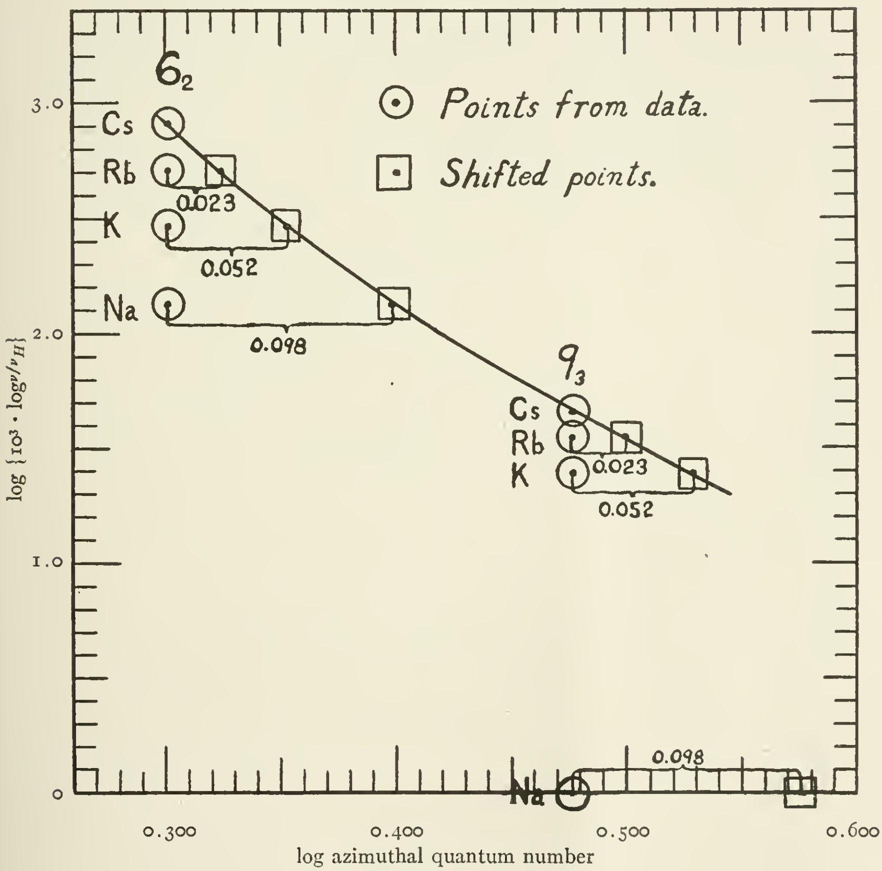

FIG. 2

of these shifts with those calculated from Figure 1 is not very impressive, unless one considers that it is rather remarkable that there should be any agreement at all for data picked in this special way unless there was some truth in the theory governing the choice. There are two good reasons for expecting failure of perfect

agreement. One is the probable deviation of the fields of the kernels from exact geometrical similarity even up to considerable distances from the center. The other is the fact that in the *p*-orbits the electrons are thought to penetrate much nearer the central structure than in the *d*-terms, probably near enough so that the assumption of a nucleus of point charge $+9e$ is no longer strictly valid. In this connection it is noted that the best agreement between the shifts in Figures 1 and 2 is found for those elements whose inner structure is, relatively, least different.

The same kind of a curve can be made for the 8_2- and 12_3-orbits. The available data are not complete, however. Using the shifts

TABLE IV

	Mean ν	log ν	log ν/ν_H	log ($10^3\cdot$log ν/ν_H)
		8_2		
Na	2150.2	3.332	0.098	1.991
K	2779.4	3.444	0.210	2.322
Rb	3845.1	3.585	0.351	2.545
Cs	5655.0	3.743	0.509	2.706
H	1714	3.234	...	...
		12_3		
Na	761.7	...	...	...
K	...	...	...	...
Rb	...	...	...	...
Cs	825.8	2.917	0.035	1.544
H	761.0	2.882	...	...

from the last curve, these points are brought into line. The 12_3 for Na (not plotted) would be badly off again. Figure 3 shows this curve.

The results of the two methods are as in Table V. The average value of $\frac{1}{2}$ log k is used to get the ratios of the sizes. The corresponding ratios of Bragg's radii of the ions in crystals and also of the radii of the nearest rare gas atoms as determined from (1) the constant b in Van der Waal's equation and (2) the mean free paths are given for comparison. Very close agreement would not be expected here because there is no theory adequate to relate the region belonging to an ion in a crystal to its actual size or to give an idea of its probable deformation in the structure of a crystal, neither do we

know just what relation the size of the sphere of influence of a rare gas atom bears to the actual size of the atom, or to that of the kernel of the following alkali metal.

We see, therefore, that not only does this theory lead to a fairly good co-ordination of the spectra of the alkali metals in terms of the assumed sizes of their atomic kernels, but also that these assumed

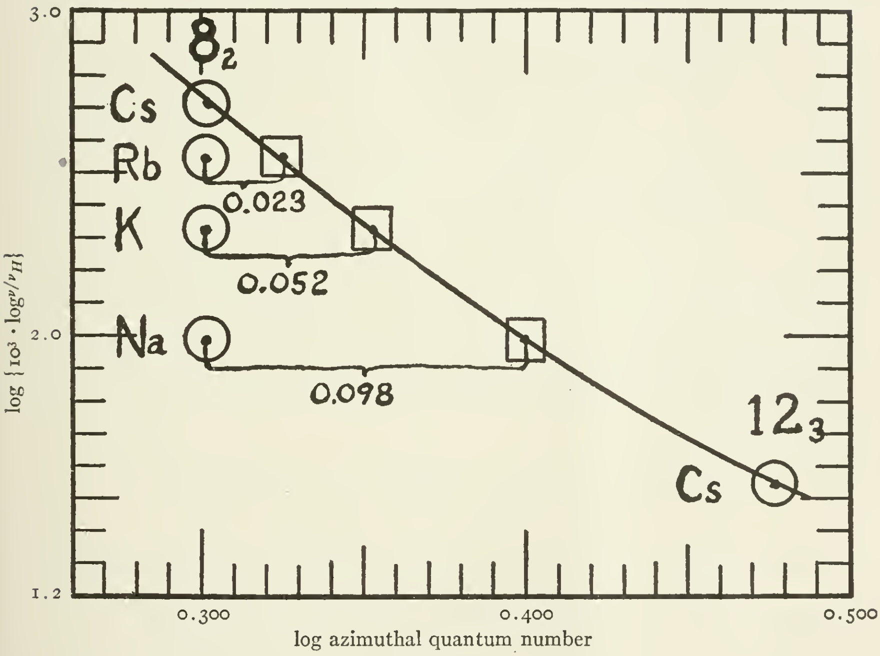

FIG. 3

sizes are reasonable ones in view of the meager data available for comparison.

This theory may be used to test the correctness of the quantum numbers assigned to the various spectral terms—a matter of interest because Bohr's and Sommerfeld's numbers differ. Such a test cannot be made from the spectra themselves, since a difference in the assignment of quantum numbers can be compensated by an equal opposite change in the value of the other term constants, whose values have not hitherto been known by any independent method. I have tried various other choices of quantum numbers

and have found no case in which the predictions of the present theory are satisfied with as good a degree of consistency as is shown in the instances cited above. These results are, therefore, a confirmation of the numbers assigned by Bohr.

It can be concluded from this work that the size of the kernel is very important in determining the difference between the homologous terms of the spectra of the alkali metal atoms and that Bohr's assignment of the quantum numbers for these terms is in agreement with this theory.

TABLE V

	$\frac{1}{2}\log k$		R_{Cs}/R	R_{Cs}/R	$R_{Xe}/R(Kr, A, Ne)$	$R_{Xe}/R(Kr, A, Ne)$
Rb.........	0.024	0.023	1.12	1.06	1.09	1.30
K...........	0.044	0.052	1.25	1.14	1.20	1.325
Na..........	0.142	0.098	1.74	1.34	....	1.85
	$3d$, $4b$	6_2, 9_3 8_2, 12_3		Bragg	Van der Waal	m.f.p.

II. THE RADII OF THE ORBITS AND THE FIELD OF FORCE AROUND THE KERNEL

According to Bohr's new theory the valence electron in an alkali metal atom describes an orbit which is much like a rotating ellipse. At its outermost position it is thought to be at quite a distance from the center of the atom as compared with the distances between the other parts of the atom. At this outermost position its velocity vector must lie in a position perpendicular to the radius vector. Call this radial distance r, the energy E, the velocity v, the mass m, the electronic charge e, Planck's constant h, and the azimuthal quantum number k. Denote the potential energy of the electron by $-pe^2/r$. e^2/r is the potential energy of an electron at a distance r from a charge of $+e$, so p is the potential energy factor or apparent charge on the equivalent point nucleus. In general p is a function of r in virtue of the second and higher order terms in the expression for the potential due to any assemblage of charges, and it approaches the value unity to the extent to which the potential can be consid-

ered as due to a single point charge. From the conditions that the energy and the angular momentum are constant one gets the two equations

$$mvr=\frac{kh}{2\pi} \qquad E=\tfrac{1}{2}mv^2-p\frac{e^2}{r}$$

which combine to give

$$E=\tfrac{1}{2}m\frac{k^2h^2}{4\pi^2m^2r^2}-\frac{pe^2}{r} \qquad \text{or} \qquad r^2+p\frac{e^2}{E}r-\frac{k^2h^2}{8\pi^2mE}=0.$$

Solving for r gives

$$r=\frac{-p\dfrac{e^2}{E}\pm\sqrt{\dfrac{p^2e^4}{E^2}+\dfrac{k^2h^2}{2\pi^2mE}}}{2}=\frac{-pe^2}{2E}\left(1\pm\sqrt{1+\frac{k^2h^2E}{2\pi^2me^4p^2}}\right).$$

This equation can be put in a form more convenient for application by putting in different constants as given by Bohr's theory of the hydrogen atom. The radius of a normal hydrogen atom is $a_0=\dfrac{h^2}{4\pi^2me^2}$. The Rydberg constant, $N=\dfrac{2\pi^2me^4}{h^3c}$. Their product $a_0N=\dfrac{e^2}{2hc}$. If ν is the wave number for a spectral term or stationary state of the atom, $E=-hc\nu$. Making these substitutions we get

$$r=\frac{pe^2}{2hc\nu}\left(1\pm\sqrt{1+\frac{-k^2h^3c\nu}{2\pi^2me^4p^2}}\right)=p\frac{N}{\nu}a_0\left(1\pm\sqrt{1-\frac{k^2\nu}{p^2N}}\right).$$

It is apparent from the derivation that these two values of r are the same as the aphelion and perihelion radii for an elliptical orbit in a hydrogen-like atom with a charge of $+pe$ on the nucleus. These two radii, in terms of the semimajor axis and eccentricity of the ellipse are $(1+e)a$ and $(1-e)a$. Hence the expression under the radical sign gives the eccentricity. It is possible to find the values of p and r for a circular orbit, since the eccentricity is equal to 0. The radical can be set equal to 0 and the value of p thus obtained used in finding r. By this method the following values for p and r have been calculated.

It is seen that these orbits have approximately the radii $9a_0$ and $16a_0$ of the corresponding hydrogen orbits, the agreement

being more exact for the outer orbit and for the atom with the smallest nucleus. Under the same conditions the potential energy factor p is nearly unity, but it increases rapidly at distances nearer the nucleus, especially for the larger atoms.

Consider one of the atoms where the value of p is a certain quantity at a distance r from the center. What will be the value of p at the corresponding distance from the center in an atom k times as large? The corresponding distances will be kr and the

TABLE VI

$a_0 = 0.532 . 10^{-8}$ cm

	Mean ν	p	r	r (cm)	Mean ν	p	r	r (cm)
	3d-Terms				4b-Terms			
Na....	12276	1.0037	8.97 a_0	$4.77 \cdot 10^{-8}$	6860.4	1.0004	15.99 a_0	$8.50 \cdot 10^{-8}$
K.....	13469	1.0513	8.57 a_0	4.61	6878.5	1.0017	15.98 a_0	8.49
Rb....	14334	1.0845	8.29 a_0	4.40	6897.6	1.0031	15.95 a_0	8.48
Cs....	16856	1.1761	7.64 a_0	4.06	6932.6	1.0057	15.90 a_0	8.46

new potential energy $1/k$ times the former. Let primed symbols refer to the larger atom. Then

$$E = -p\frac{e^2}{r} \qquad E' = -p'\frac{e^2}{r'}$$

$$E' = \frac{E}{K} \qquad r' = rK \qquad \therefore \quad p' = p$$

Hence p is the same for both, as would be expected from the principle of dimensions. If then we plot values of p as ordinates and values of r as abscissas there will be a curve for each atom and these curves will be transformable one into the other by multiplying the radii by the proper size ratio factors as found before. For reasons as before, it is better to plot log $\{(p-1) \cdot 10^4\}$ and log r. The curves will then all be the same but shifted along the log r axis. The data plotted are as in Table VII.

Figure 4 shows the curves obtained using the size factors found from the d and b terms in Part I. These curves give, within limits, the way in which the potential energy factor varies with distance from the center for the various atoms. They permit a calculation

of the aphelion distances of any orbits which lie within these limits of distance.

Fues[1] has shown that the spectrum terms of Na can be accounted for by assuming that the electron orbits are in a central field of force which varies with distance from the center in approximately the same way as the field about a central nucleus of charge $+9e$ surrounded by eight electrons at the corners of a cube varies along a radius perpendicular to a cube face. It is of interest to see whether the field of force about the kernel which these curves give is of that sort. The equation for the potential energy at a distance

TABLE VII

	$\log \{(p-1)\cdot 10^4\}$	$\log r$	$\log \{(p-1)\cdot 10^4\}$	$\log r$
	d		*b*	
Na	1.568	0.952	0.602	1.203
K	2.710	0.933	1.230	1.203
Rb	2.927	0.918	1.491	1.202
Cs	3.246	0.883	1.756	1.201

r from the center along a line perpendicular to the face of the cube of side $2a$ is

$$V=-\frac{9e^2}{r}+\frac{4e^2}{\sqrt{(r-a)^2+2a^2}}+\frac{4e^2}{\sqrt{(r+a)^2+2a^2}}.$$

Expanding in powers of a/r by the binomial theorem gives

$$V=-\frac{e^2}{r}\left\{1+\frac{84}{3}\frac{a^4}{r^4}\right\}$$ plus higher power terms negligible for small a/r.

To a first approximation then the expression for p is

$$p=1+\frac{84}{3}\frac{a^4}{r^4}.$$

It follows that

$$\log\{10^4(p-1)\}=\log\left\{\frac{84}{3}\cdot 10^4\cdot a^4\right\}-4\log r$$

so the $\log\{10^4(p-1)\}-\log r$ curves should be straight lines of

[1] E. Fues, *Zeit. für Phys.*, 11, 6, p. 364, 1922.

slope -4. The curves are a fair approximation to straight lines. The slope of the line drawn with the Cs curve on Figure 4 is -5.9. The value of p falls off more rapidly with increasing ν than for the cube. The slope of the straight line through the two Na points, however, is -3.85, in fair agreement with Fues's assumption. Apparently the assumption of geometrical similarity of the field

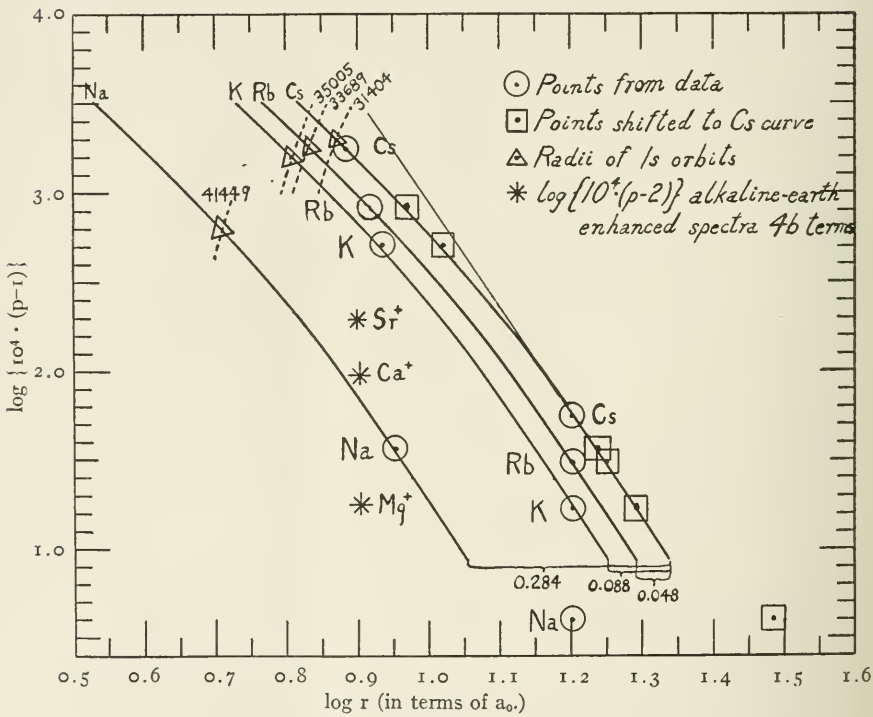

FIG. 4

about the kernels is a fair approximation for K, Rb, and Cs, but is not very accurate when Na is compared with one of the others.

With these curves it is now possible to find the radii of the electron orbits for the normal state of the atom, corresponding to the $1s$ spectroscopic level, since it is found that the electron comes into the region of the field represented on these curves. The quickest way to do this is to take a value of ν for a $1s$-term, such as 44149 for Na and by use of the equation $r=p\frac{N}{\nu}\left(1+\sqrt{1-\frac{k^2}{p^2}\frac{\nu}{N}}\right)a_0$

find corresponding pairs of p and r for this ν. That is, taking a fixed value for ν, assign various values to p and calculate the corresponding values of r. Plot these points on the same sheet with the $p-r$ curve for the Na atom and the intersection of the two curves will give the desired radius, since the values of p and r at that point fit both the field of the atom and the particular value of ν. I have plotted these curves as dotted lines on Figure 4, using the logarithms of $\{10^4 \cdot (p-1)\}$ and r. The radii obtained for the atoms are as follows. The one for Li is obtained on the assumption that $p=1$. The curve for $\nu=41449$ is steep so that the value of log r for the intersection would not be changed much even though the curve for the field around the Na kernel may have a smaller slope.

TABLE VIII

Metal	log r	r	r(cm)
Li	....	$4.48a_0$	$2.381 \cdot 10^{-8}$
Na	0.709	5.12	2.721
K	0.809	6.45	3.43
Rb	0.831	6.78	3.61
Cs	0.870	7.41	3.94

These values should be closely proportional to the radii of the atoms in the gaseous state. Unfortunately there are no available data on these radii.

Crystal analysis experiments indicate that metallic Na and K crystallize with the atoms arranged in a body-centered cubic structure with cube edges of 4.30 and 5.20 A respectively. The distance between nearest atoms in such structures are 3.72 and 4.50 A. These orbital radii are of the same order of magnitude as the internuclear distances in the crystal structures, making it seem highly improbable that in the crystal the atoms are in the neutral state, each with its own valence electron. It seems to indicate that the atoms must be present as positive ions, the electrons fitting into the structure between the ions rather than following their usual orbits.

Making the assumption that the 4*b*-terms of the enhanced spectra of the alkaline-earth metals correspond to circular orbits of quantum number 4, it is possible to calculate r and p for these

orbits as above. This term for Ba shows an anomalous behavior in that it is less than the one for Sr, so I have not used it. It may correspond to an elliptical orbit of quantum numbers 5_4. In this case, the value of $(p-2)$ represents the change of the field because of the extended structure of the nucleus. I have plotted corresponding values of log $\{10^4(p-2)\}$ and log r for these terms, on Figure 4. They show clearly that the kernel is much smaller for

TABLE IX

Element	$\nu-4b$	p	r	log $\{10^4(p-2)\}$	log r
Mg+........	27467.4	2.0018	7.99 a_0	1.255	0.902
Ca+........	27686	2.0097	7.96 a_0	1.986	0.901
Sr+.........	27960.4	2.0198	7.91 a_0	2.297	0.898
Ba+........	25288.8	....	....	....	....

any alkaline-earth metal atom than for the corresponding alkali metal atom. The curve for any alkali metal atom lies to the left of the curve for any one which has a larger kernel. Each of these alkaline-earth metal points lies well to the left of the curve for the corresponding alkali metal. Such shrinkage would be expected with the increase of nuclear charge. The data are as in Table IX.

In conclusion, I wish to express my indebtedness to Professor K. T. Compton for valuable advice and help in this work.

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WAVE-LENGTHS OF LINES IN THE SPECTRA OF STARS OF CLASS Me[1]

By PAUL W. MERRILL

ABSTRACT

Wave-lengths of emission and absorption lines in the spectra of numerous long-period variables.—An intercomparison of the measurements of the *bright lines* $H\beta$, $H\gamma$, $H\delta$, $H\zeta$, $\lambda\lambda$ 4571 Mg, 4308 Fe, 4202 Fe, and 3905 Si, shows that the relative wave-lengths of these lines are nearly the same in the variables as in the laboratory. An additional list of 14 bright lines is given, each of which has been measured on 5 or more spectrograms. Most of these lines have not been identified. Ninety-five *absorption lines* are listed, over half of which are identified with low temperature lines of metals. The temperature of the reversing layer near the time of maximum light appears to be about 2200°.

As by-products of an investigation of the radial velocities of long-period variables,[2] numerous wave-length determinations of the lines used for velocity and of other lines measured incidentally have been accumulating. Since a part of the data for class S stars has been published,[3] the present paper will deal principally with lines in spectra of class Me.

The measurements reported here were all made from spectrograms taken with one-prism dispersion. The Mount Wilson spectrograms upon which are based all of the results for the absorption lines, and most of those for the emission lines, have a dispersion as follows: at $H\beta$, 56.6; at $H\gamma$, 36.1; at $H\delta$, 28.2 A per mm. Measurements with higher dispersion are of course desirable, but the observational work involved would be heavy, because of the many long exposures required to go over the ground already covered by one-prism observations. The one-prism results are therefore published now, as it will probably be several years before they are supplanted by measurements with higher dispersion. A list of the stars observed, with the dates of observation and various details, will be found in *Mount Wilson Contributions* No. 264.

[1] *Contributions from the Mount Wilson Observatory*, No. 265.

[2] *Mt. Wilson Contr.*, No. 264; *Astrophysical Journal*, 58, 215, 1923.

[3] *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, 56, 457, 1922.

EMISSION LINES

Table I contains a list of the emission lines used for radial velocity, the wave-lengths being recent laboratory determinations.

In order to ascertain whether the wave-lengths of these lines have the same relative values in the variable stars as in the laboratory, data showing the mean differences in kilometers between velocities from individual lines have been collected in Table II. The results from the Ann Arbor and Mount Wilson observations (two spectrographs for the latter) are given separately. The figures following a difference indicate the number of plates upon which the difference depends.

TABLE I

LABORATORY WAVE-LENGTHS OF EMISSION LINES

I. A.	Identification
3835.36	$H\eta$
3889.05	$H\zeta$
3905.51	Si
3970.08	$H\epsilon$
4101.74	$H\delta$
4202.03	Fe
4307.91	Fe
4340.47	$H\gamma$
4571.11	Mg
4861.33	$H\beta$

TABLE II

AGREEMENT OF EMISSION LINES

	CLASS Me						CLASS Se					
	100-inch		60-inch		Ann Arbor		100-inch		60-inch		Ann Arbor	
	km	No.	km	No.	km	No.	km	No.	km	No.	km	No.
$H\beta-H\gamma$	−0.5	40	−1.2	18	−0.6	28	−0.3	43	−5.3	19	+2.	7
$H\gamma-H\delta$	+0.4	180	−0.6	75	+2.8	107	+4.8	31	+6.4	11	......	......
$H\delta-H\zeta$	−5.0	14	−5.	2	−10.0	23	......	......	......	......	......	......
$H\gamma-4308$	−3.9	21	−3.4	5	......	......	−1.8	5	......	......	......	......
$\frac{H\gamma+H\delta}{2}-4202$	−0.6	33	+1.0	10	− 5.8	5	−2.	2	......	......	......	......
$H\gamma-4571$	−0.6	20	−5.2	6	......	......	−1.	2	......	......	......	......
$H\zeta-3905$	−2.4	7	−5.5	2	+ 6.4	5	......	......	......	......	......	......

Slight systematic differences between different lines may arise from various observational causes such as atmospheric dispersion, photographic or bisection errors depending on line intensities,

TABLE III

WAVE-LENGTHS OF EMISSION LINES IN SPECTRA OF CLASS Me

I. A.	P.E.	No. Spectrograms	No. Stars	Probable Identification
3852.63	±0.03	5	1	
3977.78	.03	6	1	
4030.51	.03	8	3	Mn 4030.76
4103.03	.01	37	15	Si 4102.95
4138.65	.01⁵	15	9	
4178.82	.01⁵	27	12	
4215.74	.04	6	4	
4233.35	.01	38	16	
4372.58	.01	19	8	
4375.84	.02	16	5	Fe 4375.93
4458.83	.03	10	5	
4511.46	.02	18	7	
4521.54	.04⁵	5	3	
4578.84	±0.03	6	4	

NOTES TO TABLE III

4030.51 This line first appears as a bright edge on the violet side of the low temperature manganese line 4030.76. The identification is probably correct in spite of the considerable discrepancy between the stellar and the laboratory wave-length.

4103.03 A close companion to Hδ. Its distance from Hδ as measured directly is, in the mean, 1.28 A. When added to the wave-length of Hδ, 4101.74, this gives 4103.02, in very good agreement with the wave-length depending on the mean velocity displacements. The measured wave-length may be too great on account of the close proximity of the very strong Hδ line. King's[1] wave-length of the silicon line to which the stellar line may correspond is 4102.95.

4233.35 This can scarcely be the enhanced iron line 4233.14.

4375.84 R Leonis gives a consistently low value for this line. Eight spectrograms of four other stars give a mean value of 4375.94.

4458.83 Marked doubtful on several measures. It is near the head of a titanium oxide band.

4511.46 Occurs also in class Se spectra.

4521.54 Occurs also in class Se spectra.

deviation of certain comparison lines from their adopted values, etc. As errors from several of these causes are necessarily small when the lines are close together, the hydrogen lines nearest to the metallic lines have been chosen for comparison.

[1] *Publications Astronomical Society of the Pacific*, 35, 171, 1923

The hydrogen lines $H\beta$, $H\gamma$, and $H\delta$ are in substantial agreement in the M-type stars. The effective wave-length of $H\zeta$ appears to be about 0.1 A greater than the standard laboratory value. This divergence may or may not be inherent in the stellar rays. In the S-type stars $H\delta$ shows a small displacement toward the violet with respect to $H\gamma$.

The line λ 4308, if correctly identified as the iron line λ 4307.91, is shifted toward longer wave-lengths by a few hundredths of an angstrom; λ 4202 agrees very closely with the assumed laboratory value Fe 4202.03; and λ 4571 Mg and λ 3905 Si agree, within reasonable limits, with their laboratory values. These identifications have recently been called in question by Baxandall.[1] The results in Table II are probably not to be considered as furnishing a conclusive answer to this question, but in so far as they have a bearing on the matter I think they tend to support the assumed identifications.[2]

A number of bright lines have been measured in addition to those in Table I. They have not been used for finding the radial velocity, but their wave-lengths have been deduced on the assumption that they have velocity displacements corresponding to the mean adopted bright-line velocity for each plate. Those lines which have been measured on five or more spectrograms have been collected in Table III.

ABSORPTION LINES

The wave-lengths of the absorption lines which were used for velocity determinations will be found in Table II of *Contribution* No. 264. These wave-lengths have recently been corrected by applying the mean systematic residuals from all the measures, and thus modified appear in Table IV, together with other lines which were not used at all for determinations of velocity but whose wave-lengths were deduced from the measurements. These additional lines were included when five or more determinations were available, provided the probable error from a single spectrogram did not

[1] *Observatory*, **46**, 82, 1923.

[2] The measurements by Adams and Joy of the bright lines of *o* Ceti (*Publications Astronomical Society of the Pacific*, **35**, 168, 1923) are in good general agreement with the results in Table II.

exceed 0.08 A. For those cases in which the adopted value differs from the laboratory wave-length of the line to which it is believed to correspond, the chemical symbol has been placed in parentheses.

TABLE IV

Corrected Wave-Lengths of Absorption Lines in Spectra of Class Me

3998.61 (V)	4147.68 Fe	4306.05
4020.56	49.76	07.75 (Fe)
26.95	52.32	14.66
30.83 (Mn)	73.99	25.89 (Fe)
33.07 Mn	74.37	30.02 V
34.59 (Mn)	79.56 (V)	32.74 (V)
35.87 (Mn)	90.73	37.50 (Cr)
41.29 (Mn)	94.90	44.42 (Cr)
44.21 (K)	97.00	47.21
45.87 (Fe)	4198.53	68.05
54.87	4200.08	75.93 Fe
57.10	02.10 (Fe)	79.38 (V)
63.76 (Fe)	03.63	83.70 (Fe)
76.36	04.94	84.24 (Fe, V)
77.81 (Sr)	06.74 (Fe)	84.71 V
88.16	15.82 (Sr, Fe)	89.5 (Fe, V)
90.50 V	26.73 Ca	91.67
4092.47 (V)	32.59 (V, Fe)	4395.22 V
4105.05 (V)?	34.10	4404.85 (Fe)
09.65 (V)	54.30 (Cr)	08.3 (V, Fe)
11.79 (V)	58.34 (Fe)	12.24
13.69	60.48 Fe	21.95
15.19 (V)	74.80 Cr	27.30 Fe
16.58 (V)	82.73 (Fe)	37.85 V
18.60	85.84 (Ti)	55.25
21.62	87.48 (Ti)	62.03
23.60 (V)	89.59 (Cr)	82.14
28.00 V	91.44 (Fe)	94.37
29.78	94.26 (Fe)	4496.57
32.07 (V)	4299.07	4548.61
34.39 (V)	4300.79	80.23
39.93 Fe	02.67	

The number of discrepancies amounting to three or more times the probable error is surprising. These large differences do not necessarily mean that the identification is wrong, as they may be due to systematic displacements of the lines, caused, for example, by

a partially bright edge or a close invisible companion line. It is possible, of course, that some lines have been erroneously identified. These may be detected later by means of more data, especially by measurements with higher dispersion. In general support of the identifications in Table IV, it may be said that groups of lines which are shown by laboratory work to belong together are fairly consistently represented in the table.

Among the most prominent absorption lines in class Me spectra are the low temperature lines of iron, vanadium, manganese, chromium, calcium, and strontium. The temperature of the reversing layer near the time of maximum light, as indicated by a comparison with King's studies of the behavior of lines in the electric furnace, is about 2200°.

It is hoped at some future time to make a detailed study of the lines present in the spectra of long-period variables, especially of their relationship to temperature, ionization, and spectral series.

Mount Wilson Observatory
May 1923

THE

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MOLECULAR SPECTRA AND HALF-QUANTA

By E. F. BARKER

ABSTRACT

Theoretical relations between half-quantum numbers and band spectra.—Half-quantum numbers designating steady states of molecular rotation have been introduced in several recent discussions of band series. Only integral transitions are considered, but the lowest steady state is assigned one-half quantum of angular momentum. This minimum motion may be attributed to the coupling between molecular and electronic rotations. An effect analogous to the Stark effect for line spectra has been shown by Hettner to depend upon the proper choice of quantum numbers for the band series, and is here suggested as a criterion for distinguishing between integral and half-quantum values. The general modification of the spectrum is of the second order and cannot be observed, but if integral numbers are assumed, one line should show a first-order effect.

Experimental evidence.—An absorption cell containing HCl between plates of platinized quartz 1 centimeter apart was subjected to a field sufficient to spark across a gap of 8 mm in air between 1-inch balls. No change in the absorption spectrum was detected. This experiment therefore supports the hypothesis of half-quanta. Other facts pointing in the same direction are the single missing line at the band center for HCl, and the position of the zero branch for CH_4.

In a very complete analysis of the spectra due to molecular nitrogen, Kratzer[1] has shown that the constants characterizing the various band series are much more consistent with one another when half-quantum numbers instead of integers are employed to describe the rotation states. A similar designation of quantum states for certain infra-red band spectra of the simplest type had previously been suggested by Einstein[2] to account for the fact that

[1] *Annalen der Physik*, 67, 127, 1922, Munich Academy, March 4, 1922.

[2] Cf. Reiche, *Zeitschrift für Physik*, 1, 283, 1920.

a single line is missing at the band center. More recently Kramers[1] has discussed the subject in some detail, and Curtis[2] has shown that the bands of helium apparently form two types, one of which involves half-quantum numbers. In each case, of course, the transitions are integral; no partial changes of state have been observed.

Planck's hypothesis in its second form involves a distribution of the molecules within the phase space such that the average action for all those below the one-quantum level is exactly one-half, the probability of the rotationless state being zero. Thus the half-steps might be attributed to the characteristic probability distribution inherent in the quantum hypothesis, rather than to the peculiar structure of certain molecules. Such an interpretation is not wholly consistent with other phases of quantum mechanics, however, and fails to explain the two types of band series in He. Both Kratzer and Kramers have preferred to attribute the angular momentum characteristic of the lowest state of molecular rotation to a coupling between the motion of the system as a whole and that of the electrons within it.

Sometime ago Hettner[3] suggested a test for the Sommerfeld-Bohr method of defining stationary states, which now becomes especially significant as a means for distinguishing between the two possibilities with respect to the choice of rotational quantum numbers for *HCl*. It involves the modifications to be expected in the absorption band when the absorbing gas is subjected to an intense electric field, and is thus analogous to a Stark effect. On the supposition of discrete stationary states of rotation, defined by integral quantum numbers, Hettner showed that for polar molecules a transverse electric field should introduce precessional motions, with resulting displacements of the energy levels. Each line in the band series would thus be resolved into two or more components, usually lying close together, and more or less symmetrically distributed about the position corresponding to zero field. A possible indication of such resolution has been found in the visible bands of

[1] *Zeitschrift für Physik*, **13**, 343, 1923.

[2] *Proc. Royal Soc.*, **101**, 38, 1922; also *Nature*, June 3, 1922.

[3] *Zeitschrift für Physik*, **2**, 349, 1920.

nitrogen,[1] but for observations in the infra-red the definition is so poor compared with that obtained photographically that there would be little hope of observing this general effect. For one particular line, however, a relatively large and very characteristic modification is predicted. If there occurs a transition corresponding to the change $1 \rightarrow 0$ in m, the molecule in its final state would be lined up with the field and there would be no precession. In this case the two energy changes (from initial states with precessions in either of the two possible directions) would each be larger than that corresponding to the normal transition with zero field, and their difference would also be relatively large. As a consequence, the first line on the low-frequency side of the band center should be separated into two components, both being displaced from the normal position toward longer wave-lengths. Hettner computed the displacement to be expected for HCl, assuming an electric moment equal to the product of the electronic charge by the distance between nuclei as determined from the moment of inertia, and a field intensity of 50,000 volts per centimeter. He found it to be nearly half the normal separation of lines in the band, and hence, under these conditions, easily observable.

Some months ago the writer, then a Fellow under the National Research Council, undertook to investigate this effect experimentally. The results were not published at that time since they proved to be negative, and it was felt that, in the absence of an adequate explanation, further observations should be attempted with larger and more precisely determined field intensities. The problem of maintaining intense and fairly uniform fields across an absorption cell of rather large dimensions containing gas at atmospheric pressure is a somewhat difficult one, and the data which have been obtained can hardly be considered as more than preliminary. It appears, however, that if Hettner's hypothesis were correct, some slight effect should have been observable under the conditions employed, even though his assumed value for the electric moment is probably much too large. On the other hand, if the rotationless state is an impossible one, the lowest quantum number being one-half, precessions will always appear when the

[1] Datta, *Astrophysical Journal*, 57, 114, 1923.

gas is subjected to an external field. The energy differences between states will then differ from those corresponding to zero field by small positive and negative amounts, and the various lines of the series will be separated into narrow groups. The integral effect appearing in the absorption curve, where these groups cannot be resolved, may depart only imperceptibly from the normal, and an analysis of the band under the influence of the field thus yield negative results. In particular, the two lines adjacent to the band center would be correlated with the

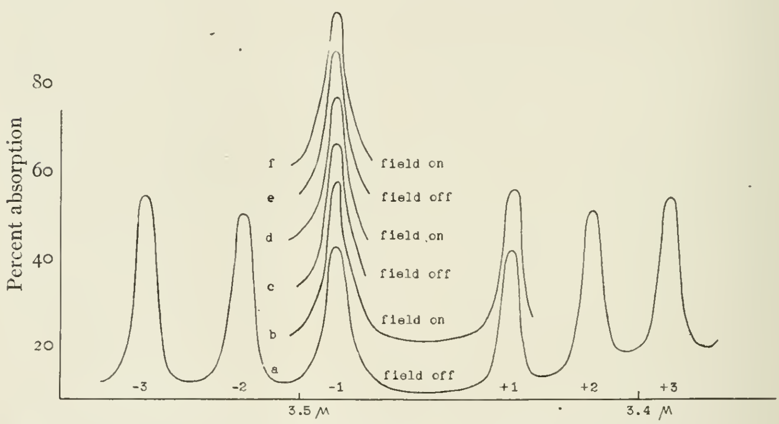

FIG. 1.—*a*. Central portion of the *HCl* absorption band, without field.
b, *c*, *d*, *e*, and *f*. Field alternately on and off, ordinates displaced upward.

same transition ($\frac{1}{2}\rightarrow\frac{3}{2}$ or $\frac{3}{2}\rightarrow\frac{1}{2}$) and no unsymmetrical displacement of one of them should be observed. This is entirely in accord with experimental results, a few of which are shown in Figure 1.

The apparatus employed is the same as that previously described,[1] except for the absorption cell. This consists of a bakelite box with mica windows, inclosing two plane parallel electrodes of platinized quartz 1 centimeter apart and 2 centimeters wide. The edges of the electrodes are rounded, and the windows placed about 2 millimeters from them to avoid leakage across the mica surfaces. The chamber is connected with a reservoir consisting of a large deflated rubber balloon, so that the windows are not

[1] Barker, *Astrophysical Journal*, 55, 391, 1922.

deflected by changes in pressure. The field applied was sufficient to produce a spark in air across an alternate gap of 8 millimeters between smooth 1-inch balls. Even with this field a brush discharge sometimes occurred within the cell. The grating was ruled with 7200 lines per inch in the echelette form, upon a surface of copper nickel alloy, with an effective area of 4 by 6 inches. Deflections of over 100 millimeters are obtained at 3.5 μ. Absorption curves were taken with the field alternately on and off, the position of the cell remaining unchanged. Since the absorbing gas was not subjected to a uniform field, a complete separation of lines into their respective components could not be expected; there is, however, no indication of a widely displaced component on the low-frequency side of -1, and indeed no observable broadening of any of the lines. This may, of course, mean that the field obtained was insufficient, or that the predicted values of the displacements were very much too large. It seems, however, that a slight distortion of the curve should have been detected if any distinctly unsymmetrical resolution had occurred. As a check upon these observations the spectrometer was set at the maximum of the line -1, and a number of observations taken with the field alternately on and off. No lowering of the absorption could be detected. Similarly, no increase in absorption occurred at the minimum position between -1 and -2. The spectrometer was then set for a point where the slope of the curve is greatest on the low-frequency side of -1, and where a very slight shift of the line would make a relatively large change in absorption. Observations here showed no noticeable effect. These results seem to be in agreement with the hypothesis of half-quanta for rotation.

It may be remarked that other experimental facts point in the same direction. The absence of a single line at the band center, instead of a pair as predicted by Reiche, is thus most simply explained. When integral values are assigned to m we must suppose that the transition $0 \rightarrow 1$ does not occur in absorption, but that $1 \rightarrow 0$ is permissible. Observations show, however, that the lines $+1$ and -1 are of almost equal intensity, which is hardly consistent with the idea that the final state for one of them is an abnormal one (see Fig. 2). As Kratzer has pointed out, if half-

quanta are used, both these lines may be correlated with the same pair of rotation states, $\frac{3}{2}$ and $\frac{1}{2}$, and the missing line, $\frac{1}{2}\rightleftarrows\frac{1}{2}$, would correspond to a reversal in sign for the minimum angular velocity without change in energy of rotation, an effect not to be expected as a result of simple absorption.

Infra-red band series of this type with a single absent line have been observed for the hydrogen halides, and possibly also for methane.[1] In the latter case the intensity distribution of the lines is the only criterion, since the band center is masked by an intense absorption due to the unresolved zero branch, which completely hides the line -1 also. The presence of this zero branch is of particular interest, however, in the determination of m values. It probably originates in vibrational transitions unaccompanied

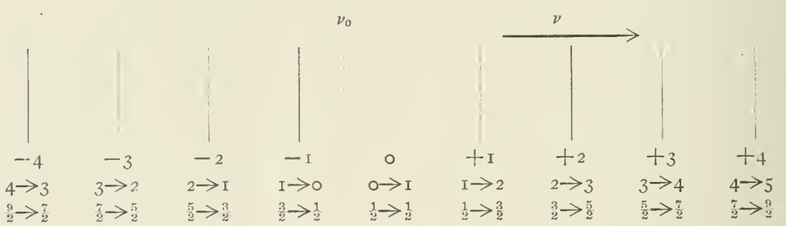

FIG. 2.—Schematic representation of a simple band series, with transitions corresponding to integral and half-quantum numbers. ν_0 is the head of the zero branch if m is integral.

by changes in m, and its locus intersects the axis of frequencies almost normally;[2] hence for small values of m there is little separation between the lines. If m is integral, the intercept ν_0 should lie midway between the two lines $0\rightleftarrows1$ of the other series. For half-values of m the frequency ν_0 coincides with that of the missing line, and the first line of the zero branch occurring at $m=\frac{1}{2}$ would have very nearly the same position. Cooley has made observations on the CH_4 band at 3.3 μ, employing reduced temperatures in order to suppress that part of the zero branch corresponding to higher quantum numbers. The maximum of the unresolved portion is thus displaced toward higher frequencies, and clearly approaches the band center rather than a position midway between it and the line -1.

[1] Cooley, *Physical Review*, 21, 376, 1923.

[2] Sommerfeld, *Atombau und Spektrallinien*.

When fractional numbers are thus admitted for the designation of stationary states, the question at once arises as to whether or not particular fractions only are permissible, and what it is that determines their values. An answer to the latter question must await the development of a satisfactory hypothesis concerning the electron configurations within the molecules in question. Kratzer suggests that the doublets observed in the nitrogen bands may be due to a slight departure of the m values from exact halves. There is no indication of such doublets in the spectra of HCl or of CH_4 although in each case the series have been observed out to the twentieth term on each side of the center. It follows that m must assume values either almost exactly integral or almost exactly halfway between the integers. It is interesting that all the molecules for which infra-red band series of the half-quantum type have been identified belong to the class involving a single octet structure, as does N_2 also, according to Langmuir. Thus the whole electron group of each is a unit in the same sense as it is for the neon or argon atom.

University of Michigan
June 14, 1923

FOCAL CHANGES IN MIRRORS[1]

By EDISON PETTIT

ABSTRACT

Method of testing focal changes in mirrors.—The mirrors were mounted on the coelostats of the 150-foot tower, the 60-foot tower, and the Snow horizontal telescope and exposed to the sun. The position of the focused image of the sun was determined at intervals over a period of several hours by means of a card moved along a scale. Each pair of mirrors was tested on three or more days and the mean curve drawn. Both long-focus lenses and concave mirrors were used to project the image. The observed focal changes are all reduced to the scale of the 150-foot tower telescope for comparison.

Focal changes observed.—Seven pairs of plane mirrors of crown glass, pyrex glass, and speculum metal, and of varying thickness were used, with and without cooling systems. The smallest changes observed were with the speculum-metal and the pyrex-glass mirrors. For the first hour of exposure the rate of change in the pyrex mirrors is about the same as in the thick crown-glass mirrors of the towers, after which it ceases entirely and the focus remains constant during subsequent exposure to the sun. The speculum-metal mirrors show the same phenomenon, with a slightly smaller focal change. The effect of wind blowing on the mirrors is to reduce and even to reverse the march of the focal plane.

Nature of changes in figure of mirrors.—All the mirrors tested, except the crown-glass mirrors of the Snow telescope coelostat and the speculum-metal mirrors, became concave on exposure to the sun. This is an anomaly difficult to explain, since one would expect a convex figure to result from the linear expansion of the surface exposed to the sun.

Seeing conditions.—The seeing conditions at the 60-foot tower were found to be greatly superior to those at the Snow horizontal telescope.

One of the principal difficulties encountered in solar investigations is occasioned by the focal changes taking place in the image-projecting apparatus when the fixed form of telescope is employed. These changes of focus, due to the exposure of the mirrors to solar radiation, are often accompanied by other optical disturbances. It is the object of the present investigation to compare systems of mirrors in order to determine their relative merits.

Seven mirror systems were employed as follows: (1) The 150-foot tower telescope; coelostat mirror 20 inches in diameter, fixed mirror 16 inches in diameter; both mirrors of crown glass, 12 inches thick, and provided with jackets through which a stream of kerosene is forced. A 12-inch lens of 150 feet focal length forms the image. (2) The same system as above without the kerosene pump in operation. (3) The same system with the crown-glass mirrors replaced

[1] *Contributions from the Mount Wilson Observatory*, No. 266.

by two mirrors of pyrex glass, 12 inches in diameter and 2 inches thick. The kerosene cooling system was not used. (4) The 60-foot tower telescope; coelostat mirrors of crown glass 17 inches in diameter, fixed elliptical mirror 13×22 inches; both mirrors 12 inches thick. A 12-inch lens of 60 feet focal length forms the image. (5) The Snow horizontal telescope; coelostat mirror of crown glass 30 inches in diameter, fixed mirror 24 inches in diameter; both mirrors 4 inches thick. A concave mirror 24 inches in diameter and 4 inches thick, of 60 feet focal length, forms the image. (6) The same system as in (5) with crown-glass mirrors replaced by mirrors of speculum metal. The coelostat and fixed mirrors were 10 inches in diameter and $1\frac{1}{2}$ inches thick. A concave mirror 6 inches in diameter and $1\frac{1}{2}$ inches thick, and of 60 feet focal length, was used to project the image. (7) Mirrors 12 inches in diameter and 1 inch thick, used in both the 150-foot tower and the Snow telescope.

The method of observing was very simple. The coelostat was directed upon the sun and the focal position of the sun's image determined by means of a white card moved along a fixed millimeter scale. Three readings were made for each determination of focal position for a given instant. For the instruments of 60-foot focus, with seeing rated at about 4 on a scale of 10, the deviation of a single reading from the mean was only 4 or 5 mm. Quite as good settings could be made on the limb as on a small spot. The scale readings, plotted against the times of observation, represent the march of the focal plane.

Since projection systems of two focal lengths were used in observing the focal changes in the various mirror systems, the observed focal changes were all reduced to values corresponding to a projection system of 150 feet focal length by the well-known reciprocal equation. Since the mirrors are sensibly flat at the beginning of a series of observations, the values of the focal distance of the image may be obtained by applying the scale readings to the focal length of the projecting system.

The curves of the focal change plotted against the time, for the seven systems described here, are given in Figure 1. Each of these curves is the mean of three to five series of observations on as many

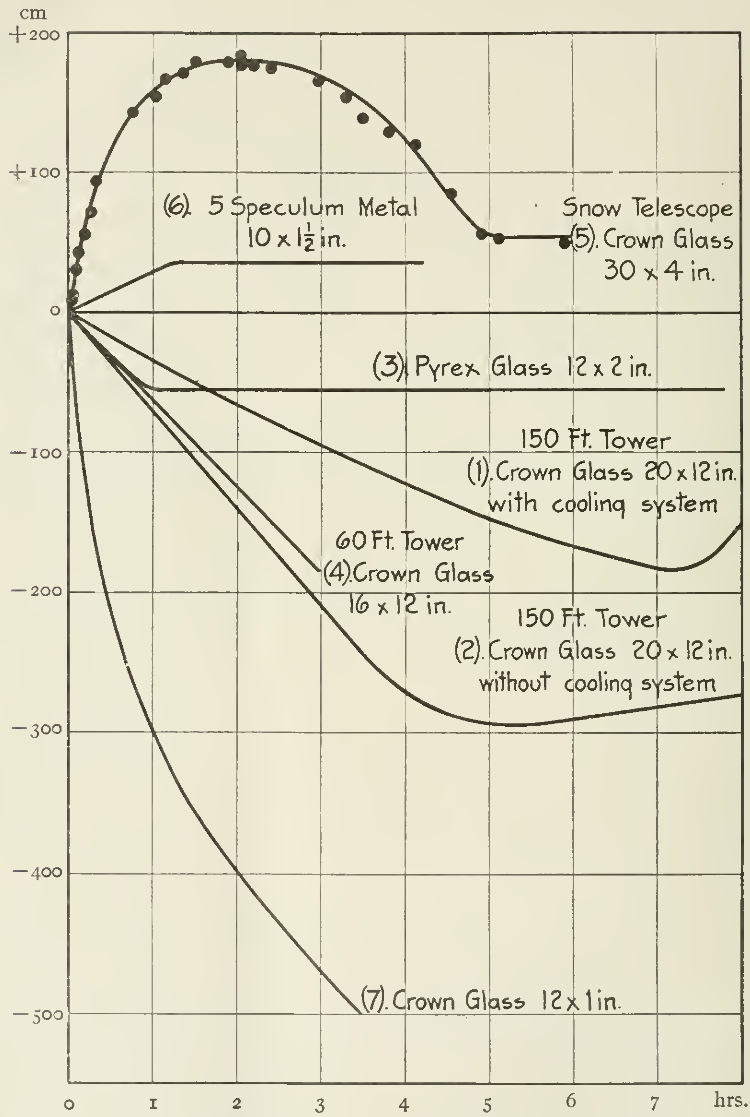

FIG. 1.—Focal changes in optical systems including coelostat and fixed mirrors of differing composition and dimensions. Ordinates: focal change in centimeters; abscissae: duration of exposure to sunlight, in hours. Positive ordinates indicate that the focal length is greater than that given by flat surfaces.

different days, so chosen that atmospheric conditions were nearly the same. As most of the observations were made in September and October, they may be taken as fairly representative of the best observing conditions of the year on Mount Wilson.

The curves are numbered according to the descriptions of the mirror systems given above. The unit for the ordinates is 1 centimeter of focal change; for the abscissae, 1 hour of time. Positive ordinates indicate that the focal length is greater than that given by flat surfaces.

Several points are significant. The mirrors of the 150-foot and 60-foot towers, which are 12 inches thick, change focus more than a centimeter per minute continuously for about 4 hours. The effect of the cooling system is to decrease this rate to one-half centimeter per minute, but this change continues for more than 6 hours. The rate of focal change in the pyrex mirrors for the first hour is about the same as that in the thick crown-glass mirrors of the towers. The focal change then ceases entirely and the focus remains practically fixed for a period of more than 6 hours. These mirrors are less affected by temperature changes than any others tested, except those of speculum metal, and should be valuable in eclipse work as well as in the routine solar observations.

With the exception of the crown-glass mirrors of the Snow telescope, and the speculum metal mirrors (6), the focal length of all those tested decreases with continued exposure. Hence the mirrors become concave when exposed to the sun. This is an anomaly difficult to explain, since one would suppose that the heated side of a poorly conducting substance like glass would become linearly larger than the unheated side and hence convex on the heated side. This, however, is not the normal behavior. That the observed effect cannot have anything to do with the form of the optical system or projecting apparatus, i.e., whether the image-forming device be a concave mirror or lens, is shown by the fact that the 12×1-inch mirrors gave the same results when used in the 150-foot tower with a lens-projecting system, when used in the Snow horizontal telescope with mirror-projecting system, and when used in the Snow telescope with the 150-foot tower lens in place of the concave mirror. The 30-inch Snow coelostat mirrors

gave sensibly the same results, whether the concave mirror or 150-foot focus lens was employed to form the image. In addition it is obvious that the rate of focal change and its direction are not dependent on the thickness of the mirrors employed, since the crown-glass mirrors of the towers, which are 12 inches thick, were

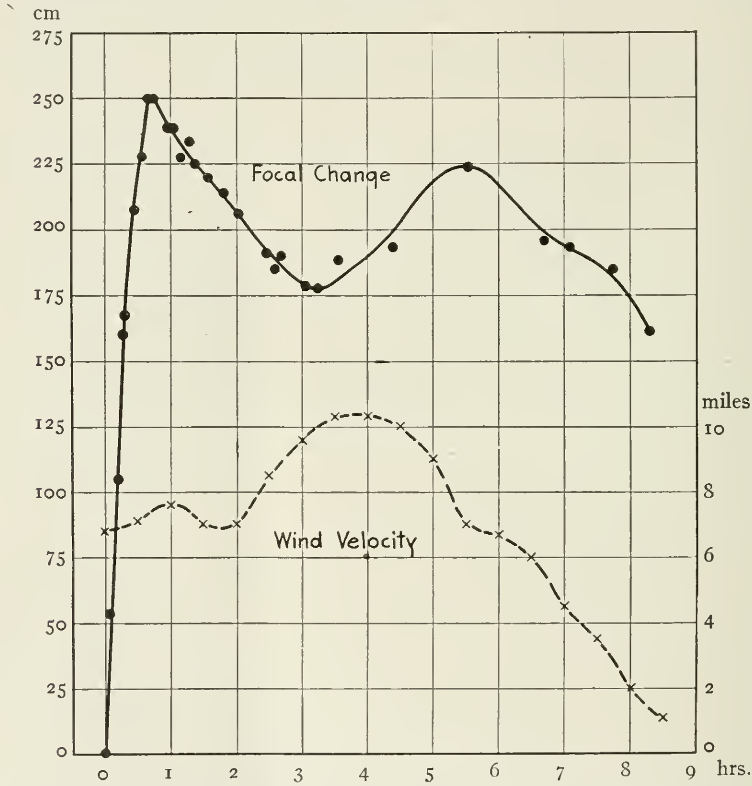

FIG. 2.—Dependence of focal change (centimeters, at the left) on wind velocity (miles per hour, right).

subject to focal changes of the same sign but of smaller rate than the crown-glass mirrors 12 inches in diameter and 1 inch thick, while, on the other hand, the Snow coelostat mirrors, which are of crown glass, are subject to focal changes of the opposite sign.

The effect of a cooling system in reducing the rate and maximum value of focal change is shown in curve 1 in Figure 1. That much the same result can be obtained by forcing a current of air

over the mirror has been pointed out by Mr. Hale.[1] The effect of the wind blowing over the mirrors is shown in Figure 2. Here the full-line curve shows the observed focal changes in the Snow coelostat hour by hour on October 17, 1921. The scale of focal change in centimeters is on the left. The broken curve represents the corresponding observed wind velocities, the scale of velocity in miles per hour being on the right. It will be noted that the effect of increased wind velocity is to reduce and even reverse the sign of the focal changes.

If we assume, for the extreme case of the two thin mirrors 12×1 inches, that both are equally affected, it can be shown that they

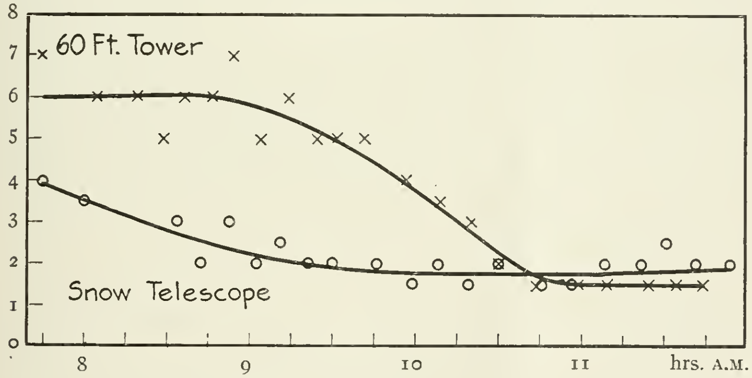

FIG. 3.—Comparison of seeing conditions at the 60-foot and 150-foot tower telescopes. Ordinates: seeing, on a scale of 10; abscissae: Pacific Standard Time.

become concaves of 737 meters focal length, corresponding to an observed focal change of 5 meters at the 150-foot tower telescope. In most cases, after an exposure of several hours, the mirrors show a tendency to return to flat surfaces, indicating an approach to a uniformly heated condition.

The use of pyrex or speculum-metal mirrors has not only the great advantage of restricting the focal changes to small values and confining them to a period of about an hour after exposure, but also the accompanying advantage of reducing the astigmatism of field. This astigmatism is caused by the fact that the incident beam of light strikes the mirrors, curved by exposure to the sun, at large

[1] *Mt. Wilson Contr.*, No. 4; *Astrophysical Journal*, 23, 6, 1905.

angles to their optical axes. This may be a seriously disturbing factor when the sun is observed at large hour angles.

While making the observations for focal change at the Snow horizontal telescope and the 60-foot tower telescope, which are of the same focal length, the atmospheric conditions were directly compared. Figure 3 shows the mean values of the seeing on a scale of 10 plotted against the time. The observations were made alternately at the two telescopes and are thus directly comparable. The superior definition of the 60-foot tower is apparent.

NOTE ADDED SEPTEMBER 21, 1923

As a result of the experiments described here, full-size pyrex mirrors have been obtained for the 150-foot tower. The coelostat mirror is 20 inches in diameter, the fixed mirror 16 inches in diameter; both mirrors are 4 inches thick. The position of the focal plane shows little change for the first hour after exposure of the mirrors to the sun, the remainder of the curve of focal change being essentially that given for the smaller pyrex mirrors in Figure 1, curve 3. The maximum focal change observed is 60 cm.

Mount Wilson Observatory
May 15, 1923

THE RADIAL VELOCITIES OF LONG-PERIOD VARIABLE STARS[1]

By PAUL W. MERRILL

ABSTRACT

Radial velocity observations of 112 long-period variables and five irregular variables have been secured at Mount Wilson since 1919 in continuation of the program begun at Ann Arbor in 1913. The observing list consisted of Md variables having maximum magnitudes brighter than 9.0. Most of the stars were observed only near the time of maximum. The *instruments* employed were one-prism spectrographs having a dispersion at Hγ of 36 A per mm. They were attached to the 100-inch Hooker telescope or the 60-inch telescope.

Measurements of the spectrograms were made by the usual method of micrometer bisections of the lines. The *emission-line velocities* depend largely upon Hγ and Hδ, but several other lines were used when available (Table I). The *absorption-line velocities* are based primarily upon the low-temperature lines of several metals. The wave-lengths were revised and additional lines added from measurements of about 50 stellar spectrograms, the new list (Table II) being used throughout. Mount Wilson measurements of the emission-line velocities for 112 stars and of the absorption-line velocities for 43 stars are given in Table III.

Discussion of results for individual stars.—Velocities at different maxima are probably the same within errors of measurement. A slight variation of emission-line velocity with phase is shown by R Leonis, R Virginis, X Ophiuchi, χ Cygni, T Cephei, and possibly by other stars. The velocities appear to have algebraically low values for a month or two after maximum light. Stars measured at more than one observatory are listed in Table IV. Collected radial velocity data for 133 stars are given in Table V together with periods and new estimates of spectral type. Forty-seven stars have *radial velocities from both bright and dark lines.* The differences of these velocities, with other data of interest, are tabulated in Table VI.

Statistical studies.—The relative displacements of the bright lines are found to increase on the average with advancing spectral type and increasing period and range. The correlation with period is used for establishing an empirical correction to be applied to the bright-line velocities to reduce them to a dark-line basis, since the displacements of the dark lines rather than those of the bright lines appear to correspond to the true radial velocities. Curves showing the *relationship between period and relative displacement* for classes Me and Se are reproduced in Figure 2. The absorption-line velocities, either measured directly or found from the bright lines by use of the curves just mentioned, are made the basis for studies of the *apparent solar motion.* The speed of the sun is almost three times that usually found for K and M stars, but the position of the apex is nearly the same. The following values are representative: $A_0=281°$; $D_0=+34°$; $V_0=53$ km; $K=+1$ km; arithmetic mean residual, 31 km. Sixty-eight stars with residuals less than 25 km give $V_0=48$ km, and 65 with larger residuals, $V_0=65$ km. This increase in V_0 furnishes an excellent illustration of the well-known velocity asymmetry of high-speed stars. The average *residual radial velocity* is found to decrease with advancing spectral type and increasing period. Very high velocities are largely confined to stars of types M2e to M5e and to stars having periods in the neighborhood of 200 days.

A very brief *general discussion* of the properties of variables which show some degree of interdependence, and of the general evolutionary problems concerning these stars, is included.

[1] *Contributions from the Mount Wilson Observatory*, No. 264.

INTRODUCTION

The spectroscopic observations of long-period variable stars, made by the writer at Ann Arbor during the years 1913–16,[1] and at Mount Wilson since 1919, were planned with two principal objects in view. One was the measurement of the radial velocities of a considerable number of variables, upon which could be based a determination of the apparent solar motion and average peculiar motion for comparison with similar data relating to other stars. The possible binary character of these variables, and the interpretation of the relative displacement of bright and dark lines,[2] were, of course, to be considered in this connection. The second object was a study of the physical and chemical conditions which prevail in the long-period variables, with especial attention to changes which occur as the brightness varies.

The determination of radial velocities has thus far had the chief place on my observing program, and has now reached such a stage that it seems wise to collect and discuss the available data, as they are sufficiently numerous to yield fairly satisfactory results for several statistical inquiries. Should further observations of velocity be undertaken, the present discussion will serve to suggest how the work may most profitably be extended. The emission lines of many faint variables could still be observed with the Hooker telescope with exposure times of two or three hours. A few rather bright variables remain unobserved for the reason that during recent years the time of maximum brightness has nearly coincided with that of conjunction with the sun. Moreover, numerous stars already observed could profitably be made the subjects of further study. The total number of long-period variables now listed[3] is about 600, including 150 whose periods are not definitely known, but which are probably long, although some may be irregular. The spectra of 460 of these have been recorded: 415 are of class M, 385 having bright lines; 25 are of class N; and 16 of class S.

Prior to 1916 the radial velocities of five long-period variables of classes M and S had been published. The writer's observations

[1] *Publications of the Astronomical Observatory, University of Michigan*, **2**, 45, 1916.

[2] Previously recognized by other observers in the spectrum of *o* Ceti and of χ Cygni.

[3] *Harvard Annals*, **56**, 197 (Table IX), 1912.

at Ann Arbor increased the number to 43. Since 1916 Paddock has reported[1] the velocity of T Centauri, and the present investigation adds measurements of 89 variables, making 133 in all.

THE OBSERVATIONS

The observing program for the present investigation, as well as that for the previous work at Ann Arbor, was based on the list of variables in *Harvard Annals*, 56, 197 (Table IX), 1912. It included Md variables having maximum magnitudes brighter than 9.0. There are 264 such objects in the Harvard list, of which 201 are north of declination $-30°$; over the whole sky 122 Md stars with maximum magnitudes of 9.0 or fainter are known.

The formation of the program for each night's observation has required much more attention than would ordinarily be necessary in the investigation of a group of stars of a certain spectral type. The faintness of these variables during the greater part of their light-cycles made it essential that nearly all of them be observed within a few weeks of the maximum phase, and after a few dozen of the brighter variables had been observed and eliminated from the program, the number available for observation on a particular night was often surprisingly small.

The predicted times of maximum in *Harvard Circulars*, Nos. 212, 220, 222, and 227, served as a general guide for the selection of stars for each night's work. As the light-curves are not uniform, however, the actual time of maximum is likely to deviate somewhat from the predicted time, and, moreover, the maximum brightness often differs very considerably from the average maximum value. Accordingly, in order to make spectroscopic work with the large reflectors as effective as possible, it was necessary to rely on a considerable number of current magnitude determinations. Usually it was not feasible for the writer to make these at Mount Wilson, but at the suggestion of Professor Bailey a very satisfactory arrangement was made by which predicted magnitudes of selected stars were sent each month from the Harvard College Observatory. These predictions were by Mr. Leon Campbell from current observations made at Cambridge and by members of the American

[1] *Lick Observatory Bulletins*, **9**, 68, 1917.

Association of Variable Star Observers, which is rendering highly important service by systematic observations of a large number of long-period variables. The photometric data in Table III were also supplied by Mr. Campbell. It is a pleasure to express my thanks to Professor Bailey, Professor Shapley, and especially to Mr. Campbell, for their kind co-operation, which greatly facilitated the present investigation.

Practically all of the radial-velocity observations in the present investigation were made with two single-prism spectrographs having camera lenses with focal lengths of 18 inches. One of these, which has been described by Mr. Adams,[1] is attached at the Cassegrain focus of the 60-inch reflector, and the other, which has nearly the same optical dimensions, is attached at the Cassegrain focus of the 100-inch Hooker reflector. The dispersion at various points in the spectrum is as follows: at $H\beta$, 56.6 A per mm; at $H\gamma$, 36.1 A per mm; at $H\delta$, 28.2 A per mm. Nearly all of the photographs were on the Seed 30 emulsion. The slit-width was usually 0.05 or 0.06 mm. In connection with the 40-inch collimator lens this gave a satisfactory degree of purity.

With the Hooker telescope, spectrograms can usually be obtained in not more than half the time required with the 60-inch telescope. The importance of this gain in speed lies not so much in the reduction of the total exposure time on a long program as in the fact that most of the long-period variables are bright enough for observation during only a few weeks of each year, and must often be photographed, if at all, at large hour angles with exposure times not exceeding one or two hours.

As is well known, the color of the long-period variables is orange or red; the color-index of the Md stars is usually taken as 1.8 magnitudes. Compared with the visual brightness, the blue and violet light is very weak, and the continuous spectrum to the violet of λ 4500 is relatively difficult to photograph. On many plates only the bright hydrogen lines $H\gamma$ and $H\delta$ are measurable, these lines usually being so strong that they can be photographed in 5 to 20 per cent of the time required for the adjacent continuous spectrum. The bright $H\gamma$ and $H\delta$ lines of a tenth-magnitude star

[1] *Mt. Wilson Contr.*, No. 59; *Astrophysical Journal*, **35**, 163, 1912.

can ordinarily be photographed with the Hooker telescope in two hours or less. Stars fainter than the ninth magnitude, however, were seldom observed except for special reasons. Under average conditions an exposure of two hours on a star of visual magnitude 9.0 usually yields a plate with the bright hydrogen lines strong, and with the continuous spectrum sufficiently well recorded between λ 4500 and λ 5000 to allow a good estimate of the spectral type, but too weak in the region λ 4100 to λ 4500 for satisfactory measurement of the absorption lines.

I desire to express my appreciation of the efficient aid rendered in the observing by all the night assistants who have taken part in it. Those who have had the largest share in the work are Messrs. William Klemann and W. P. Hoge.

THE VELOCITY MEASUREMENTS

The velocity determinations have been carried out by the method of micrometer bisections of the lines on the spectrograms, with measuring machines of the usual type. All of the plates have been measured twice and a small number three or four times. Different measurements of the same plate usually agree well. Altogether about 800 plate measurements have been made, 60 per cent of them by the writer, and nearly all of the remainder by Miss Florence MacCreadie, Mr. T. S. Jacobsen, and Miss Cora G. Burwell. An interval of at least three months was allowed to elapse between two measurements of a plate by the same person.

Velocities from emission lines.—The velocities from the emission lines depend on the laboratory wave-lengths in Table I. The last two columns, which give, respectively, the numbers of plates upon which each line has been measured at Mount Wilson for the present investigation, and at Ann Arbor, show that $H\gamma$ and $H\delta$ have been used for the velocity measurements far more frequently than any of the other lines. $H\beta$ and $H\zeta$ have occasionally been omitted, even when visible on the plates, on account of poor focus. A spectrograph adjusted for the λ 3900 region and having optical parts more transparent in this region than those employed in the present investigation would give a greater relative frequency for the $H\zeta$ and $H\eta$ lines.

The bright lines offer very definite marks for the setting of the micrometer wire as they have been noticeably broadened in very few instances. The agreement between individual lines is generally good,[1] and the velocities for each plate are more accurate than one might expect from the small number of lines. Each bright line has as much weight for a velocity determination as four or five average absorption lines in the same stars.

Velocities from absorption lines.—A preliminary table for the absorption lines was formed by taking the available laboratory

TABLE I

LABORATORY WAVE-LENGTHS OF EMISSION LINES

I. A.	Number of Stellar Spectrograms	
	Mt. Wilson	Ann Arbor
3835.36 Hη........	10	4
3889.05 Hζ.........	15	24
3905.51 *Si*.........	9	5
3970.08 Hε.........	5	0
4101.74 Hδ.........	316	115
4202.03 *Fe*.........	57	5
4307.91 *Fe*.........	41	0
4340.47 Hγ........	345	113
4571.11 *Mg*........	38	0
4861.33 Hβ........	133	35

measurements of the low-temperature lines of *Ca*, *Cr*, *Fe*, *Mg*, *Mn*, *Sr*, *Ti*, and *V*. After about fifty stellar plates had been measured, the lines which had been used less than five times were rejected. The velocities, residuals, and probable errors were then computed for the remaining lines, and the list of wave-lengths was further revised as follows: (1) All lines showing a probable error for a single plate greater than 0.10 A were rejected; (2) wave-lengths of the remaining lines were corrected by amounts corresponding to the average residuals when these exceeded two and a half times their probable errors, otherwise the original laboratory values were retained; (3) lines not in the preliminary table, but measured on five or more plates were added if the probable error from a single plate was not in excess of 0.08 A.

[1] *Mt. Wilson Contr.*, No. 265; *Astrophysical Journal*, **58**, 195, 1923.

These rules obviously favor the original laboratory wave-lengths rather than those obtained by measurement of stellar spectrograms. This was thought desirable in order that the velocities might depend as directly as possible upon laboratory wave-lengths, and not be unnecessarily subject to systematic errors introduced through the use of wave-lengths derived from a limited number of stellar measurements. This "Second Revised Table of Absorption Lines," containing sixty-five lines from λ 4026 to λ 4580, is the basis of the

TABLE II

WAVE-LENGTHS OF ABSORPTION LINES—CLASS ME

(*Second Revised Table*)

I. A.	Element	I. A.	Element	I. A.	Element
4026.88	....	4147.68	*Fe*	4325.93	(*Fe*)
4030.85	(*Mn*)	4149.78	....	4330.02	*V*
4033.07	*Mn*	4179.62	(*V*)	4337.56	*Cr*
4034.58	(*Mn*)	4200.07	....	4344.43	(*Cr*)
4045.87	(*Fe*)	4206.70	*Fe*	4347.27	....
4054.86	....	4215.70	(*Sr*)	4375.93	*Fe*
4077.85	(*Sr*)	4226.82	(*Ca*)	4379.39	(*V*)
4090.50	*V*	4234.08	....	4383.64	(*Fe*)
4092.53	(*V*)	4254.35	*Cr*	4384.19	(*Fe, V*)
4105.03	....	4274.80	*Cr*	4389.44	(*Fe*)
4109.67	(*V*)	4282.62	(*Fe*)	4389.60	(*Fe, V*)
4111.76	*V*	4285.86	(*Ti*)	4395.22	*V*
4115.16	*V*	4287.49	(*Ti*)	4404.82	(*Fe*)
4116.58	(*V*)	4289.57	(*Cr*)	4408.24	(*V*)
4118.58	....	4291.47	*Fe*	4408.42	*Fe*
4121.65	....	4294.28	(*Fe*)	4412.22	....
4123.59	(*V*)	4296.04	....	4427.30	*Fe*
4128.00	*V*	4299.03	....	4455.39	....
4129.85	....	4300.79	....	4461.94	(*Fe?*)
4131.97	*V*	4302.63	....	4482.09	....
4134.40	(*V*)	4306.08	....	4580.29	....
4139.93	*Fe*	4307.76	(*Fe*)		

absorption-line velocities in Table III. It is reproduced in Table II. Lines having the chemical identification in parentheses are those for which slight changes from the laboratory values have been introduced as outlined above. Lines without identifications were derived directly from the stellar spectrograms.

When all the measurements had been completed, the residuals were again formed and the process of correcting the table was repeated. This led to the list printed as Table IV in *Mt. Wilson Contribution* No. 265. A logical step would have been to re-reduce

all the plates, using this final table, but this was not done because the resulting changes would probably have been too small to justify the additional labor. Table IV in *Contribution* No. 265 is, of course, the one recommended for future measurements of the spectra of long-period variables of class M, having dispersion comparable with that used in this investigation.[1]

Mount Wilson radial-velocity data.—Table III gives the data for the radial velocities of 111 long-period variables and six irregular variables observed at Mount Wilson. The first column contains the name of the star and the Harvard designation, of which the first four figures indicate the hours and minutes of right ascension for 1900, and the last two figures the degrees of declination, numbers in italics representing southern declinations. Dates of observation are given in the second column. Spectrograms on dates marked with an asterisk were made with the 60-inch telescope, all others with the 100-inch telescope. The column headed "Phase" gives the number of days before (−) or after (+) the nearest maximum. In the column "Absorption Velocity" are given the measured velocities in kilometers per second, and the number of lines. The last column, "Emission Velocity," gives the measured velocities and the particular lines used, the Greek letters referring to hydrogen lines in the Balmer series. The individual velocities are printed to the nearest kilometer only, but in forming the means the original values to a tenth of a kilometer were used.

[1] 36 A per mm at $H\gamma$.

TABLE III

RADIAL-VELOCITY OBSERVATIONS OF LONG-PERIOD VARIABLES AT MOUNT WILSON

Star	Date	Mag.	Phase Days	Velocity			
				Absorption		Emission	
S Sculptoris *001032*......	1921 Nov. 12	7.9	− 31	....		+ 11	γδ
	14	7.8	− 29	....		+ 15	γδ
						+ 13.2	
X Androm. 001046......	1921 Nov. 13	9.5	− 16	....		− 16	βγ
	14	9.5	− 15	....		− 20	βγ
						− 18.2	
T Androm. 001726......	1919 Sept. 3	9.0	+ 21	− 90	8	− 95	γδζ
	1921 Jan. 29*	8.5	− 9	....		− 95	γδ
				− 90		− 94.8	
T Cassiop. 001755......	1919 Aug. 29	9.1	−132	− 15	19	− 27	γδζ 3905
	31	9.0	−130	....		− 26	δ
	Sept. 5	8.9	−125	(− 9)	23		
	7	8.8	−123	....		− 22	δ
	Oct. 3*	8.2	− 97	....		− 26	δ
	14	8.1	− 86	− 7	32	− 22	γδ
	Nov. 9	8.3	− 60	− 13	15	− 25	γδ 4202
				− 11.0		− 24.6	
Androm. 001838......	1919 Oct. 3*	7.5	− 25	....		− 44	βγδ
	15	7.5	− 13	− 6	11	− 34	βγδ
	16	7.5	− 12	− 9	23	− 31	βγδ
				− 8.2		− 36.3	
S Ceti *001909*......	1922 Nov. 6	9.0	− 42	....		+ 20	γδ
U Cassiop. 004047......	1921 Nov. 12	8.2	− 4	− 45	21	− 54	βγδ
	13	8.2	− 3	....		− 54	βγδ
	1922 Sept. 8	9.3	+ 19	....		− 61	βγδ
						− 56.6	
V Androm. 004435......	1923 Jan. 6	9.1	+ 16	....		+ 8	βγδ
	7	9.2	+ 17	....		+ 8	βγδ
						+ 8.0	
S Cassiop. 011272......	1922 Jan. 13*	8.9	+ 22	....		− 54	βγ
Y Androm. 013338......	1920 Jan. 9	8.6	− 1	....		− 16	γδ
	16	8.6	+ 6	....		− 17	γ
						− 16.8	
U Persei 015354......	1922 Feb. 12	7.9	− 2	+ 15	21	+ 9	βγδ
	13	7.9	− 1	....		+ 11	γδ
						+ 9.0	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption		Velocity: Emission	
R Arietis 021024......	1921 Oct. 11	7.9	− 16	+114	24	+106	γδ
	12	7.9	− 15	+116	12	+101	βγδ
				+114.4		+103.5	
W Androm. 021143a.....	1920 July 31	7.9	− 19	− 22	8	− 44	γδ
	31	7.9	− 19	− 33	10	− 47	γδ
	31	7.9	− 19	....		− 47	γδ
	1921 Oct. 12	8.5	+ 33	− 29	20	− 43	γδ 4202
				− 28.5		− 45.0	
R Ceti *022000*......	1919 Sept. 4	8.0	− 2	+ 46	23	+ 33	γδ
	6	8.0	0	+ 39	21	+ 30	γδ
	8*	8.1	+ 2	....		(+ 28)	γ
				+ 42.5		+ 31.6	
U Ceti *022813*......	1920 Dec. 27	7.6	+ 18	....		− 37	γδ
	29*	7.7	+ 20	....		− 45	βγδ
	1922 Nov. 6	7.4	− 6	− 27	18	− 35	γδ
	6	7.4	− 6	....		− 41	γδ
						− 39.4	
R Trianguli 023133......	1919 Oct. 14	7.0	− 30	+ 68	45	+ 60	γδ
	15	7.0	− 29	+ 67	40	+ 59	γδ
	17	6.9	− 27	+ 66	39	+ 60	βγδ
				+ 66.7		+ 59.6	
R Persei 032335......	1919 Oct. 14	8.6	− 18	− 82	10	− 90	βγδ
	16	8.5	− 16	− 74	16	− 88	βγδ
				− 78.2		− 89.1	
T Eridani *035124*......	1921 Sept. 22	8.9	+ 32	....		+ 36:	γδ
	23	8.9	+ 33	....		+ 33	βγδ
						+ 34	
W Eridani *040725*......	1921 Nov. 14	8.6	−36±	....		+ 14	γδ
	Dec. 15	8.5	− 5±	....		+ 9	γδ
						+ 11.4	
R Tauri 042209	1920 Oct. 27	8.7	− 13	....		+ 18	γδ
	28	8.7	− 12	....		+ 20	γδ
						+ 19.2	
T Camelop. 043065......	1922 Aug. 9*	8.8	+ 64	....		− 19	βγ
X Camelop. 043274......	1919 Sept. 9*	8.0	− 13	....		− 2	βγδ
	9*	8.0	− 13	....		− 4	βγδ
	Oct. 3*	7.8	+ 11	....		− 11	βγδ
						− 5.8	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity		
				Absorption	Emission	
T Leporis						
050022	1922 Feb. 12	8.1	+22	....	− 21	γδ
	1923 Feb. 6	8.6	+ 1	....	− 14	γδ
					− 18.	
V Orionis						
050003	1919 Oct. 15	8.8	+ 9	....	+ 13	βγδ
	16	8.8	+ 10	....	+ 17	βγδ
	17	8.8	+ 11	....	+ 12	βγδ
					+ 13.9	
R Aurigae						
050953	1917 Nov. 23*	8.1	+ 35	....	− 18	γδ
	1921 Sept. 20*	9.0	+ 20	....	− 13	γδ 4202
					− 15.1	
T Columbae						
051533	1921 Nov. 12	8.0	+ 8	....	+ 52	γδ
	13	8.0	+ 9	....	+ 55	γδ
					+ 53.4	
U Orionis						
054920	1919 Sept 4	8.0	+ 65	− 17 21	− 33	γδζη 4202, 3905
	1920 Sept. 2	7.0	+ 49	....	− 41	γδ 4202
	26	8.0	+ 73	....	− 39	γδ 4202
	Oct. 26	8.6	+103	....	− 36	γδ 4571, 4308, 4202
					− 37.2	
X Aurigae						
060450	1919 Dec. 12*	8.7	− 24	....	− 24	γ
	13*	8.6	− 23	....	− 28	γδ
					− 26.2	
X Gemin.						
064030	1920 Apr. 8	8.6	+ 25	....	+ 68	γδ
	10	8.7	+ 27	....	+ 66	γδ
					+ 67.2	
X Monocer.						
065208	1919 Nov. 9	7.4	+ 32	+157 15	+152	γδ
	13*	7.5	+ 36	....	+146	γ
	14*	7.6	+ 37	....	+142:	γ
					+147.5	
R Gemin.						
070122*a*	1919 Oct. 3*	7.4	− 27	....	− 58	βγδ
	14	7.0	− 16	....	− 54	βγδ
	15	7.0	− 15	....	− 56	βγδ
	17	7.0	− 13	....	− 56	βγδ
	1920 Sept. 28	7.2	− 30	....	− 52	βγ
	Dec. 27	8.3	+ 60	....	− 60	βγδ
	1921 Sept. 21	8.1	− 50	....	− 58	βγ
	Oct. 10*	7.2	− 31	....	− 62	βγδ
	Nov. 13	7.2	+ 3	....	− 59	βγδ
	Dec. 15	7.3	+ 35	....	− 55	βγδ
	1922 Nov. 6	6.6	+ 10	− 36 29	− 56	γδ
	1923 Jan. 6	7.6	+ 71	....	− 62	βγδ
					− 57.4	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity			
				Absorption		Emission	
R Can. Min.							
070310......	1920 Mar. 3	8.0	− 10	....		+ 42	βγδ
	4	8.0	− 9	....		+ 34	βγδ
	7*	8.0	− 6	....		+ 32	βγ
	1921 Feb. 25	8.3	+ 20	....		+ 27	βγδ
	Mar. 26	8.4	+ 49	....		+ 26	βγ
	1922 Jan. 12	8.5	− 33	....		+ 33	βγ
	Dec. 8	8.1	− 42	....		+ 41	βγδ
						+ 33.3	
V Gemin.							
071713......	1920 Feb. 7	8.5	− 18	....		+ 14	γδ
	Mar. 3	8.4	+ 7	+ 22	12	+ 7	βγδ
						+ 10.6	
S Can. Min.							
072708......	1922 Apr. 13	9.6	− 72	....		+ 54	δ
	14	9.6	− 71	....		+ 56	δ
						+ 54.6	
Z Puppis							
072820b.....	1921 Feb. 26	7.9	+ 22	....		+ 13	γδ
	27	7.9	+ 23	....		+ 12	γδ
						+ 12.6	
T Gemin.							
074323......	1921 Mar. 28	9.1	− 23	....		+ 15	βγδ
	Apr. 29	8.6	+ 9	....		+ 12	βγδ
	1923 Jan. 8	9.9	+ 65	....		+ 4	βγ
						+ 11.8	
R Cancri							
081112......	1920 Mar. 4	7.0	− 36	+ 38	37	+ 23	γδ
	5	7.0	− 35	+ 38	32	+ 24	γδ
	5	7.0	− 35	+ 38	19	+ 26	γδ
	6*	6.9	− 34	+ 27	30	+ 12	γδ
	7*	6.9	− 33	....		+ 13	γδ
	7*	6.9	− 33	....		+ 16	γδ
	1921 Mar. 26	7.6	− 13	....		+ 17	γδ
	Apr. 26*	8.5	+ 18	....		+ 16	γδ
				(+ 32.1)		+ 18.4	
V Cancri							
081617......	1919 Nov. 9	8.0	+ 10	....		− 6	βγδ
	13*	8.0	+ 14	....		− 17	βγδ
	14*	8.0	+ 15	....		− 13	βγδ
	1921 Apr. 27*	7.9	− 3	....		− 17	βγδ
	1922 Jan. 13*	7.9	− 12	....		− 16	βγδ
	Feb. 11	7.8	+ 17	....		− 11	βγδ
						− 13.5	
RT Hydrae							
082405......	1919 Dec. 13*	9.0	−100	....		+ 34:	γ
	1920 Mar. 5	8.0	− 17	+ 40	17	+ 36	γδ
						+ 35	
U Cancri							
083019......	1921 Oct. 13	11.3	+ 44	....		+ 68:	γδ
	14	11.3	+ 45	....		+ 59	βγδ
						+ 61	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption	Velocity: Emission	
X Urs. Maj. 083350......	1921 Mar. 28	9.2	0	....	− 90	βγδ
S Hydrae 084803......	1920 Mar. 6*	8.0	− 11	....	+ 62	βγδ
	1922 Apr. 14	8.2	− 21	....	+ 70	βγδ
	1923 Jan. 5*	7.7	− 3	....	+ 66	βγδ
					+ 66.1	
W Cancri 090425......	1921 Nov. 13	8.5	− 24	....	+ 36	γδ
	14	8.4	− 23	....	+ 33	γδ
					+ 34.7	
R Leo. Min. 093934......	1922 Feb. 12	7.8	− 30	+ 8 47	+ 1	γδ
	Mar. 18	7.3	+ 4	....	− 2	γδ
	18	7.3	+ 4	+ 12 19	− 3	γδ
				+ 9.5	− 1.5	
R Leonis 094211......	1920 Jan. 16	9.0	− 59	....	+ 3	δ
	Apr. 8	6.4	+ 24	+ 12 19	− 2	γδ 4202
	11*	6.5	+ 27	....	− 4	γδ 4202
	11*	6.5	+ 27	....	− 5	γδ 4202
	May 5*	7.4	+ 51	+ 16 14	− 2	γδεζη 4571, 4308, 4202, 3905
	11*	7.6	+ 57	+ 16 10	− 1	γδεζη 4571, 4308, 4202, 3905
	30	8.3	+ 76	....	+ 2	γδ 4571, 4308, 4202
	Dec. 26	5.8	− 15	+ 14 47	0	γδ
	1921 Jan. 28	6.2	+ 18	+ 10 42	− 2	γδ
	Feb. 25	6.9	+ 46	+ 11 31	+ 1	γδεζη 4202, 3905
	26	6.9	+ 47	....	+ 1	γεζηθι 3905
	Mar. 26	7.7	+ 75	....	− 2	γδ 4571, 4308, 4202
	27	7.7	+ 76	+ 18 14	+ 3	γδεζη 4571, 4308, 4202, 3905
	Apr. 28	8.9	+108	....	+ 2	γδ 4571, 4308, 4202
	1922 Dec. 8	7.2	+ 71	....	+ 2	γδ 4202
				+ 13.8	0.0	
V Leonis 095421......	1920 June 1	9.0	− 13	....	− 29	γδ
	2	9.0	− 12	....	− 32	γδ
					− 30.8	
Z Urs. Maj. 115158......	1920 Mar. 3	....	......	− 53 34	− 60	γδ
	4	....	......	− 52 14	− 57	γδ
				− 52.6	− 58.4	
R Comae 115919......	1919 June 9*	7.9	− 7	....	− 22	γδ
	10*	7.9	− 6	....	− 22	γδ
					− 22.0	
T Urs. Maj. 123160......	1922 Jan. 13*	8.8	+ 8	....	−102	γδ

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity			
				Absorption		Emission	
R Virginis 123307......	1920 May 1	9.1	− 41	....		− 26	γδ
	1922 Feb. 12	8.5	+ 32	− 34	22	− 43	βγδ
	May 15	9.0	− 32	....		− 24	γδ
						− 31.	
S Urs. Maj. 123961......	1920 Apr. 8	8.5	− 31	....		− 4	βγ
	10	8.5	− 29	....		− 7	βγ
	June 3*	8.2	+ 25	....		− 7	βγ
	Dec. 29*	8.4	+ 25	....		− 10	βγδ
	1921 Jan. 28	8.8	+ 48	....		− 7	βγ
	1922 Feb. 11	8.5	− 28	....		+ 4	βγ
	Mar. 19	7.6	+ 8	....		− 4	βγδ
						− 5.0	
U Can. Ven. 124238......	1920 Mar. 5	....	....	....		− 43	γδ
	10	....	....	....		− 44	γδ
						− 43.7	
U Virginis 124606......	1920 Mar. 6*	9.1	− 37	....		− 61	γδ
	Apr. 8	8.2	− 4	− 42	16	− 56	βγδ
						− 58.1	
RT Virginis 125705*a*.....	1920 Mar. 3	....	....	+ 21	24		
SW Virginis *130802*......	1920 Mar. 4	....	....	− 12	19		
V Virginis *132202*......	1921 Feb. 26	9.5	+ 7	....		+ 23	γδ
	27	9.6	+ 8	....		+ 27	γδ
						+ 25.0	
R Hydrae *132422*......	1920 May 31	9.0	−162	....		− 18	4571, 4308
	July 8	7.9	−124	....		(− 22)	δ
	1921 Jan. 30*	6.7	+ 82	....		(− 27)	γδ
	30*	6.7	+ 82	....		− 18	γδ 4571, 4308, 4202
	30	6.7	+ 82	....		− 25	γδ 4571, 4308, 4202
	Feb. 25	7.7	+108	....		− 24	βγδ 4571, 4308, 4202
	Mar. 27	8.6	+138	....		− 19	βγ 4571, 4308, 4202
	Apr. 29	9.3	+171	....		− 20	4571, 4308, 4202
	May 26	9.6	+198	....		− 21	4571, 4308
	1922 Feb. 13	5.6	+ 52	....		− 21	γδ 4202
	1923 Jan. 4*	5.6	− 38	− 11	46	− 19	γδ
	5*	5.5	− 37	− 8	40	− 21	γδ
				− 9.3		− 21.4	
S Virginis *132706*......	1920 Jan. 16	8.0	− 28	....		− 4	δ
	Mar. 4	7.7	+ 20	....		0	γδ
	6*	7.8	+ 22	....		− 10	γδ
	1922 Mar. 20*	7.9	+ 14	....		(− 8)	δ
						− 4.7	
T Centauri *133633*......	1921 Apr. 28	6.4	+ 20	+ 22	5		
	1922 July 12	6.3	+ 13	+ 27	9	+ 14	βγ
				+ 24.6			

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption		Velocity: Emission	
W Hydrae *134327*......	1921 Feb. 28	(7.3)	......	+41	19	+26	γδ 4202
	Mar. 28	(6.5±)	......	....		+24	γδ 4571, 4308, 4202
	May 24	7	......	....		+24	γδ 4571, 4308, 4202
	June 20	8	......	....		+25	βγ 4571, 4308, 4202
	1922 Apr. 13	7	......	+43	35	+29	γδ 4202
	May 15	7.2	......	+44	19	+27	βγδ 4571, 4308, 4202
	June 11	7.5±	......	....		(+39)	γ 4571
	15	7.5±	......	....		+28	γδ 4571, 4308, 4202
	1923 Jan. 8	7 ±	......	....		+31	γδ
				+42.3		+26.4	
R Can. Ven. 134440......	1922 Apr. 13	7.7	−9	−6	46	−20	γδ
	14	7.7	−8	....		−24	γδ
	May 16	8.4	+24	−12	40	−25	γδ 4202
				−8.9		−23.1	
U Urs. Min. 141567......	1919 Aug. 6*	7.7	+9	....		−44	γδ
	6*	7.7	+9	....		−43	γδ
						−43.3	
S Boötis 141954......	1919 July 9*	8.8	−19	....		−24	βγδ
	10*	8.8	−18	....		−26	βγδ
						−25.0	
RS Virginis 142205......	1919 June 9*	7.8	+35	....		−43	γδ
	10*	7.8	+36	....		−37	γδ
						−40.0	
R Camelop. 142584......	1919 June 9*	8.2	+7	....		−50	βγ
	10*	8.2	+8	....		−40	βγ
	July 8*	8.9	+36	....		−41	βγ
	1922 May 17*	8.2	−18	....		−49	βγ
	1923 Feb. 5*	8.2	−24	....		−28	βγ
						−42.9	
V Boötis 142539......	1919 June 9*	7.8	+7	....		−47	γδ
Y Librae *150605*......	1921 May 25	9.2	+13	....		−18	γδ
	June 21	10.2	+40	....		−9	γδ
	1922 Mar. 18	9.6	+45	....		−15	γδ
						−15.0	
S Librae *151520*......	1922 Apr. 13	8.5	−18	....		+288	βγδ
	14	8.5	−17	....		+283	βγδ
	May 15	8.5	+14	....		+283	βγ
						+284.6	
S Serpentis 151714......	1920 July 30	7.8	−6	....		0	γδ
	31	7.8	−5	....		−3	γδ
						−1.9	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption	Velocity: Emission	
RS Librae *151822*......	1920 July 6	8.0	− 7	....	− 9	γδ
	1921 Feb. 26	7.4	0	....	− 18	γδ
					− 15.1	
RU Librae *152714*......	1921 July 21*	8.5	− 8	....	− 62	βγδ
	24	8.4	− 5	....	− 57	βγδ
					− 59.5	
S Urs. Min. 153378......	1920 Sept. 4*	8.3	− 30	....	− 55	γδ
	1921 Aug. 14*	8.3	− 12	....	− 50	γδ
	15*	8.3	− 11	....	− 53	γδ
					− 52.7	
R Serpentis 154615......	1920 May 1	7.6	+ 46	....	+ 3	γδ
	30	8.8	+ 75	....	+ 6	βγδ 4571, 4308, 4202
	June 1	8.8	+ 77	....	+ 4	βγδ 4571, 4308, 4202
	July 8	10.7	+114	....	+ 3	γδ 4571, 4308, 4202
	1922 Mar. 8	7.1	+ 12	+ 22 44	+ 7	γδ
	1923 Feb. 5*	7.0±	− 9	....	+ 10	γδ
					+ 5.3	
ST Herculis 154748......	1920 Mar. 3	....	......	− 33 40		
RR Librae *155018*......	1920 Apr. 10	9.2	+ 20	....	− 40	γδ
	1922 July 11	8.8	− 12	....	− 41	γδ
					− 40.6	
X Herculis 155947......	1920 Mar. 3	6.9	......	− 92 45		
	4	6.9	......	− 92 40		
	7*	6.9	......	− 91 38		
	1921 Apr. 26*	6.5	......	− 88 35		
				− 90.1		
R Herculis 160118......	1921 June 20	8.6	+ 11	....	− 40	βγδ
	22*	8.7	+ 13	....	− 44	γδ
					− 42.1	
U Serpentis 160210......	1920 Sept. 3	8.3	− 11	....	− 37	βγδ
	4	8.2	− 10	....	− 42	βγδ
					− 39.8	
RU Herculis 160625......	1920 July 6	8.8	− 18	....	− 33	γδ
	7	8.8	− 17	....	− 42	γδ
					− 37.6	
W Cor. Bor. 161138......	1921 Feb. 25	8.4	+ 1	....	+ 12	βγδ
	26	8.4	+ 2	....	+ 9	βγδ
					+ 10.5	
G Herculis 162542......	1920 May 1	5.2	......	+ 2 49		

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption		Velocity: Emission	
R Draconis 163266	1919 July 9*	8.9	−30	...		−143	γδ
	Aug. 2*	8.0	−6	...		−151	γδ
	8*	8.0	0	−138	21	−143	βγδ
	Sept. 8*	8.8	+31	...		−143	γδ
	9*	8.8	+32	...		−142	γδ
	1922 Apr. 16*	7.9	−8	...		−144	γδ
						−144.2	
RR Scorpii *165030*	1920 July 31	6.6	−3	...		−47	γδ
	1921 Mar. 28	7.4	−45	...		−43	γδ
						−45.1	
SS Ophiuchi *165202*	1922 Aug. 9	8.7	+1	...		−38	βγδ
	10	8.7	+2	...		−48	βγδ
						−42.9	
RS Herculis 171723	1920 Sept. 3	9.5	+50	...		−54	γδ
	4	9.5	+51	...		−48	γδ
						−50.8	
RY Herculis 175519	1919 Aug. 11	9.1	−18	−39	15	−50	γδζ
	12	9.1	−17	−39	11	−50	γδ
				−38.9		−50.1	
T Herculis 180531	1922 May 17*	8.1	−2	...		−136	βγδ
W Lyrae 181136	1919 Oct. 14	8.5	−34	−174	44	−180	γδ
	15	8.5	−33	...		−177	γδ
						−178.5	
RY Ophiuchi 181103	1922 July 11	8.4	+6	...		−71	βγδ
	12	8.4	+7	...		−72	βγδ
						−71.8	
X Ophiuchi 183308	...	...	...	−70.6		−83.4	
AE Herculis 183922	1922 July 7	9.5±	...	...		−58	βγδ
	12	9.5±	...	...		−62	βγδ
						−60.4	
R Aquilae 190108	1920 Oct. 26	5.9	−4	+30	32	+22	γδ
	28	5.9	−2	...		+23	γδ
	1921 Aug. 11	6.3	−20	+33	38	+24	γδ
				+31.5		+22.8	
T Sagittarii *191017*	1921 June 20	8.5±	+39	...		−19	βγδ
	1922 July 13	9.1	+65	...		−20	βγ
						−19.4	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity		
				Absorption	Emission	
R Sagittarii						
191019......	1919 July 8*	7.7	0	....	− 50	$\beta\gamma$
	8*	7.7	0	....	− 54	$\beta\gamma\delta$
	Aug. 13	8.6	+ 36	....	− 52	$\gamma\delta$
					− 52.3	
R Cygni						
193449......	1921 Mar. 26	7.2	+ 30	....	− 49	$\beta\gamma\delta$
	27	7.3	+ 31	....	− 47	$\beta\gamma\delta$
	Apr. 28	8.9	+ 63	....	− 45	$\beta\gamma\delta$ 4308
	May 26	9.2	+ 91	....	− 44	$\beta\gamma\delta$ 4571, 4308, 4202
	June 19	9.7	+115	....	− 44	$\beta\gamma\delta$ 4571, 4308, 4202
	July 23	10.6	+149	....	− 42	$\beta\gamma$ 4571, 4308, 4202
	1922 May 17*	6.8	+ 4	....	− 49	$\beta\gamma\delta$
	June 14*	8.0	+ 32	....	− 51	$\beta\gamma\delta$ 4308
	July 11	8.9	+ 59	....	− 44	$\beta\gamma\delta$ 4308
					− 46.2	
χ Cygni						
194632......	1920 May 30	8.2	− 31	....	− 15	$\gamma\delta$
	June 1	8.1	− 29	....	− 17	$\gamma\delta$
	30*	5.2	− 1	− 1 29	− 21	$\beta\gamma\delta$
	July 29	5.8	+ 28	− 5 33	− 20	$\beta\gamma\delta$ 4308, 4202
	Sept. 3	6.5	+ 64	0 18	− 18	$\beta\gamma\delta$ 4571, 4308, 4202
	26	8.1	+ 87	....	− 19	$\beta\gamma\delta$ 4571, 4308, 4202
	Oct. 26	9.6	+117	....	− 14	$\beta\gamma\delta$ 4571, 4308, 4202
				− 2.1	− 17.8	
Z Cygni						
195849......	1919 Aug. 6*	8.6	− 3	....	−168	$\gamma\delta$
	7*	8.6	− 2	....	−177	γ
	8*	8.6	− 1	....	−177	$\gamma\delta$
	10	8.7	+ 1	....	−173	$\gamma\delta$
					−173.1	
S Cygni						
200357......	1920 June 2	8.9	− 2	....	− 34	$\beta\gamma\delta$
	1921 May 25	9.0	− 10	....	(− 27)	$\gamma\delta$
					− 33.7	
Z Aquilae						
200906......	1919 June 10*	8.8	+ 2	....	(− 1)	$\beta\gamma$
	Oct. 16	8.8	0	....	− 9	$\gamma\delta$
	17	8.8	+ 1	....	− 12	$\beta\gamma$
					− 10.3	
R Delphini						
201008......	1919 June 9*	8.5	+ 9	....	− 56	$\gamma\delta$
	1921 Sept. 20*	8.7	− 7	....	− 54	$\gamma\delta$
					− 54.8	
V Aquarii						
204102......	1919 Nov. 13*	....	......	....	− 49	$\gamma\delta$
	14*	....	......	....	− 57	$\gamma\delta$
					− 52.9	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity: Absorption		Velocity: Emission	
T Aquarii *204405*......	1919 July 9*	7.8	− 16	....		− 58	$\beta\gamma\delta$
	9*	7.8	− 16	....		− 55	$\beta\gamma\delta$
	Aug. 6*	7.9	+ 12	....		− 54	$\beta\gamma\delta$
	11	8.0	+ 17	− 41	12	− 52	$\gamma\delta$
	12	8.0	+ 18	− 36	18	− 54	$\gamma\delta$
				− 38.2		− 54.9	
X Delphini 205017......	1921 Oct. 12	8.7	− 9	− 58	21	− 64	$\gamma\delta$
	13	8.7	− 8	− 55	10	− 62	$\gamma\delta$
	13	8.7	− 8	....		− 63	$\gamma\delta$
				− 56.6		− 62.9	
T Cephei 210868......	1917 Nov. 3*	7.6	− 74	....		− 29	δ
	Dec. 3*	6.8	− 44	− 9	12	− 21	$\gamma\delta$
	1918 Jan. 2*	5.8	− 14	− 8	16	− 26	$\beta\gamma\delta\zeta\eta$ 3905
	Oct. 24*	7.2	−107	− 12	23	− 21	$\gamma\delta$
	1920 June 3*	8.0	+ 77	....		− 27	$\gamma\delta$ 4571, 4308, 4202
	July 5*	8.9	+109	....		− 24	γ 4571, 4308, 4202
	28*	9.4	+132	....		− 19	4571, 4308, 4202
	Sept. 5*	10.1	+171	....		(− 19)	4571, 4308, 4202
				− 9.6		− 24.1	
R Equulei 210812......	1921 June 21	9.4±	− 18	....		− 61	$\gamma\delta$
	July 24	9.0±	+ 15	....		− 63	$\gamma\delta$
						− 62.0	
RR Aquarii *210903*......	1922 Aug. 9	10.0	−34±	....		−187	$\beta\gamma\delta$
	10	10.0	−33±	....		−191	$\beta\gamma\delta$
	Sept. 8	9.2	− 4±	....		−195	$\beta\gamma\delta$
						−191.0	
T Pegasi 220412......	1922 July 12	10.0	+ 42	....		− 22	$\gamma\delta$
	13	10.0	+ 43	....		− 26	$\gamma\delta$
						− 23.9	
RS Pegasi 220714......	1921 July 24	9.5	+ 22	....		− 38	$\gamma\delta$
	25	9.5	+ 23	....		− 42	$\gamma\delta$
						− 40.0	
RT Aquarii *221722*......	1921 Aug. 12	9.5±	+ 14	....		− 43	$\gamma\delta$
S Lacertae 222439......	1919 Aug. 29	8.3	− 14	− 53	16	− 63	$\gamma\delta$
	31	8.2	− 12	− 61	22	− 67	$\gamma\delta\zeta$
	Sept. 4	8.0	− 8	− 58	40	− 65	$\gamma\delta\zeta\eta$ 3905
	5	8.0	− 7	− 61	38	− 66	$\gamma\delta\zeta$
	7	7.9	− 5	− 62	30	− 67	$\gamma\delta\zeta\eta$
				− 59.7		− 65.9	
R Lacertae 223841......	1921 Sept. 22	10.0	+ 29	....		+ 10	$\gamma\delta$
	23	10.0	+ 30	....		+ 7	$\gamma\delta$
						+ 8.2	

TABLE III—*Continued*

Star	Date	Mag.	Phase Days	Velocity Absorption		Velocity Emission	
S Aquarii							
225120......	1919 Sept. 4	8.0	− 6	....		− 59	γδ
	9*	7.9	− 1	....		− 67	βγδ
	1922 Nov. 6	9.2	+ 29	....		− 74	γδ
						− 66.4	
V Cassiop.							
230759......	1919 Aug. 6*	7.5	+ 2	....		− 53	γδ
	7*	7.5	+ 3	....		− 47	βγδ
	8*	7.6	+ 4	....		− 51	βγδ
	10	7.7	+ 6	− 27	40	− 45	γδζ
	12	7.9	+ 8	− 34	38	− 46	γδζη 4202
	1922 Oct. 10*	7.9	+ 8	− 31	16	− 43	βγδ
				− 30.5		− 47.6	
R Aquarii							
233815......	....	....	......	− 19		− 33	
V Ceti							
235209......	1921 July 25	8.8	− 4	....		+ 47	βγδ
	Aug. 11	8.8	+ 13	....		+ 40	βγδ
						+ 43.1	
R Cassiop.							
235350......	1920 Sept. 27	8.6	+ 57	....		+ 6	γδ
	Oct. 28	9.2	+ 88	....		+ 5	γδ 4202
	1923 Jan. 4*	7.4	+ 5	....		+ 5	γδ
	8	7.5	+ 9	....		+ 13	γδ
						+ 6.3	
SV Androm.							
235939......	1921 Sept. 21	9.4	− 6	....		− 97	γδ
	22	9.4	− 5	....		−100	γδ
	23	9.3	− 4	....		−100	γδ
						− 98.8	

NOTES TO TABLE III

001755, T Cassiopeiae: On the plate of Aug. 29, 1919, the comparison spectrum is imperfect. On the plate of Sept. 5, 1919, the comparison spectrum is slightly shifted, owing to a change in the temperature of the spectrograph during the exposure. The definition of the stellar spectrum, however, is good and the measured difference of the velocities from the bright and the dark lines is considered reliable. The difference on this plate was found to be 15.5 km, which, when applied to the adopted velocity for the bright lines from the other plates, gives −9.1 km as the velocity from the absorption lines. Bright Hγ is very weak on the first plates, and increases in strength during the series. This is in harmony with the characteristic behavior as outlined in *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, 53, 185, 1921. Hγ does not become a very conspicuous line on any of my plates, all of which, however, were taken well before maximum. In the notes to *Henry Draper Catalogue*, Miss Cannon has remarked upon the lack of strength of the bright Hγ and Hδ lines. The bright line λ 4202 is beginning to appear on the last two Mount Wilson plates. The presence of λ 3905 as a bright line on the first plate is interesting as showing its occurrence in considerable strength so long before maximum. The close bright companion to Hδ on the red side is well marked on most of the plates. Bright λ 4512 is especially strong in this star. The continuous spectrum from Hγ to λ 3900 is surprisingly strong compared with that from Hγ to Hβ.

004047, U Cassiopeiae: On the last plate the titanium bands are decidedly stronger than on the first two, and bright Hδ is stronger relatively to Hγ. These changes in the spectra of class S stars will be discussed more fully in another contribution.

022000, R Ceti: The plate of Sept. 3 is very poor.

065208, X Monocerotis: The bright hydrogen lines are less intense than in most variables. This star has been considered irregular (*Henry Draper Catalogue, Harvard Annals*, 92, 308, 1918), and it was not included in the computations in this paper which relate to the period. A recent Harvard *Bulletin* (No. 787), however, gives the period as 155.3 days, and this value has been entered in Tables V and VI and the star has been added to the plot in Figure 2. The short period is in harmony with the star's high velocity and its spectrum.

074323, T Geminorum: The M-type bands are weaker on the second plate than on the other two. The third plate is poor and the velocity derived from it is assigned half-weight.

081112, R Cancri: The proper value for the mean velocity from the emission lines and especially that from the absorption lines is doubtful to the extent of a few kilometers, owing to the systematic divergence of the first three plates from the others. The discrepancy may be connected with the fact that the first three plates are strongly exposed, the bright hydrogen lines being much overexposed. In general, however, strong and weak exposures do not show any decided systematic differences, although there seems to be some obscure effect, which operates only occasionally, tending in a few instances to cause spectrograms made with the 100-inch telescope to yield algebraically larger velocities than spectrograms of the same star made with the 60-inch telescope. The adopted emission-line velocity for R Cancri is the simple mean of all the individual results; to obtain the absorption velocity, the average value, *absorption minus emission*, for the first four plates, 13.7 km, was added to the mean emission-line velocity.

115158, Z Urs. Maj.: The bright hydrogen lines are weak.

115919, R Com. Ber.: Not in the *Henry Draper Catalogue*.

123307, R Virginis: The apparent variation in velocity is probably larger than the errors of observation. Both the Ann Arbor and the Mount Wilson measurements give the largest negative velocity *after* light maximum. The star has a short period and considerable range in magnitude; hence, if the light variations are dependent on changes of a geometrical nature, we might expect to find in this star an unusually large range in apparent radial velocity.

124606, U Virginis: On the first plate $H\beta$ is not seen, but on the second it is a strong bright line.

125705*a*, RT Virginis: No definite bright lines were seen on the plate of March 3, 1920, or on another, unlisted plate, taken four days later. The continuous spectrum from λ 4030 to λ 4227 is strong compared to that at longer wave-lengths. This relative strength of the continuous spectrum at short wave-lengths was noted by Miss Cannon. A remark in the *Henry Draper Catalogue* states that "The portion of the continuous spectrum between $H\delta$ and $H\epsilon$ appears like a bright band, and the region between $H\beta$ and $H\gamma$ is very faint."

130802, SW Virginis: The spectrum is much like that of RT Virginis (see preceding note) but an absorption line at λ 4535 is stronger. There appear to be absorption lines on either side of the position of $H\gamma$. Possibly the narrow space between them should be interpreted as a bright $H\gamma$. If so, it gives about the same velocity as the absorption lines. Remark in the *Henry Draper Catalogue:* "The brightest portion of the spectrum lies between $H\epsilon$ and 4226.9."

132422, R Hydrae: In forming the mean of the velocities from the emission lines, the last five plates were given unit weight; the weights of the others were, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, 2 2, 3, 2, respectively.

133633, T Centauri: Bright lines are seen on the plate of July 12, 1922, only, and are certainly stronger than on the other plates. The continuous spectrum also shows decided changes. On the first plate, which is too weak in the violet for measurement, the titanium bands are strongly marked in the region λ 4500 to λ 5000; the type is estimated as M4–5. The spectrum on the second plate is very similar to that of α Orionis and is of class M2. The absorption spectrum of the third plate more nearly resembles that of α Scorpii and is of class M7, having bright hydrogen lines superposed. Miss Cannon has noted in the remarks in the *Henry Draper Catalogue* that the spectrum varies from K2 having bright hydrogen lines to Ma having no bright lines. A comparison of the Mount Wilson measurements with those by Paddock[1] at Santiago, Chile, suggests that the radial velocity as well as the relative displacement of the bright lines may be variable. An extensive study of the variable spectrum of this star would be valuable.

134327, W Hydrae: The plate of June 11, 1922, was taken under poor conditions and is probably affected by some instrumental error. The velocity from this plate was not used in forming the mean. On the last plate the bright $H\gamma$ line is weak, showing that the phase is considerably before maximum. Bright $H\delta$ is strong and the close companion line on the red side is visible. The velocity from this plate was given small weight in forming the mean. This variable has a very small magnitude range, but appears nevertheless to exhibit the changes in spectrum characteristic[2] of the long-period variables. From the behavior of the spectrum in 1921, it is estimated that maximum occurred about February 20. Combined with Chandler's date of maximum, February 27, 1889, the period is found to be 389 or 377 days accordingly as 30 or 31 periods are assumed to have elapsed in the interval. By comparing the spectrograms taken in 1922 with those of 1921 a period of approximately 384 days is found.

142584, R Camelopardalis: The last plate was taken with a wide slit; the velocity from it has half-weight.

150605, Y Librae: The second plate is given one-half weight as the velocities from $H\gamma$ and $H\delta$ do not agree.

151822, RS Librae: The first plate is given one-half weight as the velocities from $H\gamma$ and $H\delta$ do not agree.

154748, ST Herculis: Bright hydrogen lines are not seen in this spectrum.

155947, X Herculis: Bright hydrogen lines are not seen in this spectrum.

175519, RY Herculis: On the first plate the comparison spectrum is imperfect, but the stellar velocity appears not to be affected.

183308, X Ophiuchi: A double star. See discussion of the velocity in *Mt. Wilson Contr.*, No. 261; *Astrophysical Journal*, **57**, 251, 1923.

194632, χ Cygni: Bright $H\beta$ is stronger than usual for a star with absorption spectrum of class M6.

200357, S Cygni: The second plate is very poor.

204405, T Aquarii: On the last two plates the comparison spectrum is slightly imperfect. On these plates the continuous spectrum is underexposed and the absorption-line velocities have small weight.

210868, T Cephei: The velocity from the last plate, which was taken with the 7-inch camera, was not used in forming the mean.

222439, S Lacertae: The velocities from the first plate were given half-weight because the comparison spectrum is imperfect.

233815, R Aquarii: In addition to the usual M8e features, the spectrum contains lines characteristic of gaseous nebulae. The velocities are taken from *Mt. Wilson Contr.*, No. 206, p. 4; *Astrophysical Journal*, **53**, 378, 1921.

235350, R Cassiopeiae: The last plate is rather poor and the velocities measured by two observers are discordant. This plate is given one-half weight in forming the mean.

[1] *Lick Observatory Bulletin*, **9**, 69, 1917.

[2] *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 1, 1921.

DISCUSSION OF RESULTS FOR INDIVIDUAL STARS

Velocities from emission lines.—An examination of the velocity measurements in Table III, as well as those obtained at Ann Arbor, appears to show that the ranges for individual stars can, in general, be accounted for by errors of observation. This may be inferred from the fact that the agreement of plates of a particular star taken on the same night or on successive nights is not decidedly better than that of plates separated by longer intervals. Moreover, the agreement of several plates of a star is in most instances about as good as could reasonably be expected from the number and internal agreement of the lines on each plate. Some apparent exceptions have been discussed in the notes to Table III, and the subject will be dealt with more fully in the following paragraphs.

In the present investigation and in the similar one previously carried out at Ann Arbor, the effort has been to determine the velocities of as many stars as possible rather than to secure extensive sets of observations of individual stars. Hence, most of the observed stars are represented by a small number of spectrograms taken near maximum. The desirability of testing the constancy of the velocity at different maxima and throughout the light-cycle has been borne in mind, however, and data for this purpose have been secured for a few stars.

The available measures show that velocities at different maxima are nearly the same. In a few instances the observed variations may exceed the errors of measurement, but the data are too meager to establish this as a fact. This question might better be studied with more powerful spectrographs as in numerous stars the bright $H\gamma$ and $H\delta$ lines could easily be photographed at maximum with a dispersion several times that which I have used.

A slight variation of velocity with phase is indicated for a number of stars, and as approximately the same behavior seems to be shown by those stars for which the data are most extensive, the effect is probably real. A study of X Ophiuchi has already been published.[1] The conclusion was that the velocities are not constant but have algebraically low values for a month or two after maximum light. The same statement seems to apply to several other stars.

[1] *Mt. Wilson Contr.*, No. 261; *Astrophysical Journal*, 57, 251, 1923.

The Mount Wilson observations of R Leonis, R Hydrae, X Ophiuchi, χ Cygni, and T Cephei are plotted in Figure 1. With

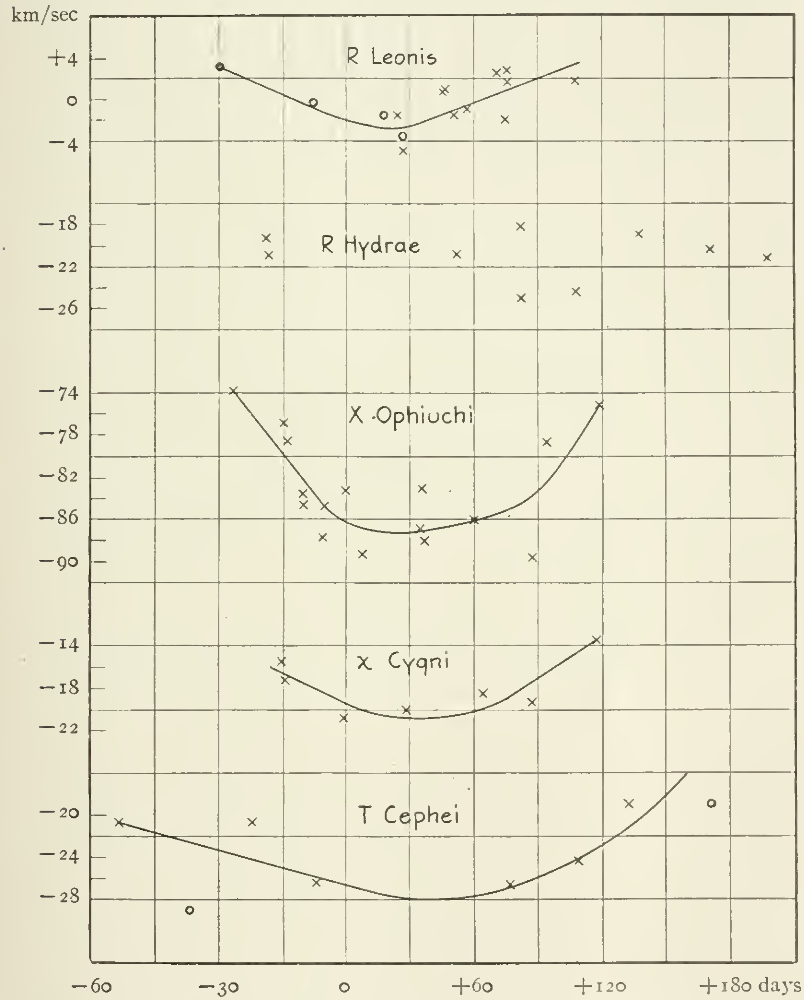

FIG. 1.—Variations in emission-line velocity. Circles represent observations of one-half weight.

the exception of R Hydrae, in which the effect is doubtful, they are better represented by a curve having a flat minimum a few weeks after maximum light than by a straight line. Several other stars

observed over shorter intervals show changes in velocity which are in harmony with a similar hypothesis. One of the most striking examples, although based on very few observations, is R Virginis.[1] Another is the S-type star R Canis Minoris.

About the time of maximum light the bright line λ 4202 becomes measurable, followed, as the light decreases, by λ 4308 and λ 4571. As these lines have been used together with the hydrogen lines in the velocity determinations, it is essential that the system of wave-lengths employed be relatively correct or apparent variations may be introduced. The wave-lengths used were the laboratory values indicated in Table I. The mean differences between the velocities from the hydrogen lines and those from the other bright lines have been computed and found to be small, showing that no large systematic errors exist.[2] For individual stars the displacements shown by the lines λλ 4202, 4308, and 4571 correspond reasonably well to those of the hydrogen lines on the same spectrograms.

It is not certain that the slight changes in the positions of the bright lines are due to variations in radial motion, but no other explanation presents itself. If the well-known displacement of the bright lines toward the violet with respect to the absorption spectrum represents an outflow of incandescent gas, it is possible that on first coming into view the gas has a low velocity and is subject to acceleration while under observation, or is replaced by other gas having a higher outward velocity, the process being reversed as the incandescent gas begins to disappear. This would indicate the existence of agencies acting during a considerable fraction of the light-period, rather than a sudden outburst followed by a gradual resumption of the previous conditions. The observation of the hydrogen lines is subject to this interpretation, but for the other gases a somewhat different state of affairs appears to exist. The lines λλ 4202, 4308, and 4571 appear later in the light-cycle than the hydrogen lines, but when they first become measurable, they have nearly the same velocities as the hydrogen

[1] See the note concerning this star on p. 235.

[2] See Table II, *Mt. Wilson Contr.*, No. 265; *Astrophysical Journal*, **58**, 196, 1923. See also measurements of *o* Ceti by Adams and Joy, *Publications Astronomical Society of the Pacific*, **35**, 168, 1923.

lines on the same plates. Later they agree with the hydrogen lines in showing a tendency toward algebraically larger velocities, as illustrated by the mean curves in Figure 1. A detailed explanation of this behavior is impossible at present, but it seems justifiable to conclude that the appearances of bright lines at different times in the light-cycle do not correspond to separate outbursts of various elements, but are manifestations of successive phases of the same disturbance by which the bright hydrogen lines were produced shortly after minimum.

Velocities from absorption lines.—Not much can be said as to possible variations in the absorption-line velocities, owing to the small number of observations obtained for individual stars. Except for a few weeks near maximum, most of the stars are so faint that it is difficult to photograph the continuous spectrum. Moreover, when the stars are faint, the absorption spectrum may not yield a satisfactory velocity, even when normally exposed, because the lines are likely to be weak and indefinite and to be interfered with by the titanium-oxide bands.

Neither my observations nor the other available data present evidence of changes in velocity, but they do not prove that changes of a few kilometers may not occur. Apparent changes in absorption-line velocities indicated by Tables III and IV are probably due in most if not in all cases to errors of measurement. Observations of *o* Ceti at different maxima agree remarkably well, as shown in Table IV.

All the stars for which either the absorption or the emission-line velocity has been measured by more than one observatory are listed in Table IV. The agreement, on the whole, is very good. Hence, even though we admit the existence of slight variations, it appears that both the bright and dark-line velocities are definite and nearly constant quantities for which essentially the same values are found by different observers using different instruments and methods of reduction. The data are, therefore, suitable for a statistical study of the motions of these objects.

The data in Table IV are taken largely from Table III and from the *Publications of the Observatory, University of Michigan*, 2, 50 ff., 1916. Professor Frost has had the kindness to send me for use in

TABLE IV

LONG-PERIOD VARIABLES OBSERVED AT MORE THAN ONE OBSERVATORY

Designation	Name	Observatory	Radial Velocity				Adopted	
			A.	Wt.	E.	Wt.	A.	E.
001755	T Cassiop.	Detroit	....	......	−27	0.5	......	−24.8
		Mt. Wilson	....	......	−25	6	......	......
001838	R Androm.	Detroit	....	......	−36	1	......	−36.3
		Mt. Wilson	....	......	−36	3	......	......
021024	R Arietis	Detroit	....	......	+101	3	......	+102.0
		Mt. Wilson	....	......	+104	2	......	......
021143	W Androm.	Detroit	....	......	−44	1	......	−44.6
		Mt. Wilson	....	......	−45	3	......	......
021403	*o* Ceti	Lick 1897–98	+62.3	14	+54	8	+64.3	+48.1
		Lick Nov. 1898	....	......	+44	10	......	......
		Lick Stebbins 3-pr	....	......	+44	8	......	......
		Lick Stebbins 1-pr	+66	1	+52	8	......	......
		Ottawa	+65.4	4	+46.1	14	......	......
		Yerkes 3-pr	+64.1	7	+44	7	......	......
		Yerkes 1-pr	+67.6	10	+49.6	27	......	......
		Bonn	+66.1	6	+51	8	......	......
		Detroit	+63.9	3	+52	4	......	......
		Cape	+63.4	10	....	......	......	......
		Mt. Wilson	+63.7	18	+46.7	18	......	......
050953	R Aurigae	Detroit	....	......	−9	1	......	−13
		Mt. Wilson	....	......	−15	2	......	......
084803	S Hydrae	Detroit	....	......	+77	1	......	+70
		Mt. Wilson	....	......	+66	2	......	......
093943	R Leo. Min.	Detroit	....	......	−6	2	......	−3.1
		Mt. Wilson	....	......	−2	3	......	......
094211	R Leonis	Yerkes	+18	2	−6.2	8	+15.0	−1.8
		Lick	+11	2	−7.	1	......	......
		Detroit	+26	1	0.0	7	......	......
		Mt. Wilson	+13.8	7	0.0	15	......	......
123160	T Urs. Maj.	Detroit	....	......	−107	3	......	−106
		Mt. Wilson	....	......	−102	1	......	......
123307	R Virginis	Detroit	....	......	−35	2	......	−33
		Mt. Wilson	....	......	−31	2	......	......
123961	S Urs. Maj.	Detroit	....	......	−1	2	......	−4.1
		Mt. Wilson	....	......	−5	7	......	......
132422	R Hydrae	Lick	−3	4	−26	2	−5.0	−22.8
		Detroit	+5	0.5	−26	3	......	......
		Mt. Wilson	−9	3	−21.4	12	......	......
133633	T Centauri	D. O. Mills	+24	2	+28	1	+24	+21
		Mt. Wilson	+25	1	+14	1	......	......
134440	R Can. Ven.	Detroit	....	......	−24	4	......	−23.6
		Mt. Wilson	....	......	−23	3	......	......
142539	V Boötis	Detroit	....	......	−41	3	......	−42.
		Mt. Wilson	....	......	−47	1	......	......
154615	R Serpentis	Yerkes	+30	1	+10	2	+27.	+6.9
		Detroit	+30	0.5	+8	4	......	......
		Mt. Wilson	+22	1	+5	7	......	......
180531	T Herculis	Detroit	....	......	−130	4	......	−131.
		Mt. Wilson	....	......	−136	1	......	......

TABLE IV—*Continued*

Designation	Name	Observatory	Radial Velocity A.	Radial Velocity Wt.	Radial Velocity E.	Radial Velocity Wt.	Adopted A.	Adopted E.
181136........	W Lyrae	Detroit	....	......	−186	3	....	−183.
		Mt. Wilson	....	......	−178	2	....	......
183308........	X Ophiuchi	Detroit	....	......	− 86	3	....	− 83.8
		Mt. Wilson	....	......	− 83.4	16	....	......
193449........	R Cygni	Lick	....	......	− 34	1	....	− 45.0
		Mt. Wilson	....	......	− 46.2	9	....	......
194048........	RT Cygni	Yerkes	....	......	−125	6	....	−125.5
		Detroit	....	......	−127	4	....	......
194632........	χ Cygni	Potsdam 1901	+ 2.4	4	− 19.8	25	− 0.5	− 19.7
		Potsdam 1902	− 2.3	4	− 21.0	15	....	......
		Detroit	....	......	− 17	2	....	......
		Mt. Wilson	− 2.1	3	− 17.8	10	....	......
210868........	T Cephei	Yerkes	−14	1	− 30	2	−10.7	− 26.1
		Detroit	....	......	− 30	2	....	......
		Mt. Wilson	− 9.6	3	− 24.1	8	....	......
235350........	R Cassiop.	Detroit	....	......	+ 9.4	5	....	+ 8.2
		Mt. Wilson	....	......	+ 6.3	3	....	......

NOTES TO TABLE IV

021403, o Ceti: Lick, *Astrophysical Journal*, **9**, 32, 1899; *Lick Observatory Bulletin*, **2**, 93, 1902; Ottawa, *Journal Royal Astronomical Society of Canada*, **1**, 53, 1907; Bonn, *Astrophysical Journal*, **27**, 304, 1908; Cape, *Astrophysical Journal*, **48**, 265, 1918; Mount Wilson, unpublished measures by Mr. Joy, to whom I am indebted for permission to include them here.

094211, R Leonis: Lick, a slightly revised value of the emission-line velocity was communicated to me by Dr. Moore.

133633, T Centauri: D. O. Mills, *Lick Observatory Bulletin*, **9**, 68, 1917.

194632, χ Cygni: Potsdam, *Astrophysical Journal*, **18**, 198, 1903.

this connection unpublished measures of o Ceti, R Leonis, R Serpentis, RT Cygni, and T Cephei made at the Yerkes Observatory. The notes indicate data from other sources.

Collected radial-velocity data.—The adopted velocities are collected in Table V, which contains other data of value for statistical studies. The first four columns require no explanation. The fifth column gives my estimates of spectral type at maximum, on the new system adopted by the International Astronomical Union in 1922. The typical stars selected for the subdivisions Mo to M6 by Mr. Joy and the writer are as follows:

Mo β Andromedae, H.D. 6860
M1 ν Virginis, H.D. 102212; α Scorpii, H.D. 148478
M2 α Ceti, H.D. 18884; α Orionis, H.D. 39801
M3 μ Geminorum, H.D. 44478
M4 ρ Persei, H.D. 19058
M5 α Herculis, H.D. 156014
M6 Boss 660, H.D. 18191

TABLE V

RADIAL VELOCITIES OF LONG-PERIOD VARIABLES

No.	Name	α 1900	δ 1900	Sp.	Period	Velocity Abs.	Velocity Em.
					days	km/sec	
1	S Sculptoris	$0^{h}10^{m}3$	−32°36′	M8e	366	(+28.4)	+ 13.2
2	X Androm.	0 10.9	+46 27	Se	346	(+1.)	− 18.2
3	T Androm.	0 17.2	+26 26	M6e	281	−90.	− 94.8
4	T Cassiop.	0 17.8	+55 14	M8e	444	−11.0	− 24.8
5	R Androm.	0 18.8	+38 1	Se	411	−8.2	− 36.3
6	S Ceti	0 19.0	− 9 53	M5e	321	(+32.)	+ 20.
7	U Cassiop.	0 40.8	+47 43	Se	276	−45.	− 56.6
8	V Androm.	0 44.6	+35 6	(M2e)	259	(+16.1)	+ 8.0
9	S Cassiop.	1 12.3	+72 5	Se	610	(− 20.±)	− 54.
10	R Piscium	1 25.5	+ 2 22	(M5e)	344	(−45.)	− 59.
11	Y Androm.	1 33.8	+38 50	(M3e)	218	(−3.8)	− 16.8
12	U Persei	1 53.0	+54 20	M6e	320	+15.	+ 9.9
13	R Arietis	2 10.4	+24 35	M3e	186	+114.4	+102.0
14	W Androm.	2 11.2	+43 50	M8e	395	−28.5	− 44.6
15	o Ceti	2 14.3	− 3 26	M6e	332	+64.3	+ 48.1
16	R Ceti	2 20.9	− 0 38	M4e	167	+42.5	+ 31.6
17	U Ceti	2 28.9	−13 35	M3e	236	−27.	− 39.4
18	R Trianguli	2 31.0	+33 50	M5e	267	+66.7	+ 59.6
19	R Persei	3 23.7	+35 20	M3e	210	−78.2	− 89.1
20	T Eridani	3 51.0	−24 20	(M7e)	252	(+43.)	+ 34.
21	W Eridani	4 7.3	−25 24	M7e	374	(+27.1)	+ 11.4
22	R Tauri	4 22.8	+ 9 56	M5e	324	(+31.3)	+ 19.2
23	T Camelop.	4 30.3	+65 57	Se	370	(+3.)	− 19.
24	X Camelop.	4 32.6	+74 56	M3e	142	(− 2.3)	− 5.8
25	T Leporis	5 0.6	−22 2	(M8e)	366	(−2.8)	− 18.
26	V Orionis	5 0.8	+ 3 58	M3e	267	(+21.9)	+ 13.9
27	R Aurigae	5 9.2	+53 28	M7e	459	(+8.)	− 13.
28	T Columbae	5 15.6	−33 49	M5e	225	(+66.4)	+ 53.4
29	U Orionis	5 49.9	+20 10	M8e	374	−17.	− 37.2
30	X Aurigae	6 4.4	+50 15	(M2e)	163	(−20.2)	− 26.2
31	V Monocer.	6 17.7	− 2 9	(M6e)	332	(+29.)	+ 16.
32	X Gemin.	6 40.7	+30 23	M5e	262	(+75.2)	+ 67.2
33	X Monocer.	6 52.4	− 8 56	M4e	155	+157.	+147.5
34	R Lyncis	6 53.0	+55 28	Se	379	(+34.)	+ 11.
35	R Gemin.	7 1.3	+22 52	Se	370	−36.	− 57.4
36	R Can. Min.	7 3.2	+10 11	Se	338	(+51.)	+ 33.3
37	L2 Puppis	7 10.5	−44 29	(M5e)	140	+52.6	+ 51.
38	V Gemin.	7 17.6	+13 17	M4e	276	+22.	+ 10.6
39	S Can. Min.	7 27.2	+ 8 33	(M8e)	330	(+67.2)	+ 54.6
40	Z Puppis	7 28.3	−20 27	M7e	516	(+35.6)	+ 12.6
41	T Gemin.	7 43.3	+23 59	Se	288	(+25.)	+ 11.8
42	R Cancri	8 11.0	+12 2	M7e	362	+32.1	+ 18.4
43	V Cancri	8 16.0	+17 36	Se	272	(−2.)	− 13.5
44	RT Hydrae	8 24.7	− 5 59	M7e	irreg.	+40.	+ 35.
45	U Cancri	8 30.1	+19 14	(M2e)	305	(+71.)	+ 61.
46	X Urs. Maj.	8 33.7	+50 30	M4e	251	(−81.)	− 90.
47	S Hydrae	8 48.4	+ 3 27	M4e	256	(+78.)	+ 70.
48	T Hydrae	8 50.8	− 8 46	(M3e)	289	(−3.)	− 12.
49	W Cancri	9 4.0	+25 39	(M7e)	385	(+51.3)	+ 34.7
50	R Leo. Min.	9 39.6	+34 58	M8e	372	+9.5	− 3.1
51	R Leonis	9 42.2	+11 54	M8e	313	+15.0	− 1.8
52	V Leonis	9 54.5	+21 44	M7e	273	(−22.7)	− 30.8
53	R Urs. Maj.	10 37.6	+69 18	(M6e)	302	+34.	+ 23.
54	Z Urs. Maj.	11 51.3	+58 25	M6e	120	−52.6	− 58.4
55	R Comae	11 59.1	+19 20	(M4e)	362	(− 7.0)	− 22.0

TABLE V—*Continued*

No.	Name	α 1900	δ 1900	Sp.	Period	Velocity	
						Abs.	Em.
					days	km/sec	
56	R Corvi	12^h14^m4	$-18°42'$	(M6e)	318	(−22.)	−34.
57	T Urs. Maj.	12 31.8	+60 2	(M6e)	257	(−98.)	−106.
58	R Virginis	12 33.4	+ 7 32	M6e	146	−34.	−33.
59	S Urs. Maj.	12 39.6	+61 38	Se	226	(+2.)	−4.1
60	U Can. Ven.	12 42.6	+38 55	(M8e)	....	(−27.)	−43.7
61	U Virginis	12 46.0	+ 6 6	M5e	207	−42.	−58.1
62	V Virginis	13 22.6	− 2 39	M6e	250	(+33.7)	+25.0
63	R Hydrae	13 24.2	−22 46	M8e	425	−5.0	−22.8
64	S Virginis	13 27.8	− 6 41	M7e	377	(+11.3)	−4.7
65	T Centauri	13 36.0	−33 6	M1e	90	+24.	+21.
66	W Hydrae	13 43.4	−27 52	M8e	384	+42.3	+26.4
67	R Can. Ven.	13 44.5	+40 2	M6e	333	−8.9	−23.6
68	U Urs. Min.	14 15.1	+67 15	(M6e)	327	(−31.0)	−43.3
69	S Boötis	14 19.5	+54 16	M4e	270	(−17.0)	−25.0
70	RS Virginis	14 22.3	+ 5 8	(M6e)	355	(−25.6)	−40.0
71	R Camelop.	14 25.1	+84 17	Se	270	(−32.)	−42.9
72	V Boötis	14 25.7	+39 18	(M6e)	256	−31.	−42.
73	R Boötis	14 32.8	+27 10	M4e	223	−46.	−57.
74	Y Librae	15 6.4	− 5 38	(M5e)	272	(−6.9)	−15.0
75	S Librae	15 15.7	−20 2	(M2e)	192	(+295.3)	+284.6
76	S Serpentis	15 17.0	+14 40	M5e	368	(+13.5)	−1.9
77	S Cor. Bor.	15 17.3	+31 44	(M8e)	361	−1.0	−22.
78	RS Librae	15 18.2	−22 33	M7e	219	(−2.1)	−15.1
79	RU Librae	15 27.7	−14 59	(M4e)	314	(−48.2)	−59.5
80	S Urs. Min.	15 33.4	+78 58	M7e	324	(−40.7)	−52.7
81	R Serpentis	15 46.1	+15 26	M7e	357	+27.	+6.9
82	RR Librae	15 50.6	−18 1	(M5e)	277	(−32.3)	−40.6
83	R Herculis	16 1.7	+18 38	M5e	318	(−30.6)	−42.1
84	U Serpentis	16 2.5	+10 12	M4e	240	(−29.3)	−39.8
85	RU Herculis	16 6.0	+25 20	M7e	486	(−15.3)	−37.6
86	W Cor. Bor.	16 11.8	+38 3	M4e	244	(+20.1)	+10.5
87	U Herculis	16 21.4	+19 7	(M8e)	403	(−22.5)	−40.4
88	W Herculis	16 31.7	+37 32	(M4e)	280	(−51.)	−59.
89	R Draconis	16 32.4	+66 58	(M5e)	246	−138.	−144.2
90	S Herculis	16 47.4	+15 7	(M5e)	308	(−10.7)	−21.3
91	RR Scorpii	16 50.2	−30 25	M7e	281	(−36.6)	−45.1
92	SS Ophiuchi	16 52.7	− 2 36	(M6e)	230	(−30.3)	−42.9
93	R Ophiuchi	17 2.0	−15 58	(M4e)	302	(−49.)	−59.
94	Z Ophiuchi	17 14.5	+ 1 37	(M2e)	348	(−79.)	−93.
95	RS Herculis	17 17.5	+23 1	(M6e)	223	(−37.8)	−50.8
96	RY Herculis	17 55.4	+19 29	M4e	222	−38.9	−50.1
97	T Herculis	18 5.3	+31 0	(M3e)	165	(−125.)	−131.
98	W Lyrae	18 11.5	+36 38	M5e	197	−174.	−183.
99	RY Ophiuchi	18 11.6	+ 3 40	M5e	153	(−67.2)	−71.8
100	X Ophiuchi	18 33.6	+ 8 44	M6e	335	−70.6	−83.8
101	AE Herculis	18 39.0	+22 54	M6e	....	(−48.)	−60.4
102	R Aquilae	19 1.6	+ 8 5	M7e	355	+31.5	+22.8
103	T Sagittarii	19 10.5	−17 0	Se	381	(+4.)	−19.4
104	R Sagittarii	19 10.8	−19 29	(M6e)	269	(−44.3)	−52.3
105	RT Aquilae	19 33.3	+11 30	M8e	326	(−42.0)	−54.2
106	R Cygni	19 34.1	+49 58	Se	426	(−15.)	−45.0
107	RT Cygni	19 40.8	+48 32	(M3e)	190	(−115.0)	−125.5
108	χ Cygni	19 46.7	+32 40	M6e	406	−0.5	−19.7
109	Z Cygni	19 58.6	+49 46	M5e	263	(−165.1)	−173.1
110	S Cygni	20 3.4	+57 42	(M2e)	323	(−21.7)	−33.7

TABLE V—*Continued*

No	Name	α 1900	δ 1900	Sp.	Period	Velocity	
						Abs.	Em.
					days	km/sec	
111	Z Aquilae	20h 9m.8	− 6° 27′	M3e	129	(−7.9)	− 10.3
112	R Delphini	20 10.1	+ 8 47	(M6e)	284	(−46.1)	− 54.8
113	V Aquarii	20 41.8	+ 2 4	M6e	246	(−43.4)	− 52.9
114	T Aquarii	20 44.7	− 5 31	M3e	203	−38.2	− 54.9
115	X Delphini	20 50.3	+17 16	M5e	281	−56.2	− 62.9
116	R Vulpeculae	20 59.9	+23 26	(M4e)	137	(−14.)	− 17.
117	T Cephei	21 8.2	+68 5	M7e	387	−10.7	− 26.1
118	R Equulei	21 8.4	+12 23	(M6e)	262	(−54.0)	− 62.0
119	RR Aquarii	21 9.8	− 3 19	M3e	180	(−182.0)	−191.0
120	W Cygni	21 32.2	+44 56	(M4e)	132	−27.	− 26.
121	T Pegasi	22 4.0	+12 3	(M6e)	374	(−8.2)	− 23.9
122	RS Pegasi	22 7.4	+14 4	(M6e)	436	(−20.0)	− 40.0
123	RT Aquarii	22 17.7	−22 34	M6e	241	(−33.)	− 43.
124	S Lacertae	22 24.6	+39 48	M5e	238	−59.7	− 65.9
125	R Lacertae	22 38.8	+41 51	(M6e)	300	(+18.1)	+ 8.2
126	S Aquarii	22 51.8	−20 53	(M6e)	280	(−57.9)	− 66.4
127	V Cassiop.	23 7.4	+59 8	M6e	230	−30.5	− 47.6
128	W Pegasi	23 14.8	+25 44	(M7e)	342	(−22.)	− 35.
129	S Pegasi	23 15.5	+ 8 2	(M6e)	318	(+5.)	− 7.
130	R Aquarii	23 38.6	−15 50	M7e+P	387	−19.	− 33.
131	V Ceti	23 52.8	− 9 31	(M3e)	261	(+51.1)	+ 43.1
132	R Cassiop.	23 53.3	+50 50	M7e	432	+30.	+ 8.2
133	SV Androm.	23 59.2	+39 33	M7e	298	(−89.1)	− 98.8

Two more subdivisions in continuation of this sequence, M7 and M8, were used, but no standard spectra not subject to variation are known to us. M9 and M10 are available for the spectra of certain stars at times other than maximum. Parentheses in the fifth column indicate that the classification is somewhat uncertain because, in most instances, of underexposure of the spectrograms. No star was included in this table unless emission lines of hydrogen had been measured in its spectrum.

The Harvard values of the period are given in the sixth column. Two stars marked as irregular, namely, X Monocerotis[1] and RT Hydrae, are included because their spectra are much like those of the periodic stars.

All the figures in the last column, and all those in the preceding column not in parentheses, are observed values and are taken from Tables III and IV or from the *Publications of the Observatory, University of Michigan*, 2, 50 ff, 1916. Slight corrections have been

[1] See note on p. 234.

made to some of the absorption-line velocities in the Ann Arbor list by using additional lines in the reductions. The absorption values in parentheses have been found from the emission velocities in a manner that will be described presently.

Relative displacement of bright and dark lines.—Spectrographic observations made at the Lick Observatory in 1897 and 1898[1] showed the effective centers of the bright $H\gamma$ and $H\delta$ lines to be displaced toward the violet with respect to the absorption spectrum. Bright lines at λ 4308 and λ 4376, if identified with iron lines, showed corresponding displacements. These results were confirmed and extended by Stebbins in 1902.[2] Similar behavior was found for the bright lines of χ Cygni by Eberhard in 1901 and 1902.[3] It seemed probable, as Eberhard remarked, that this is typical of long-period variables having the same type of spectrum. The present investigation, including the Ann Arbor measurements, makes it certain that this is the case, and also indicates the same effect for three S-type stars. Moore has found the same phenomenon in the spectrum of one N-type variable, U Cygni.[4] The occurrence of this relative displacement in such diverse types of spectra is very interesting and should be held in mind in considering the general problems of long-period variables of the three spectral classes.

The relative displacements of the bright lines have been measured for 47 variables of classes M and S. The results, computed from data in Table V, together with the spectral class, the period, and the magnitude range are collected in Table VI. Omission of the decimal of a kilometer in the column A. – E. denotes considerable uncertainty, except for the stars S Coronae Borealis and R Aquarii, in which cases the record does not show the decimal. As a matter of fact, no value in Table VI is reliable to a tenth of a kilometer, but the decimal has been used when available to prevent the accumulation of errors in plotting and computing.

[1] *Astrophysical Journal*, **9**, 31, 1899.

[2] *Lick Observatory Bulletin*, **2**, 93, 1902.

[3] *Astrophysical Journal*, **18**, 198, 1903.

[4] *Lick Observatory Bulletin*, **10**, 166, 1922.

TABLE VI

MEASURED DISPLACEMENTS OF ABSORPTION AND EMISSION LINES

No.	Name	Spect.	Period	Mag. Range	A.—E.	Wt.
			days		km	
3......	T Androm.	M6e	281	6.0	+ 4.8	1
4......	T Cassiop.	M8e	444	5.8	+13.8	2
5......	R Androm.	Se	411	8.4	+28.1	2
7......	U Cassiop.	Se	276	8.0	+11.6	1
12......	U Persei	M6e	320	3.9	+ 5.1	1
13......	R Arietis	M3e	186	5.9	+12.4	2
14......	W Androm.	M8e	395	7.5	+16.1	2
15......	*o* Ceti	M6e	332	7.6	+16.2	2
16......	R Ceti	M4e	167	6.2	+10.9	1
17......	U Ceti	M3e	236	6.1	+12.4	1
18......	R Trianguli	M5e	267	6.7	+ 7.1	2
19......	R Persei	M3e	210	5.9	+10.9	1
29......	U Orionis	M8e	374	7.0	+20.2	1
33......	X Monocer.	M4e	155	2.0	+ 9.5	1
35......	R Gemin.	Se	370	7.4	+21.4	1
37......	L_2 Puppis	(M5e)	140	2.8	+ 1.6	2
38......	V Gemin.	M4e	276	6.5	+11.4	1
42......	R Cancri	M7e	362	5.3	+13.7	2
44......	RT Hydrae	M7e	irreg.	1.8	+ 5.	1
50......	R Leo. Min.	M8e	372	6.0	+12.6	2
51......	R Leonis	M8e	313	5.9	+16.8	2
53......	R Urs. Maj.	(M6e)	302	6.5	+11.	1
54......	Z Urs. Maj.	M6e	120	1.5	+ 5.8	2
58......	R Virginis	M6e	146	5.7	− 1.	1
61......	U Virginis	M5e	207	6.0	+16.1	1
63......	R Hydrae	M8e	425	5.8	+17.8	2
65......	T Centauri	M1e	90	2.7	+ 3.	1
66......	W Hydrae	M8e	384	1.3	+15.9	2
67......	R Can. Ven.	M6e	333	6.6	+14.7	1
72......	V Boötis	(M6e)	256	4.1	+11.	1
73......	R Boötis	M4e	223	6.3	+11.	1
77......	S Cor. Bor.	(M8e)	361	7.3	+21.	2
81......	R Serpentis	M7e	357	7.4	+20.1	1
89......	R Draconis	(M5e)	246	5.7	+ 6.2	1
96......	RY Herculis	M4e	222	5.3	+11.2	2
98......	W Lyrae	M5e	197	4.9	+ 9.	1
100......	X Ophiuchi	M6e	335	5.2	+13.2	2
102......	R Aquilae	M7e	355	>6.2	+ 8.7	2
108......	χ Cygni	M6e	406	9.5	+19.2	2
114......	T Aquarii	M3e	203	6.3	+16.7	1
115......	X Delphini	M5e	281	>5.	+ 6.7	2
117......	T Cephei	M7e	387	5.4	+15.4	1
120......	W Cygni	(M4e)	132	1.7	− 1.	1
124......	S Lacertae	M5e	238	4.5	+ 6.2	2
127......	V Cassiop.	M6e	230	5.3	+17.1	2
130......	R Aquarii	M7e	387	4.8	+14.	2
132......	R Cassiop.	M7e	432	7.5	+21.8	2

STATISTICAL STUDIES

Relative displacement of bright lines.—Inspection of Table VI shows that the relative displacements (absorption *minus* emission-line velocity) for different stars differ by amounts greater than the errors of measurement. Studies were therefore made of the relationship of the displacement to spectral type, period, and range, respectively.

The mean displacements for each spectral type are shown in Table VII. The second column gives the average displacement (absorption *minus* emission) when each star is counted as one; the

TABLE VII

DISPLACEMENT OF EMISSION LINES AS RELATED TO SPECTRAL TYPE

Type	Equal Weights		Weights 1 and 2	
	Mean Displ.	No.	Mean Displ.	Wt.
M1e	3. km	1	3. km	1
M2e	....	0	....	0
M3e	13.1	4	13.0	5
M4e	8.7	6	9.2	7
M5e	9.0	7	7.3	10
M6e	10.6	11	11.8	16
M7e	14.1	7	15.7	10
M8e	16.8	8	16.2	14
Se	20.4	3	22.3	4

fourth column gives the displacements with weights 1 and 2, as indicated in Table VI, except that L_2 Puppis and S Coronae were given weight 1 because of some uncertainty in the determination of the spectral type. The table shows that the displacement increases with advancing spectral type, although class M3e is an exception, possibly because of the small number of stars included. The three stars of class Se have high values, but Figure 2 shows an influence depending on the period. The curve defined by the three S-type stars lies about 6 km above the corresponding portion of the curve for Me stars.

The stars were then divided into three nearly equal groups according to the size of the displacements, omitting the three stars of class S. The resulting mean displacements and types are given

in Table VIII. The correspondence between advancing type and increasing displacement is again shown.

If the stars with known periods are divided into three groups according to the period, we find the results given in the first portion of Table IX. If the same stars are grouped according to the

TABLE VIII

MEAN DISPLACEMENTS AND MEAN TYPES BY GROUPS

Group	No.	Mean Displ.	Mean Type
1	15	5.6 km	M 5.0
2	15	12.5	M 5.5
3	14	17.8	M 6.9

TABLE IX

MEAN DISPLACEMENT AND MEAN PERIOD BY GROUPS

	Group	No.	Mean Period	Mean Displ.
			days	km
Arranged by period	1	15	180	9.2
	2	15	297	10.3
	3	15	394	17.9
Arranged by displacement	1	15	219	5.7
	2	15	314	12.7
	3	15	356	19.0

TABLE X

MEAN DISPLACEMENT AND MEAN RANGE BY GROUPS

	Group	No.	Mean Range	Mean Displ.
			mag.	km
Arranged by range	1	15	3.6	9.4
	2	15	5.8	12.2
	3	15	7.5	17.5
Arranged by displacement	1	15	4.1	5.5
	2	15	5.7	12.7
	3	15	6.7	19.0

size of the displacement, we obtain the figures in the second portion of Table IX. In both cases the period increases with the displacement.

Proceeding in the same way for displacement and magnitude range, we have the figures in Table X, which show that the displacement increases with the range.

It thus appears that the displacement increases, on the average, with spectral type, with period, and with magnitude range. The accordance of individual stars for the period relationship is fair, but for spectral type and especially for range it is poor, the plotted results being scattering. We might expect the displacements to be closely correlated with spectral type, but this is not the case, although a statistical relationship is undoubtedly present.

The stars with ranges of less than five magnitudes have in general small displacements, and those with ranges above seven magnitudes have large displacements, but for the numerous stars having ranges between five and seven magnitudes the dispersion in displacement is large. One star, W Hydrae, deserves special mention as the outstanding exception to the rule that stars with small ranges have small displacements. Its period and spectroscopic behavior are typical of stars of type M8e, but it has the remarkably small range of 1.3 magnitudes. It is barely possible that, like X Ophiuchi, it is a double star. R Aquarii also has a magnitude range somewhat smaller than is typical of its period and spectral type. This may be due to the influence of the portion of the star connected with the emission of the nebular lines.

The best accordance shown by individual stars is in the case of the period relationship. Even here the deviations of some stars appear to exceed the errors of observation, although more material is perhaps necessary to establish this beyond doubt. In any event the period relationship offers the best method available of reducing the measured bright-line velocity of a star to a dark-line basis.

We must now face the question whether the displacements of the bright lines, or of the dark lines, or neither, yield the true radial velocity. In the case of one star, X Ophiuchi, a direct answer is available in favor of the absorption lines. The evidence is fully stated in another paper[1] and need not be repeated here. In R Aquarii the velocities from the nebular lines more nearly coincide with those from the absorption lines than with those from the bright lines connected with the M-type spectrum.[2] In both of these stars the Me spectrum seems to be a normal one. Statistical investigations of the motions also favor the absorption lines as yielding

[1] *Mt. Wilson Contr.*, No. 261; *Astrophysical Journal*, **57**, 251, 1923.

[2] *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 375, 1921.

essentially the true radial velocities. A solution for the sun's motion with respect to 83 Me variables, using emission-line velocities, gave a K term of -11.7 km, but a solution from the dark-line velocities of 133 stars (119 of class Me and 14 of class Se) gave a K term of $+3.9$ km; omitting one very high velocity star, S Librae, the K term came out -0.2 km. The K term for 76 of the slower moving variables (using absorption-line velocities) was $+1.3$ km. Another selection of 68 slow-moving variables gave a K term of $+1.1$ km. It seems clear therefore that for a study of the motions of these stars the absorption-line velocities are to be preferred to those from the emission lines.

In order to reduce the emission-line velocities to an absorption-line basis, the measured displacements were plotted against the periods; a curve drawn to represent the resulting points is shown in Figure 2. The observations seem to demand a maximum near 220 days, but it is uncertain what physical meaning should be attached to this feature. From the appearance of the plot we might infer that there exists a group of stars with periods in the neighborhood of 200 days, which have larger displacements than called for by the general progression shown by stars of longer and shorter periods. We might accordingly continue the general run of the curve across the interval from 140 to 260 days, but for the purpose of making an empirical determination of the displacement corresponding to a given period, it seemed best to draw the curve as indicated. For those stars for which emission lines only have been measured, the hypothetical dark-line velocities were obtained by applying the displacements read from the curve. The S-type stars were treated separately; only 3 were available as standards and these are marked by the letter S in Figure 2. The dark-line velocities determined in this manner are given in the next to the last column in Table V, *in parentheses* to distinguish them from values depending directly upon measurement.

In the earlier work at Ann Arbor it was noticed that two stars with especially short periods, namely L_2 Puppis and W Cygni, showed practically zero displacements. From a study of the Ann Arbor data Ludendorff was led to state[1] that "the magnitude of

[1] *Astronomische Nachrichten*, **212**, 483, 1921.

the displacement of the hydrogen emission lines depends on the period of the star." In spite of the meager material at his disposal, this conclusion is essentially correct, being substantiated by the more extensive data reported in the present paper. It has seemed best, however, not to assume a linear relationship between displacement and period, as was done by Heiskanen and Ludendorff,[1] to reduce emission-line velocities to a dark-line basis.

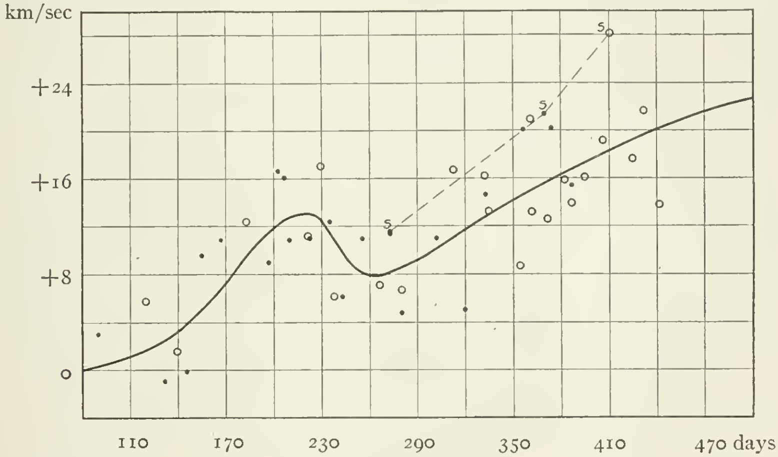

FIG. 2.—Relative displacement of emission lines (Abs.—Em). Observations represented by dots have one-half the weight of those represented by circles.

Group-motion of long-period variables.—The highly specialized character of the light variations and of the spectra of long-period variables leads us to examine their motions to ascertain whether they differ in any way from the average motions of other stars. We may consider first the common or group-motion of the variables, and second their random or individual motions.

The Ann Arbor data[2] strongly suggested a group-motion in a general direction opposite to that in which the sun is moving with respect to other stars. Heiskanen and Ludendorff[3] emphasized the fact that the result depended to a considerable extent on the

[1] *Ibid.*, 213, 297, 1921.

[2] *Publications of the Observatory, University of Michigan*, 2, 63, 1916.

[3] *Astronomische Nachrichten*, 213, 297, 1921.

influence of a few rapidly moving stars, and thought that no real stream-motion existed. With the additional velocity determinations now available, we are in a position to discuss this question to much better advantage than before.

The following computations have been based on the absorption-line velocities as tabulated in the next to the last column in Table V. The positions and velocities of the 133 stars included in this table are charted in Figure 3. The deficiency of observed stars in the

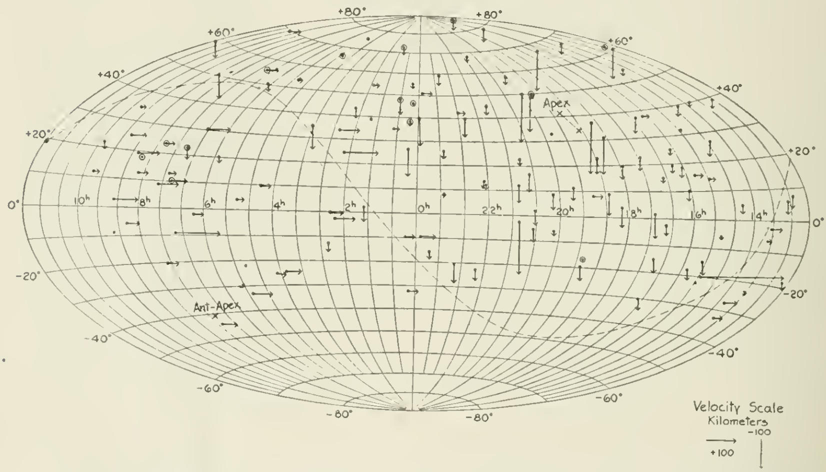

FIG. 3.—Chart of long-period variables whose radial velocities have been measured. Class Se stars are indicated by circles. The position of the solar apex for the 133 stars is marked by the cross just below the word "apex"; another cross, to the right and below, indicates the computed position when the most rapidly moving star, S Librae, is omitted. The dotted curve is the great circle whose poles are the apex and ant-apex.

southern hemisphere is, of course, an unsatisfactory feature. It is hoped that before long data will be available for additional southern stars. Other irregularities in the chart correspond more or less closely to the actual distribution of variables in the sky. As has been frequently pointed out, this is by no means uniform, a decided lack of stars existing near right ascension eleven hours.

In the first place, a least-squares solution for the solar motion with respect to all the 133 variables was made with the following results:

$$A_0 = 287^\circ\!.1,\ D_0 = +41^\circ\!.0,\ V_0 = 55.0 \text{ km/sec.},\ K = +3.9 \text{ km/sec.}$$

The position of the apex is about the same as that given by several investigators for stars of classes K and M, but the speed of the sun is nearly three times as great as that usually found. As the variables have high random motions,[1] we may recognize in the facts above a phase of the dependence of solar motion upon the speed of the stars, as brought out by Boss,[2] Adams and Joy,[3] and Strömberg.[4]

In computations concerning the motions of the stars the questions of grouping the material and of rejecting apparently exceptional stars are often troublesome, especially when, as in the present investigation, the total number of stars treated is small. If the stars are divided into strictly homogeneous groups, the number in each group may become so small that the results are untrustworthy. For this reason I have avoided the use of more than two groups in solutions for the solar motion. Several methods of grouping are possible, but the only one used here is that based on the random motion of the individual stars as a criterion of selection. The following computations are those which seemed the most suggestive, but they do not constitute an exhaustive treatment of the data. Other investigators may find further computations profitable.

One star, S Librae, has such a high velocity that it stands quite by itself in the plot in Figure 4. A solution for the solar motion with this star omitted is shown in Table XI. The large effect of this star is, of course, due to the fact that the determining factors are the *squares* of the residuals. Some other system of solution would probably be preferable. The arithmetic mean residual for the 132 stars has not been computed, but it is estimated to be between 30 and 31 kilometers.

The small value of the K term gives us confidence that the method of reducing the emission-line velocities to a dark-line basis is reasonably accurate.

In order to bring out the possible dependence of the computed solar motion upon the peculiar velocities of the stars involved, the variables were divided into two groups according to the numerical size of their residual velocities, which were found by applying

[1] The arithmetic mean residual from this solution is 33.4 km.

[2] *Astronomical Journal*, **35**, 26, 1923, and references there given to earlier papers.

[3] *Mt. Wilson Contr.*, No. 163; *Astrophysical Journal*, **49**, 179, 1919.

[4] *Mt. Wilson Contr.*, No. 245; *Astrophysical Journal*, **56**, 265, 1922.

to the absorption-line velocities a correction for the solar motion computed from the 133 variables themselves, as given on page 252. Sixty-eight stars were found to have residual velocities equal to or less than 25 km, and 65 to have residual velocities exceeding this figure. Separate solutions for the solar motion were based on these

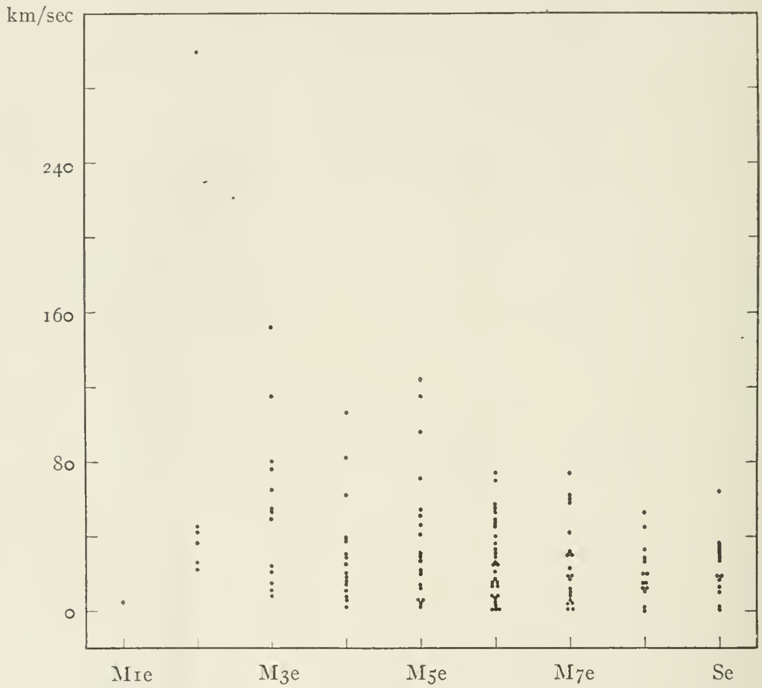

FIG. 4.—Residual radial velocity and spectral type. Spectroscopically class Se is not a continuation of the sequence M1e–M8e; it is placed at the end of the diagram for convenience and to show the correspondence in velocity to classes M7e and M8e.

two groups. The resulting data are shown in Table XI. A previous solution based on the emission-line velocities of 83 stars[1] is included for comparison. The solution based on 76 stars with low residuals will be described presently.

The increase in V_0 in going from the 68 slow-moving stars to the 65 rapid ones is very marked, and seems to be a fine illustration of the velocity asymmetry of high-speed stars.

[1] These stars were not selected for small velocity but included all that were available when the solution was made.

The 68 slow-moving stars give a value of V_0 much greater than that usually found, and, on the face of it, this indicates a strong group-motion prevailing among the variables independently of their high speed. These figures may be somewhat misleading, however, as a little consideration will show that the two solutions lead of necessity to results much like those based on all the stars. As far as the 68 stars are concerned, they have been selected in such a way that their motions must nearly equal that called for by the general solution, and hence they are not likely to yield very different results. On the other hand, the group of 65 rapidly moving stars will contain the high residuals which, since the method of solution is that of least *squares*, counted most heavily in the general

TABLE XI

SOLUTIONS FOR SOLAR MOTION

	All	Omitting S Librae	Low Residuals		High Residuals	Emission-Line Velocities
Number....	133	132	68	76	65	83
A_0........	287°.1	280°.7	283°.5	263°.9	289°.2	274°.4
D_0........	+41.0	+33.2	+35.6	+34.4	+46.2	+44.0
V_0........	55.0km	52.5km	47.9km	29.0km	64.6km	55.9km
K.........	+ 3.9	− 0.2	+ 1.1	+ 1.3	+ 7.9	−11.7
Arith. Mean Resid.	33.4	....	11.1	13.3	56.8	30.9

solution, and hence will tend to reproduce the same figures. We must therefore exercise caution in interpreting similarities in the results of the three solutions. The same reasoning, however, leads us to rely on *differences* in the figures as probably having a real meaning. The increase in V_0 with speed, for example, is scarcely to be explained except as a physical fact.

Another procedure was adopted to test the systematic motions of the slowly moving variables. The residuals from the ordinary solar motion, $A_0=270°$, $D_0=+30°$, $V_0=20$ km, were first computed. Seventy-six stars having residuals less than 30 km were then made the basis of a new solution. The results are in the fifth column of Table XI. Although the mean residual, regardless of sign, 13.3 km, is not greater than that for stars of classes F, G, K, and M, the solar motion nevertheless is increased from 20 to 29 km. Moreover, reasoning similar to that outlined above shows

that the method of selection tends to make this difference unduly small. Hence the conclusion seems to be justified that long-period variables have a general group-motion, largest, it is true, for the high-speed stars, but not zero for the slow ones.

Random motions.—It is well known that the average random motion varies for groups of stars of different spectral types or different absolute magnitudes. The general rule is that it increases with advancing spectral type, and decreases with increasing luminosity. The residual motions of the individual variables have been computed from several solutions for the group-motion, and have been grouped and tabulated in various ways in an attempt to bring out possible systematic relationships to other quantities.

TABLE XII

SPECTRAL TYPE AND RESIDUAL VELOCITY

Type	Number	Arith. Mean Residual
		km
M1e	1	5
M2e	6	78
M3e	13	56
M4e	16	32
M5e	19	40
M6e	30	28
M7e	20	25
M8e	14	21
Se	14	24

First, the residuals from the ordinary solar motion were found, assuming $A_0 = 270°.0$, $D_0 = +30°.0$, $V_0 = 20.0$ km. The arithmetic mean residual is 35.5 km. This is the largest value found for any group of stars selected on the basis of spectral type.[1] It may be reduced to 35.3 km by applying an arbitrary correction of +3.6 km to all the residuals. For a solar motion based on the 133 variables themselves the arithmetic mean residual velocity is 33.4 km. By omitting one star this would be reduced about 2.5 km.

The relationship of residual motion to spectral type was next considered. The mean residual velocities for each type from the solution for 133 stars were found with the results shown in Table XII.

[1] The corresponding value for 102 planetary nebulae is 36 km (*Publications of the Lick Observatory*, 13, 168, 1918). Is the agreement accidental?

Here we find a strong tendency for the random motion to decrease with advancing type, the velocity of the Se stars corresponding to that of classes M7e and M8e. The residuals from the other solutions have also been grouped according to spectral type, the results being exhibited in Table XIII.

The most rapidly moving stars show an evident tendency to favor the earlier spectral classes. Among the slowly moving stars one could not expect to find a decided progression with type, but in the case of the 68 stars selected by residuals from the solution based on the variables themselves, there appears to be a trace of it. In the case of the 76 stars selected on the basis of the residuals from the ordinary solar motion, the selection has little or nothing

TABLE XIII

MEAN SPECTRAL TYPE AND RESIDUAL VELOCITY BY GROUPS

Types	All			High Residuals			Low Residuals					
							68 Stars			76 Stars		
	Mean Type	No.	Mean Resid.	Mean Type	No.	Mean Resid.	Mean Type	No.	Mean Resid.	Mean Type	No.	Mean Resid
			km			km			km			km
M1e–M4e.	3.5	36	47	3.1	20	74	3.4	16	14	3.1	17	10
M5e–M6e.	5.6	49	32	5.6	25	53	5.7	24	11	5.7	23	15
M7e–M8e.	7.4	34	24	7.4	13	46	7.4	21	10	7.4	26	15
Se.......	.....	14	24	.....	7	40	.....	7	11	.....	10	12

to do with characteristic properties of variable stars and the mean residuals probably have no significance. The correlation of high speed with early spectral type is evidenced, however, by the *numbers* of stars involved in the various groups. Among the high residuals the ratio of the number of stars in classes M1 to M4 to those in classes M7 and M8 is 1.5, while the same ratio for the stars with low residuals is 0.8 in one case and 0.7 in the other. If the stars are divided into several groups according to size of the residual velocity, the mean spectral type of each group tends to decrease with increasing velocity. Both methods of grouping exhibit irregularities in the correlation between type and velocity, however, showing that the relationship is of a general statistical nature and not binding on individual stars.

The relationship between light-period and velocity was next examined. The stars involved in each solution were arranged in order of period and divided into equal or nearly equal groups. In the main solution for 133 stars, seven groups were formed; in the solution for 68 and 65 stars, five groups each. The mean periods and the mean residuals for these groups are tabulated in Table XIV.

On the whole there is a decided tendency toward slower motion with increasing period. This appears most clearly among the stars with high residuals. Among the stars with low residuals the

TABLE XIV

PERIOD AND RESIDUAL VELOCITY

Group	All		High Residuals		Low Residuals	
	Mean Period	Mean Residual	Mean Period	Mean Residual	Mean Period	Mean Residual
	days	km	days	km	days	km
1	155	64	174	90	171	13
2	227	27	254	62	242	12
3	260	37	293	50	291	10
4	289	36	328	38	350	7
5	323	21	380	48	428	15
6	359	31	....	....	....	....
7	418	24	....	....	....	....

total range of velocity is so small that a strong progression cannot be expected. The fact that long-period stars tend to have small residuals is indicated, however, by the fact that the last two groups have a larger mean period than the corresponding groups of the stars with high residuals.

The correlation between period and velocity is incomplete in that stars of low velocity may have any length of period. It seems clear, however, that very high velocities are largely confined to stars having short periods.[1]

Heiskanen and Ludendorff, in a study based on 44 variables,[2] called attention to the probable dependence of velocity upon period, and pointed out that at least the dispersion of the radial velocities

[1] Averaging a little over 200 days. The stars with the very shortest periods, 90–150 days, do not appear, from present data, to have extremely high velocities.

[2] *Astronomische Nachrichten*, 213, 297, 1921.

decreases with increasing period. This is confirmed by the more extensive data of the present article.

That the apparently fainter variables are moving more rapidly than the brighter ones, as indicated by the Ann Arbor measurements, appears to have been an accidental result due to high velocities of comparatively few stars, as the present data do not afford much evidence of such a relationship.

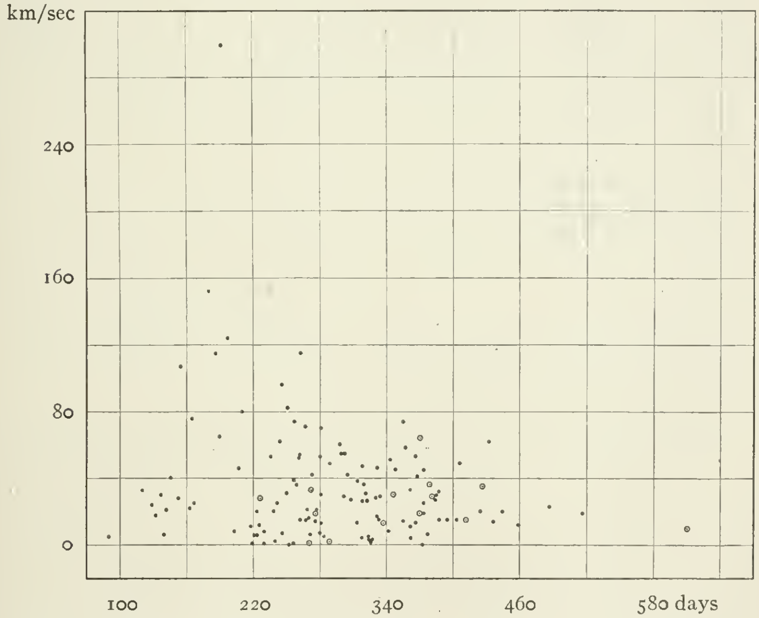

FIG. 5.—Residual radial velocity and period. Class Se stars are indicated by circles.

It is a pleasure to acknowledge the able assistance of Miss Cora G. Burwell in making the computations described in the foregoing pages.

GENERAL DISCUSSION

The main purpose of this article, which is to present observational results concerning the radial velocities of long-period variables, has been accomplished in the preceding pages, which contain also some computations and tabulations exhibiting certain statistical relationships. Some features of the data are very puzzling and

seem quite out of harmony with the more general facts of stellar motions and with plausible hypotheses as to the place of the variable stars in stellar evolution. No attempt is made to explain these phenomena and to co-ordinate them with other astronomical facts, and the article is concluded with only a brief recapitulation of the general results and a cursory examination of some of the problems involved.

The very extensive photometric and spectroscopic data concerning long-period variables of class Me gathered by many observers, but chiefly by those of the Harvard College Observatory, have shown that the average values of the following characteristics vary together as indicated:

1. Advancing spectral type[1]
2. Increasing period
3. Increasing magnitude range

The spectrographic observations discussed in this paper enable us to add to the list:

4. Increasing displacement of bright lines relative to absorption lines
5. Decreasing random velocity

It is not easy to decide which of these relationships arise directly from physical causes, and which are incidental. The correlation of magnitude-range with relative displacement of the bright lines and with random velocity appears to have little direct significance. The relative displacement of the bright lines seems to be most clearly connected with the period, while the random velocity is correlated about equally well with type and with period. But in all these correlations neglected factors are evidently present, which in some stars are more potent than the general statistical tendencies and cause a considerable spread in values from individual stars.

One might expect the relative displacement of the bright lines to vary more directly with type than with period, but this is not the case, and it is really not surprising in view of the fact that S- and

[1] The correlation of the subdivisions of class Me with period becomes quite clear when the S-type stars are removed from the list. See *Mt. Wilson Contr.*, No. 252; *Astrophysical Journal*, 46, 472, 1922.

N-type stars exhibit similar displacements and that the S stars also show a progression with period. Some fundamental property such as mass or density probably influences both period and displacement.

The decrease of average random velocity with advancing type and increasing period is difficult to understand if we make the natural assumption that the Me stars continue the main sequence of types from B to M. According to this view, there is a great increase in velocity in passing from the ordinary M stars to those of classes M3e to M5e and then a decrease for classes M6e to M8e. This is so improbable that one can scarcely avoid the conclusion that we have to deal with something more complex than a single evolutionary sequence of identical objects passing from M8e to K. It may well be that individual stars differ in certain properties; e.g., mass or chemical composition, or that their cosmical environments are not sufficiently uniform to cause all stars to pursue precisely the same train of behavior.

The periods and the random velocities of the S-type stars are nearly the same as those of types M7e and M8e, but their spectra certainly do not belong at the end of the sequence M1e-M8e. Judging from its general appearance, the typical S spectrum more nearly corresponds to that of Class M1 or M2.

From twenty-five stars of class N, Moore[1] found a mean residual velocity of 18.0 km, which is decidedly less than the corresponding value for class Me, and somewhat less than that for class Se. The solar motion from the N stars does not resemble that from the Me stars, so that we may say that radial velocity data do not suggest a close connection between variables of classes M and N.

The average residual radial velocity of the Me stars is nearly the same as that of 102 planetary nebulae.[2] Whether or not this fact has any physical significance is a question for the future.

If the Me variables form a part of the main evolutionary sequence, they apparently represent either an initial or a final stage; their high velocities, however, make it difficult to consider

[1] *Lick Observatory Bulletins*, 10, 160, 1922.

[2] *Publications of the Lick Observatory*, 13, 168, 1918.

them very young stars just beginning their visible careers, while, on the other hand, it is very improbable that they are aged and highly condensed objects about to sink into obscurity.

In spite of our very considerable store of data concerning these objects, it seems plain that we cannot cope with the fascinating problems which they present without securing many more facts of various kinds.

The discussion of the velocity determinations may profitably be extended by using the proper motions which, chiefly through Wilson's work,[1] are now available for about 90 of the 133 stars having measured radial velocities. It is the intention of Dr. Strömberg and the writer to do this in another contribution.

Mount Wilson Observatory
June 1923

[1] *Astronomical Journal*, 34, 183, 1923.

NOTE ON THE TOTAL SOLAR ECLIPSE OF SEPTEMBER 10, 1923

The total solar eclipse of September 10, 1923, proved a sore disappointment to the American astronomers assembled along the southwestern coast of our continent. Cloudy conditions, quite unusual at the season, prevailed for several hundred miles along the track of totality and completely frustrated the extensive preparations of most of the expeditions.

At Puerto Libertad, on the Gulf of California, Professor A. E. Douglass, of the University of Arizona, was able to carry out his program.

Near Yerbanis, in the State of Durango, Mexico, were located the joint party from the Swarthmore and Allegheny observatories, under Professors J. A. Miller and Heber D. Curtis, and the official Mexican party, under Sr. J. Gallo, director of the Observatory of Tacubaya. In spite of a heavy downpour not long before the eclipse, the sky cleared sufficiently for successful observations. Not far away was a party of German astronomers, including Messrs. H. Ludendorff, R. Schorr, and A. Kohlschuetter, who had a clear sky.

The sky was fairly favorable at Lompoc, northwest of Santa Barbara, California, and photographs of the corona were secured, but there were few observers in this vicinity on account of the short duration of totality.

Mt. Wilson Observatory had a large party stationed on Point Loma, near San Diego. Director W. S. Adams and Dr. C. E. St. John were planning to make spectroscopic observations on Mt. Wilson, where the eclipse was within 2 per cent of being total, but unfavorable sky nullified these plans.

A party from the McCormick Observatory of the University of Virginia, under Professor S. A. Mitchell, had a station at Fort Rosecrans, near Point Loma, and also at Lakeside, where the Mt. Wilson party had a second station.

The Lick Observatory had planned an extensive program for Ensenada, Mexico, and not far away were the stations of the Lowell Observatory and of the observatory of the University of Indiana.

The station of the Yerkes Observatory was situated on Santa Catalina Island. The expedition was made possible by the generous gift of $5,000 from Mr. William Wrigley, Jr., who holds the principal financial interest in the island. The site chosen for the station was 1300 feet above the sea, and about three miles from the city of Avalon. During the forty days prior to September 10 that the

camp had been occupied, the sky was very favorable at the hour of the eclipse on thirty-five days, and unfavorable on five, being completely covered on only two days at 0^h54^m Pacific Standard Time. The elevation of the sun would have been nearly 58°, and the conditions as regards wind were very favorable. The program included:

1. Photography of the sun on a large scale with the coelostat as used in 1900 and 1918;
2. The photography of the flash spectrum with a train of objective prisms on a motion picture film:
3. The photography of the infra-red flash spectrum on films sensitized with dicyanin, a small concave grating being used;
4. An attempt to measure the rotation of the corona by the use of an autocollimating spectrograph supplied with coronal light from points east and west of the sun;
5. The measurement of the brightness of the corona, visually with the Hartmann microphotometer, and photographically on plates taken with objective-prisms placed over the Zeiss U.-V. doublet and over a 6-inch reflector;
6. The determination of the color temperature of the corona with an optical pyrometer;
7. The photography of the successive stages of the eclipse and of the corona, with a Hawkeye lens of 20 inches focus attached to a Universal moving picture camera, an image from a standard light being impressed upon each picture;
8. Photography with short-focus camera of the sky around the sun;
9. An autochrome photograph of the corona;
10. Photography of the shadow bands with a very rapid lens, and with cinematographic camera; besides various visual observations of the corona and attendant phenomena with several telescopes.

Among those composing the party of the Yerkes Observatory were thirty-seven scientific men and women, including representatives of some twenty-five astronomical observatories and colleges. A more detailed account of this expedition will soon appear in *Popular Astronomy*.

Northwestern and Harvard Universities had instruments within Camp Wrigley. Immediately adjacent were the parties from the University of Wisconsin, Drake University, and Carleton College.

About twenty-five miles distant, at the "Isthmus" on Catalina Island, Professor Frank P. Brackett of Pomona College had established a station, where observations were to be made by a number of observers.

E. B. F.

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THE

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SPECTROGRAPHIC OBSERVATIONS OF MIRA CETI, R LEONIS, T CEPHEI, AND R SERPENTIS

By EDWIN B. FROST AND FRANCES LOWATER

ABSTRACT

Reference is made to the results published prior to 1917 by other observers of the spectra of Mira Ceti and R Leonis.

Velocities of Mira Ceti, R Leonis, T Cephei, and R Serpentis.—The velocities of these stars were determined from (*a*) absorption lines, (*b*) bright hydrogen lines, and (*c*) bands of titanium oxide on spectrograms obtained chiefly during the years 1906–1917 at the Yerkes Observatory. Table II exhibits these velocities. A dispersion of three prisms was used for five plates of Mira Ceti, but that of one prism for all of the other thirty-eight plates measured. The velocities obtained from dark lines are approximately constant for each star. The average difference in the velocity of these stars determined from bright and from dark lines is −19.4 km, in close agreement with the differences obtained by other observers (Table III).

A possible explanation of the striking characteristics of the spectra of Md stars.—The difference in the velocities determined from bright and from dark lines, the variation in magnitude and in the maxima may possibly be accounted for by outbursts of hydrogen recurring with considerable regularity.

Bright lines.—The wave-lengths of twenty-four bright lines in addition to those of hydrogen have been determined; although the identification of their sources remains uncertain, their origin would seem to be at comparatively low temperature.

The physical constitution of the stars of class Md, characterized by spectra having dark lines, bright lines and dark bands, offers an interesting field for investigation. One peculiar feature of this class is that the radial velocities determined from the displacement of the dark lines differ systematically from those determined from the bright hydrogen lines, by a quantity of the order of 20 km per second. Another is that the variation in the intensity

of the bright hydrogen lines implies that they disappear when the star's magnitude becomes a minimum. A complete explanation of these phenomena has not yet been found.

Some of the stars of this class were placed on the observing program of the Bruce spectrograph as early as 1906. While no novel results have been obtained, it has seemed desirable to print our study of those for which satisfactory plates are available, particularly Mira Ceti and R Leonis. It will be recalled that the period of Mira is about 331 days, with a range of magnitude from 2.0 to 9.6; and that of R Leonis is 313 days, with a range of magnitude from 4.6 to 10.2. Their maxima and minima are not constant, however, but no regularity in their variation has yet been determined.

Investigations on the spectra of Mira Ceti and R Leonis are found distributed over a period of nearly thirty years. They were assigned to class Md by the Harvard observers.[1]

The most important early papers on these stars are the following:

Harvard Annals, **28**, 45, 98, 108, 1897.

Annie J. Cannon, *ibid.*, **56**, 73, 104, 1912.

W. W. Campbell, *Astrophysical Journal*, **9**, 31, 1899.

W. Sidgreaves, *Monthly Notices*, **58**, 344, 1898; **59**, 505, 1899; **60**, 579, 586, 1900.

A. L. Cortie, *ibid.*, **67**, 534, 1907.

Joel Stebbins, *Lick Observatory Bulletin*, **2**, 78, 1903; *Astrophysical Journal*, **18**, 341, 1903.

V. M. Slipher, *Astrophysical Journal*, **25**, 66, 235, 1907.

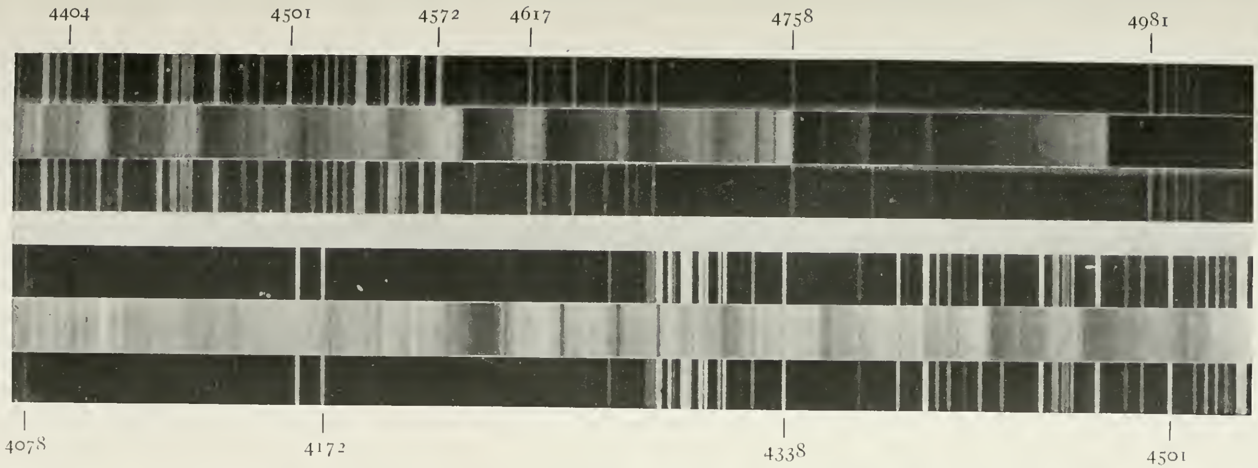
J. S. Plaskett, *Journal of the R.A.S. of Canada*, **1**, 45, 1907.

P. W. Merrill, *Publications of the Astronomical Observatory of the University of Michigan*, **2**, 45 ff., 1916; *Astrophysical Journal*, **41**, 247, 1915.

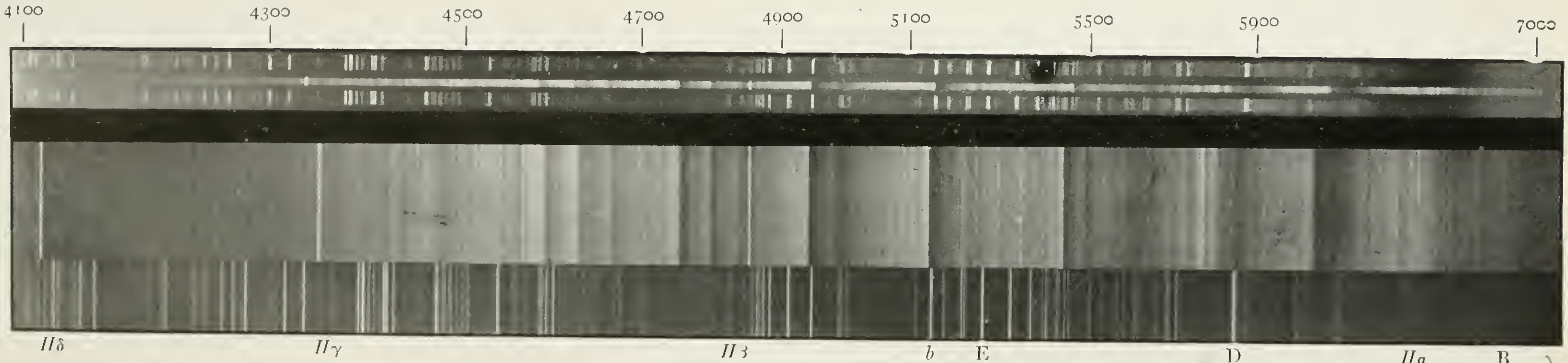
The spectrum of Mira Ceti is shown in Plate V, which is reproduced from this *Journal* (**25**, 236, 1907, *q.v.*) where it illustrated a short paper on Mira Ceti by Dr. V. M. Slipher, who has cordially assented to its reproduction here. The spectrum of R Leonis is reproduced, also on Plate V, from a vertical enlargement by Professor S. B. Barrett of our one-prism spectrogram No. IB 1004,

[1] According to the modified Harvard classification adopted at the Rome meeting (*Transactions of the International Astronomical Union*, **1**, 103, 1922; reprinted in this *Journal*, **52**, 65, 1923), the spectrum of Mira should be designated as M6e at maximum, and as Mvep! at minimum.

PLATE V



Spectrum of R Leonis taken by Parkhurst and Sullivan. Comparison Spectrum of *Ti* and *Fe*



Spectrum of Mira Ceti taken by V. M. Slipher at Lowell Observatory

The upper spectrum is a five-fold direct enlargement of the original negative; the lower one is a vertical enlargement of the same on a slightly smaller horizontal scale. The comparison lines are of *V*, *Fe*, and *Na*.

taken by Parkhurst and Sullivan about a week after maximum; for a comparison, the spark spectrum of titanium and iron was used.

The velocities of these stars, determined from their dark spectral lines, are approximately constant, that of Mira being about +67 km per second, and that of R Leonis +18 km per second. The velocities obtained by different observers are collected in Table III.

Among the points dealt with in the above investigations, aside from determination of velocity and structure of bright lines, are the following.

The wave-lengths of the heads of the bands in the spectrum of Omicron Ceti and of some other stars were determined by Stebbins and by Plaskett. (At that time sufficient evidence for the identification of these bands was not available.)

The difference in the displacement of the bright lines and of the dark lines, the former being approximately 0.25 A less toward the red than the latter, was noted by all of the observers.

Stebbins made a study of the changes in the intensity of the bright lines and of the continuous spectrum as the stars grew fainter, which indicated that the intensity of the bright hydrogen lines decreased with the decrease in magnitude of the star, while that of a few other bright lines increased; the later plates furnished evidence which supported the view that the bright hydrogen lines disappeared when the star was at minimum.

In measuring the spectrograms secured here our plan was: To determine the radial velocity of the stars from the displacement (*a*) of absorption lines, (*b*) of bright lines, and (*c*) of absorption bands; to find, if possible, the explanation of the difference in the velocity determined from these three kinds of spectra; and to identify the sources of numerous bright lines and of a few unknown absorption lines.

The spectra were photographed by different observers with the Bruce spectrograph attached to the 40-inch refractor; for the majority of the plates, the dispersion of one prism was used; for a few, three prisms were employed.

The region well covered by the three-prism plates extends from λ 4338 to λ 4885; by the one-prism plates from λ 3860 to λ 5036.

The magnitudes given in the last columns of Tables I and II are from the observations published in the *Memoirs* of the British Astronomical Association, 17–18, 1910–1912. For 1915 to 1919 the magnitudes are taken from the monthly reports of the American Association of Variable Star Observers, published in *Popular Astronomy*.

Table II gives the results of measurement of thirty-one plates of Omicron Ceti, eight of R Leonis, two of T Cephei, and two of R Serpentis, which include practically all the plates of these stars suitable for accurate measurement which were available in 1917 (later continued through 1920). The columns headed "Number of Lines" or "Number of Bands" show the number of lines or bands used in the determination of the velocity. Of course all velocities have been reduced to the sun.

The plates were measured with the Gaertner comparator and reduced by the method ordinarily practiced here of calculating the constants of a Hartmann formula for each spectrogram.

It is evident from the fourth and sixth columns of Table II that no material difference in the velocity of Mira Ceti has been introduced by the use of different spectral lines in the greater extension to the violet of the plates of low dispersion. The mean velocity of Mira obtained from dark lines, when equal weight is given to all the 18 plates, is $+67.1\pm0.53$ km; from bright hydrogen lines on 30 plates, it is $+49.2\pm0.59$ km. The number of plates is too small to warrant us in attaching significance to the difference in the velocity from dark lines, $+64.2$ km, from the three-prism plates taken in December 1906, and the velocity, $+70.0$ km, from the one-prism plates taken in January 1907. However, there may be some physical significance in the difference in the velocity of $+45.4\pm0.73$ km, obtained from bright lines soon after the maximum at magnitude $2^{M}_{.}3$, in December 1906, and the velocity of $+53.1\pm0.27$ km obtained from bright lines soon after the fainter maximum at $3^{M}_{.}6$ in February 1915; and again in the difference of the former velocity and the velocity of $+51.5\pm0.89$ km after the fainter maximum of $3^{M}_{.}6$ in September 1917. This point will be discussed later.

TABLE I

Journal of Observations

MIRA CETI

No. of Plate	Date	G.M.T.	Duration	Hour Angle	Quality	Observers	Magnitude
IIIB 677*........	1906, Dec. 22	13^h02^m	150^m	E 1^h10^m	f.	F, P, S	2.3
679........	23	13 27	206	E 0 37	g.	F, P, S	
686........	24	12 54	40	E 1 05	v.g.	F, P, B	
687........	25	11 43	18	E 2 13	v.ft., L.g.		
688........	25	12 20	12	E 1 36	f.	F, I	
IB 942†........	1907, Jan. 5	14 55	5	W 1 42	f.	F, Fox, S	2.5
943........	5	15 09	18	W 1 57	g.	F, Fox, S	2.5
944........	5	15 29	0^m40^s	W 2 16	L.g.	F, Fox, S	2.5
952........	25	13 54	30	W 2 02	g.	F, S	3.0
953........	25	14 17	6	W 2 24	L.g.	F, S	3.0
954........	25	14 24	2^m30^s	W 2 31	L.g.	F, S	3.0
972........	Feb. 8	13 03	42	W 2 05	L.f.	F	4.0
3993........	1915, Jan. 29	12 50	75	W 1 12	v.g.	My, S	3.8
3994........	29	13 41	10	W 2 03	v.g.	My, S	3.8
3998........	Feb. 8	13 10	64	W 2 13	g.	F, S	3.5
3999........	8	13 52	10	W 2 55	f.	F, S	3.5
4009........	16	12 55	62	W 2 29	v.g.	F	3.8
4357........	1916, Jan. 17	12 05	35	E 0 19	v.g.	B, H	3.3
4381........	23	12 06	30	W 0 06	L.g.	H, F	3.7
5095........	1917, Nov. 16	15 11	52	E 1 17	g.	Wk, S, B	4.0
5103........	23	15 08	75	E 0 53	v.g.	Wk, S	4.0
5115........	Dec. 10	14 18	100	E 0 34	g.	B, Wk	4.6
5145........	1918, Jan. 4	11 50	48	E 1 22	L.g.	Wk, B	5.2
5378........	Sept. 16	21 07	60	W 0 37	v.g.	Wk, S	3.7
5389........	30	20 15	61	W 0 42	g.	Wk, S	3.9
5575........	1919, Aug. 18	20 22	45	E 2 04	g.	Wk, S	3.3
5581........	29	21 36	45	E 0 10	L.g.	B, S	3.4
5591........	Sept. 12	18 34	50	E 2 12	f.	B, S	
5595........	15	19 19	62	W 0 45	L.g.	B, Pv, S	3.9
5894........	1920, Aug. 2	21 21	100	E 1 57	L g.	Bk, J, S	
5955........	25	21 17	90	E 0 36	f.	B, S	4.8

Maxima of Mira.... 1906, Dec. 18, $2^{M}3$; 1915, Feb. 8, 3.5; 1915, Dec. 22, 3.0; 1916, Nov. 8, $3^{M}6$; 1917, Oct. 15, 3.6; 1918, Sept. 16, 3.6; 1919, Aug. 8, $3^{M}0$; 1920, July 5 (Too near sun)

R LEONIS

No. of Plate	Date	G.M.T.	Duration	Hour Angle	Quality	Observers	Magnitude
IB 738........	1906, Apr. 21	14^h30^m	65^m	W 0^h48^m	v.ft., L.g.	P, S	6.1
739........	21	16 20	35	W 2 35	L.g.	P, S	6.1
741........	23	15 21	64	W 1 50	L.v.g.	F, S	6.0
747........	27	15 50	105	W 2 32	L.g.	B, S	6.3
752........	28	15 45	223	W 2 32	f.	P, S	6.3
782........	June 2	15 00	78	W 4 09	p.	P, S	6.9
1004........	1907, Feb. 24	19 51	210	W 2 00	v.g.	P, S	5.7
1011........	Mar. 31	18 08	210	W 3 05	g.	F, P	6.5

Maximum of R Leonis: 1906, April 1, $5^{M}6$.

T CEPHEI

No. of Plate	Date	G.M.T.	Duration	Hour Angle	Quality	Observers	Magnitude
IB 783........	1906, June 2	17^h00^m	120^m	E 5^h20^m	ft.	P, S	6.0
785........	9	16 28	225	E 5 23	f.	P, S	6.3

Maximum about May 24, 1906, at $6^{M}3$

R SERPENTIS

No. of Plate	Date	G.M.T.	Duration	Hour Angle	Quality	Observers	Magnitude
IB 788........	1906, June 24	18^h11^m	221^m	W 2^h42^m	f.	P, S	7.2
790........	30	16 04	172	W 0 59	f.	P, S	7.2

Maximum about July 14, 1906, at $6^{M}9$

Symbols: f.=fair; ft.=faint; g.=good; L.=bright lines of hydrogen; p.=poor; v.=very.

Observers: B=Barrett; Bk=Miss Block; F=Frost; H=Hubble; I=Ichinohe; J=M. F. Jordan; My=Maney; P=Parkhurst; Pv=Paraskévopoulos; S=Sullivan; Wk=Miss Wickham.

* Plates IIIB 677 to 688 were obtained with three prisms.

† Plates IB 942 to 5955 were obtained with one prism.

TABLE II

Radial Velocities for Mira Ceti

Plate	Date	Dark Lines				Bright Lines of Hydrogen		Bands of Titanium Oxide		Meas. By	Magnitude	Maxima
		All Suitable		Lines Common*								
		No. of Lines	Km	No. of Lines	Km	No. of Lines	Km	No. of Bands	Km			
IIIB	1906											1906
677.......	Dec. 22	24	+63.7	3	+71	1	+54	......	.....	F	2.3	Dec. 18
679.......	23	22	63.2	4	64	.....	....	......	.....	F	.....	2.3
686.......	24	24	65.1	4	67	2	44	4	+49	L	.....	....
687.......	25	7	64.9	.....	.....	2	44	......	.....	F	.....	....
688.......	25	.....	....	.....	.....	2	39	......	.....	F	.....	....
Mean (3 prisms)...		.....	+64.2	.....	+67	.....	+45	......	+49	......	.....	....
IB	1907											
942.......	Jan. 5	6	+67	3	+68	3	+47	4	+54	L	2.5	....
943.......	5	6	74	4	74	3	46	4	44	L	.....	....
944.......	5	.....	....	.....	.....	3	45	......	.....	L	.....	....
952.......	25	9	69	.....	.....	3	47	4	47	L	3.0	....
953.......	25	.....	....	.....	.....	3	43	......	.....	F	.....	....
954.......	25	.....	....	.....	.....	3	44}	......	.....	F	.....	....
954.......	25	.....	....	.....	.....	3	46}	......	.....	L	.....	....
972.......	Feb. 8	.....	....	.....	.....	3	45	......	.....	F	4.0	....
Mean (1 prism)....	....	.....	+70	.....	+71	.....	+45	......	+48	......	.....	....
IB	1915											1915
3993.......	Jan. 29	18	+66.9	4	+65	2	+53	3	+41	F	3.8	Feb. 8
3994.......	29	.....	....	.....	.....	3	50}	......	.....	F	.....	3.5
3994.......	29	.....	....	.....	.....	2	56}	......	.....	L	.....	....
3998.......	Feb. 8	18	68.4	4	69	2	53	4	53	L	3.5	....
3999.......	8	.....	....	.....	.....	2	54}	......	.....	F	.....	....
3999.......	8	.....	....	.....	.....	2	55}	......	.....	L	.....	....
4009.......	16	16	74.7	3	77	2	52	4	56	L	3.8	....
Mean......	....	.....	+70	.....	+70	.....	+53	......	+50	......	.....	....
IB	1916											1915 Dec. 22
4357.......	Jan. 17	8	+66	4	+67	3	+46	4	+44	L	3.3	3.0
4381.......	23	.....	....	.....	.....	3	47	......	.....	H	3.7	....
IB	1917											Sept. 27
5095.......	Nov. 16	.....	....	.....	.....	2	52	......	.....	L	.....	3.6
5103.......	23	14	+64}	.....	.....	4	54}	......	.....	Wk	.....	....
		17	69}	.....	.....	3	53}	8	62	L	4.0	....
5115.......	Dec. 10	16	64}	.....	.....	4	48}	......	.....	Wk	.....	....
		29	67}	.....	.....	3	50}	6	53	L	4.6	....
	1918											
5145.......	Jan. 4	.....	....	.....	....	2	56	......	.....	Wk	.....	....
Mean......	....	.....	+66	.....	.....	.....	+53	......	+58	......	.....	....
IB	1918											Sept. 16
5378.......	Sept. 16	16	+69	.....	.....	2	52	......	.....	L	3.7	3.6
5389 (2)....	30	25	66	.....	.....	2	52	6	+64	L	3.9	....
Mean......	....	.....	+68	.....	.....	.....	+52	......	+64	......	.....	....

* This includes only lines common to both three-prism and one-prism plates.

TABLE II—*Continued*

Plate	Date	Dark Lines: All Suitable, No. of Lines	Dark Lines: All Suitable, Km	Dark Lines: Lines Common*, No. of Lines	Dark Lines: Lines Common*, Km	Bright Lines of Hydrogen, No. of Lines	Bright Lines of Hydrogen, Km	Bands of Titanium Oxide, No. of Bands	Bands of Titanium Oxide, Km	Meas. by	Magnitude	Maximum
IB	1919											Aug. 8
5575.......	Aug. 18	20	+69	.....	.....	3	+45	......	.....	L	3.3	3.0
5581.......	29	.....	....	.....	.....	3	44	......	.....	L	3.4	....
5591.......	Sept. 12	8	71	.....	.....	3	50	5	46	L	.....	....
5595.......	15	.....	....	.....	.....	2	49	......	.....	L	3.9	....
Mean......	....	.....	+70	.....	.....	.....	+47	......	.....	......	.....	....
IB	1920											
5894.......	Aug. 2	.....	....	.....	.....	2	59	......	.....	L	.....	....
5955.......	25	13	+61	.....	.....	.....	+56	......	58	L	4.8	
						R LEONIS						
IB	1906											
738.......	Apr. 21	.....	....	.....	.....	2	− 6.3	......	.....	F	.....	....
739.......	21	.....	....	.....	.....	2	4.5	......	.....	F	.....	....
739.......	21	.....	....	.....	.....	2	2.4	......	.....	L	.....	....
741.......	23	.....	....	.....	.....	2	7.0	......	.....	F	.....	....
741.......	23	.....	....	.....	.....	2	7.6	......	.....	L	.....	....
747.......	27	.....	....	.....	.....	2	6.7	......	.....	L	.....	....
752.......	28	11	+19	.....	.....	2	6.2	......	.....	L	.....	....
782.......	June 2	.....	....	.....	.....	2	10.8	......	.....	F	.....	....
	1907											
1004.......	Feb. 24	11	16	.....	.....	2	7.5	......	.....	L	.....	....
1011.......	Mar. 31	.....	....	.....	.....	2	1.5	......	.....	F	.....	....
Mean......	....	.....	+18	.....	.....	.....	− 6.0	......	.....	......	.....	....
						T CEPHEI						
IB	1906											
783.......	June 2	5	−15	.....	.....	2	−33	3	−83	L	.....	....
785.......	9	7	−13	.....	.....	2	−28	3	−69	L	.....	....
Mean......	....	.....	−14	.....	.....	.....	−30	......	−76	......	.....	....
						R SERPENTIS						
IB	1906											
788.......	June 24	6	+32	.....	.....	2	+15	3	−69	L	.....	....
790.......	30	5	+28	.....	.....	2	+ 6	3	−45	L	.....	....
Mean......	....	.....	+30	.....	.....	.....	+10	......	−57	......	.....	....

The velocity has been computed from the displacements of the bands of titanium oxide, with the use of the wave-lengths given by A. Fowler[1] when he proved this compound to be the chemical origin of the bands characteristic of the spectra of stars

[1] *Proc. Roy. Soc.*, A, 79, 509, 1907.

of class Md. We see from Table II that this velocity differs little from that obtained from bright hydrogen lines, being $+51.6 \pm 1.3$ km. However, some uncertainty must be attached to the velocity determined from the heads of bands on account of the difficulty of setting on the head or the corresponding parts of a head on the stellar absorption plate and the laboratory emission plate; for more reliable values we need many plates of high dispersion and with exposure of the stellar plates most suitable for absorption bands.

A further attempt to find the velocity from the bands of titanium oxide was made by Miss Lowater while working recently under

TABLE III

COMPARISON OF VELOCITIES OBTAINED BY DIFFERENT OBSERVERS

Star	Velocity from		Difference Bright−Dark	Observers	Epoch
	Dark Lines	Bright Lines			
	km	km	km		
Mira Ceti......	+62 (7)	+44 (7)	−18	Campbell and Wright	1897–98
	+66 (7)	....	....	Stebbins	1902
	+63 (1)	+48 (1)	−15	Campbell and Stebbins	1914
	+67 (3)	+52 (4)	−15	Merrill	1914
	+65 (2)	+46 (14)	−19	Plaskett	1906–7
	+67 (18)	+49 (30)	−18	F. and L.	1906–7, 1915, 1916, 1917, 1918, 1919, 1920
R Leonis.......	+25 (2)	0 (7)	−25	Merrill	1914
	+18 (2)	− 6 (8)	−24	F. and L.	1906, 1907
R Serpentis.....	+32 (1)	+ 8 (4)	−24	Merrill	1914
	+30 (2)	+10 (2)	−20	F. and L.	1906
T Cephei.......	....	−30 (2)	....	Merrill	1914
	−14 (2)	−30 (2)	−16	F. and L.	1906
		Mean	−19.4		

Mean of differences for all stars by all observers = −19.4 km
Mean of differences for all stars by F. and L. = −19.5 km
The numbers of plates used in forming the mean are given in parentheses after the velocities.

Professor Fowler in the Astrophysical Laboratory of the Imperia College of Science, London. The emission spectrum of titanium oxide was photographed with a Littrow spectrograph, giving resolution of the same order as that of the stellar plates. A negative enlargement of the three-prism laboratory plate to represent absorption was made on the same scale as an enlarged positive of

the corresponding stellar spectrogram. Comparison of these enlargements showed more numerous coincidences of band lines in the stellar and laboratory plates than was previously suspected. For determination of wave-lengths of band lines the stellar spectrogram was not very suitable, but a study of these enlargements strongly suggests that the displacement, due to velocity of the lines of the band cannot be very different from that of the metallic lines common to the two spectra.

The results of the present investigation confirm the conclusions of other observers:

1) that the velocity of these stars is practically constant;

2) that the velocity determined from bright hydrogen lines has a much smaller positive value than that from dark lines, of the order of 20 km per second.

The recession of Mira Ceti being 67 km and that of the radiating hydrogen 49 km, the star may be considered to be at rest and the luminous gases to be leaving the star with a velocity of 18 km relatively to its surface. A possible explanation of the difference in the radial velocity, according as it is determined from the displacement of dark lines or bright hydrogen lines, would thus be found in the eruption or shooting outward of hydrogen, as is well known to occur in the eruptive solar prominences. The evidence in support of this explanation is as follows:

1. Velocities far in excess of 20 km per second have been observed in the ascent of eruptive solar prominences. If hydrogen is erupted from the stars in the direction of the line of sight, the velocity determined from its lines would have a smaller positive or a larger negative value than that obtained from dark lines. Hydrogen erupted in directions inclined to the line of sight at angles ranging from zero to 90° would have a component of its velocity varying from the maximum to zero kilometer per second. This variation in velocity would cause a broadening of the bright hydrogen lines, which is in evidence on the plates, although probably lost in the intrinsic breadth of these bright lines. The widths of the bright hydrogen lines in Mira are shown in Table IV. Thus this explanation finds support in the order of magnitude of the difference in velocity obtained from dark and bright lines.

TABLE IV

WIDTHS OF BRIGHT HYDROGEN LINES

Plate	Width in km per sec.		
	λ 3889.201	λ 4101.890	λ 4340.634
IB 5095	....	94	91
5145	....	51	69
5103	57	114	105
5115	62	104	99

2. It receives support in regard to periodicity from the evidence obtained by Stebbins, that the bright hydrogen lines disappear at a minimum, combined with the fact that eruptive prominences of the sun are most numerous at the times of sun-spot maxima; since these are periodic, the frequency of eruptive prominences must be of a periodic character.

3. Not only is there need of an explanation of the difference in velocity from dark and bright lines, but also for the variation in the magnitude of the stars, and for the variation in the values of their maxima. The variation in the brightness of Mira has not been accounted for by the presence of any companion. Stebbins has expressed the opinion that the star's variation is due to the action of internal forces. Since it is probable that the eruption of hydrogen is periodic and there is evidence of the hydrogen lines disappearing at a minimum, it is possible that the variation in magnitude itself is due to the eruption of luminous hydrogen, the magnitude increasing with the larger quantities or the greater heights of ascent of the gas. The variation in the maxima may depend on the speed with which this incandescent gas is projected. Presumably the greater its speed, the greater will be the distance outward that it will travel through the heavier envelopes of the star, and then the light which it emits will be subject to less general absorption than when its speed is less. This idea is suggested by the occurrence of a smaller difference in the velocity from dark and bright lines in January and February, 1915, when it was 17 km, and in November and December, 1917, when it was 14 km, than in January, 1907, when it was 25 km; at these times the maxima were 3.8, 3.6, and 2.3 respectively, differing by 1.5 and 1.3 magnitudes.

4. The spectrum of the solar prominences contains bright lines of hydrogen, helium, and some metals; the spectra of Mira Ceti and R Leonis also contain lines of those elements.

It is not at all probable that the difference in displacement of the dark lines and the bright hydrogen lines is due to pressure, for the pressure in the sun's atmosphere has recently been shown to be about one thousandth of an atmosphere, and that of the stars' is probably of the same order of magnitude.

BRIGHT LINES

Hydrogen lines.—In the spectrum of *o* Ceti, $H\beta$ and $H\gamma$ appeared on all but one of our thirty-one plates; on that one, IB 4009, $H\beta$ may have been masked by continuous spectrum. $H\delta$ was present on all the one-prism plates, but was not within the range of the three-prism plates. $H\zeta$ was on all the one-prism plates except those which were given short exposures and showed bright lines only. $H\epsilon$ appeared unmistakably on one plate only, namely, IB 952, taken on January 25, 1907, or about forty days after the star's maximum.

In the spectrum of R Leonis $H\beta$ and $H\epsilon$ were entirely invisible; $H\gamma$ and $H\delta$ were present on all the plates, but $H\zeta$ was visible on only four of the eight plates measured.

Other bright lines.—The wave-lengths of the other bright lines are contained in Table V; they have been corrected for the displacement due to the velocity determined from the bright hydrogen lines. In seeking to identify their sources we have compared their wave-lengths with Rowland's "Preliminary Table of Solar Spectrum Wave-lengths," with corrections and additions to date; with Exner and Haschek's *Codex*, and with the table of iron lines in Kayser's *Handbuch der Spectroscopie*, 6; also with the tables in Kayser's *Handbuch*, 5 and 6, containing the wave-lengths of lines of the elements enumerated in Table VI; with data by Eder and Valenta; with wave-lengths of flame lines by Sir Norman Lockyer and by others; also with those of electric furnace spectra by King, and of enhanced lines by Fowler and by Baxandall.

The evidence from a comparison of stellar and laboratory lines, as far as its goes, might suggest that most of the bright lines arise

from the rarer elements, namely, Tm, Ce, Wo, Y, Ta, Tr, Kr, Pr, Ru; Ti and V are also suggested, as are the more common elements, Fe, Ca, Mn, Ni, Cr, and H giving its secondary spectrum.

TABLE V

BRIGHT LINES IN THE SPECTRA OF *o* CETI AND R LEONIS, EXCEPT HYDROGEN LINES

Weighted Mean Wave-Length	Weighted Mean Wave-Length
3905.78	4511.73
4103.15	4521.54
4138.80	4559.88
4166.03	4562.15
4202.01	4584.08
4206.68	4633.72
4233.42	4634.94
4375.98	4639.26
4454.41	4756.72
4457.06	4801.33
4458.79	4803.18
4461.44	4838.68

TABLE VI

He Ne Ar Kr Xe	Mg Ca Sr Ba	Sc Y La	C Si Ti Zr Ce	N V Nb Ta	O Cr Wo	Mn	Ru

Also the rarer elements, of which the place in the Mendeléjef table is not yet fixed, namely, Pr, Nd, Sa; Eu, Gd, Tb; Dy, Nh, Er, Tm, Ny, Lu.

The correction applied to the wave-lengths of the unknown bright lines may not have reduced their measured wave-lengths correctly, for the difference in Mira's velocity determined from the bright hydrogen lines and from the absorption lines is presumably due to motion of the gas, and the sources of the other bright lines may move with a velocity less than that of hydrogen. If that be so, the wave-lengths of lines from different sources would have errors of different magnitudes, but for the lines from any one source we should have approximately equal errors, equal differences between the stellar and laboratory values of the wave-lengths. This test has been carefully applied to the lines of many metals and gases, but no consistent differences found.

Further, in cases where the wave-lengths of a few stellar lines approximate fairly consistently to the laboratory values of metallic lines, they represent only the weaker lines of the metal, the prominent lines of corresponding temperature not being found. For example, if we have the low temperature iron lines λλ 4202.2, 4216.4, 4376.1, and 4461.8, we should also find the intense lines λλ 4383.7, 4404.9, 4415.3, also of low temperature. The same criticism applies to two lines approximating those of nitrogen, λ 4511.73 and λ 4634.94. Again, our line λ 4166.03 is temptingly near the barium line λ 4166.161, and if it represents barium we should also find the line λ 4130.68, a member of the same triplet, but of twice the intensity of the former line; of that, however, we have no record.

In regard to temperature as a guide in identification, the recent work by A. S. King,[1] "On the Production of the Titanium Oxide Bands in the Electric Furnace," indicates a state of comparatively low temperature in the star, rather than high. Work by Nicholson and Pettit[2] leads to the same conclusion. This evidence is confirmed by work done by Miss Lowater on the bands of titanium oxide produced in the arc, showing that it is necessary to keep the current small, from two to three amperes, in order to maintain the bands comparatively free from titanium lines. This evidence of low temperature is in direct opposition to the existence of enhanced lines, as for instance, the iron lines λ 4233.3 and λ 4584.0 in the spectra of stars of type M. These enhanced lines characterize B-type stars in the spectra of which, as is well known, no bands of titanium oxide are found. The evidence is that the bands of titanium oxide cannot co-exist with enhanced lines. If the bright lines are high temperature lines, it would seem that they must arise from a body separated from the source of the bands.

We are strongly inclined to the opinion that identification of these bright lines will not be found among terrestrial metals, rather more likely among gases. It may, however, be an unknown spectrum.

The identification will surely remain uncertain until we can obtain more accurate determinations of the wave-lengths of the

[1] *Publications of Astronomical Society of the Pacific*, **34**, 348, 1922.

[2] *Ibid.*, **34**, 132, 1922.

bright stellar lines. A comparison of the tables of Stebbins,[1] of Adams and Joy,[2] and of ourselves, makes this point very evident: within the region common to the three tables Stebbins has four lines, and Adams and Joy eight, which we have not; we have nineteen lines not in the list of Adams and Joy, and another not in Stebbins' list; further, among those lines which we have in common, the differences in wave-length of lines apparently the same are quite inconsistent and cannot be accounted for by the application of different corrections or by the use of different standards. Can these differences be accounted for by changes in the star, which are accompanied by the change in magnitude?

It may be of interest to call attention to the appearance, on plates of long exposure of Mira and of R Leonis, of a bright line of comparatively small intensity on the less refrangible side of $H\delta$; its appearance and proximity to $H\delta$ suggested to us long ago the name "Fratello." Its mean wave-length is $\lambda\,4103.15\pm0.015$, when corrected for the star's velocity determined from the bright hydrogen lines. Comparison of the two silicon lines given by Rowland at $\lambda\,3905.660$ and $\lambda\,4103.097$ with our lines $\lambda\,3905.78$ and $\lambda\,4103.15$, shows a discrepancy of 0.07 A in the difference in wave-length of the two solar lines and of the two stellar lines. It is just such a discrepancy which makes their identification with silicon uncertain.[3]

Absorption lines.—The elements represented by the dark lines from which the velocity was determined are: Mg, Ca, Sr, Ba; Ti, Zr; V; Cr; Mn; Fe and Ni.

SUMMARY

The results of the present measurements may be summarized as follows, the first and second items apparently confirming the results of other observers:

1. The radial velocities of the stars *o* Ceti and R Leonis are constant.

[1] *Lick Observatory Bulletin*, 2, 78, 1903.

[2] *Publications of Astronomical Society of the Pacific*, 30, 193, 1918.

[3] But see the recent comments on the identification of these lines by F. E. Baxandall in the *Observatory*, 46, 226, July, 1923, which seem to establish the certainty of their identification with silicon.

2. The difference in the displacement of the dark lines and of the bright hydrogen lines is equivalent to a negative velocity of the order of 20 kilometers per second.

3. Possibly the variation in the magnitude of these stars and in their maxima may be accounted for by outbursts of hydrogen, occurring with considerable regularity as to time; possibly also these may account for the difference in the velocities determined from bright and dark lines.

This paper was prepared for publication in essentially its present form in 1917, but was withheld in the hope of obtaining some satisfactory explanation of the phenomena involved; and was further delayed by the prospect of Miss Lowater's work in London, hence it might without impropriety be dated 1917. It thus does not discuss the interesting observations, chiefly made at Mount Wilson since that date. Since the paper was in type, Joy and Humason have detected a faint companion to Mira of magnitude about 10, with a spectrum of type B or earlier and showing bright lines. This companion has been seen and measured by Aitken, by Van Biesbroeck, and by others. But this does not appear to account for the spectrum of Mira at maximum, although greatly assisting in explaining the anomalous spectrum found at Mount Wilson when it is near minimum.

We wish to express our thanks to Professor A. Fowler for his valuable suggestions pending the publication of this paper while Miss Lowater has been working on closely related subjects in his laboratory at the Imperial College of Science and Technology during 1922–1923.

Yerkes Observatory
November 1923

ARC AND SPARK SPECTRA OF ALUMINUM, ZINC, AND CARBON IN THE EXTREME ULTRA-VIOLET

By R. O. HUTCHINSON

ABSTRACT

Description of apparatus and methods employed.—A grating spectrograph, inclosed in a metal tube which could be evacuated, was used. Evacuation of the tube was effected with a mercury diffusion pump. The arc was struck by means of an electromagnet at definite intervals and exposure completed in about twenty minutes, the arc being operated at less than 110 volts. The spark was produced by an induction coil operated as a transformer on the 110-volt alternating potential. The exposure required about two or three hours.

Tables of lines measured.—The arc spectra of aluminum, zinc and carbon to λλ 696, 768, and 595, and the corresponding spark spectra to λλ 678, 467, and 460 were measured and the intensities of the lines compared. This represented an extension of the arc spectra of metals from λ 1376 to λ 595.

INTRODUCTION

The spark spectra of aluminum, zinc, carbon, and many other substances have been measured in this laboratory and extended far into the extreme ultra-violet by the work of Millikan, Sawyer, and Bowen[1] using the vacuum spectrograph. Various attempts have been made to extend the arc spectra of metals into the extreme ultra-violet. Wolff,[2] using a prism vacuum spectrograph and a vacuum arc separated from the spectrograph by a fluorite window, obtained the spectrum of zinc to λ 1376. McLennan, Ainslie, and Fuller,[3] using a fluorite prism spectrograph, measured the spectrum of aluminum to λ 1605, of carbon to λ 1464, and of zinc to λ 1445. Saunders[4] used a vacuum grating spectrograph but had his source of light and his photographic plate both outside, so that the light had to pass through two quartz plates. He photographed the arc and spark spectra of aluminum, zinc, and a few other substances

[1] *Astrophysical Journal*, **52**, 286, 1920; **53**, 150, 1920. *Proceedings of the National Academy of Sciences*, **7**, 289, 1921.

[2] Lyman, *The Spectroscopy of the Extreme Ultra-Violet*, p. 96. 1914.

[3] *Proceedings of the Royal Society*, **95**, 316, 1919.

[4] *Astrophysical Journal*, **43**, 234, 1916.

above λ 1670. McLennan and Lang,[1] using a vacuum grating spectrograph with the arc on the inside, were able to get the arc spectrum of carbon as far as λ 584, but were unable to go lower than λ 1427 with a metallic arc. They said they could get no lines below λ 2000 with an intermittent arc. They do not claim any great accuracy for their measurements.

The present investigation was started for the purpose of extending the arc spectra to shorter wave-lengths and in particular to compare the arc and spark spectra of aluminum, zinc, and carbon. Carbon was chosen because with it an arc is easily formed; aluminum, because it furnishes a good standard line of short wave-length; and zinc, because it is rich in lines in the spark spectrum in the extreme ultra-violet and was expected to be in the arc spectrum. It was also hoped that better photographs of the spark spectra would be secured by the substitution of curved plates for the straight ones which had been previously used in this laboratory.

Since this investigation was started, Simeon[2] has published the results of his research in which he photographed the arc spectrum of carbon to λ 651, using a vacuum grating spectrograph.

APPARATUS

The vacuum chamber of the spectrograph consisted of a brass tube (large end, steel) 120 cm long and with diameters of 14 cm and 21 cm at the two ends. The ends were closed with brass plates and rubber gaskets screwed to the flanges of the tube. A window, *W*, in the large plate served for observing the arc or spark.

The concave grating was ruled with 480 lines per mm and had a radius of curvature of 83.9 cm, giving a dispersion of about 24.7 A per mm. For work with such short wave-lengths it is found desirable not to cut away all the surface of the grating, and this grating was ruled with a very light touch. This gave a very bright first-order spectrum and no observable second order.

Schumann plates on celluloid, prepared by Hilger & Company, which could be bent to the desired curvature, were used. They were $1'' \times 5\frac{1}{2}''$ and covered the whole region from the principal image

[1] *Proceedings of the Royal Society*, **95**, 248, 1919.

[2] *Proceedings of the Royal Society*, (A) **102**, 484, 1923.

to λ 3000. The plates were developed in a special hydrochinone-metol developer recommended by the manufacturers. During the development the developer was kept on ice.

The grating was mounted in the small end of the tube (see Fig. 1) and was provided with all the necessary adjustments for moving it along the tube and for rotating it about three mutually perpendicular axes. Near the other end of the spectrograph was a diaphragm which served to hold the slit and plate holder, and also to prevent stray light from the arc or spark from reaching the grating. An

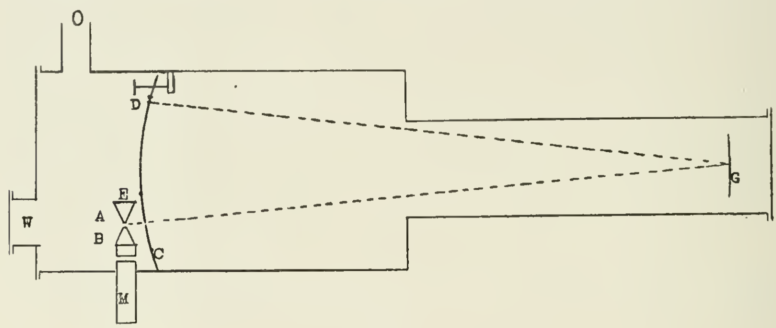


Fig. 1

auxiliary opening through the diaphragm avoided the necessity of evacuating through the slit. The plate holder *DE* formed the arc of a circle passing through the slit *S*, and both could be rotated together about a pivot at *C*, by means of a screw at *D*. The interior of the tube was painted black with a dull finish to prevent reflections.

The spectrograph was evacuated through the opening *O* which was connected to the pump by a one-inch glass tube in which were connected a liquid air trap and a mercury trap for cutting off connection between the spectrograph and pump. This trap made it possible to hold a high vacuum in the spectrograph after the pumps were stopped. A double-stage mercury diffusion pump with a Cenco Hyvac fore-pump was used for evacuation.

One of the chief difficulties encountered was in so placing the source of light that the cone of light would completely fill the grating—an essential condition for sharp lines. This was accomplished in the case of the arc by placing it very close to the slit, not more than

3 mm away. But it was necessary to keep the spark gap about three times as far away to prevent the spark from jumping to the slit and ruining it. The spark was made to pass between terminals in a vertical position which were connected to the source of high potential through large brass electrodes ground to fit bakalite insulating plugs. The same electrodes served for the arc which was used in a horizontal position.

Various attempts were made to get an arc to burn in the high vacuum maintained, but with little success until an electro-magnet and spring were arranged for striking the arc. One electrode was connected to terminal *A* which was fixed. The other was connected to *B* through a spring which held *B* against *A* when there was no current through the coil of the electromagnet *M*. An armature was attached to the terminal *B* and, when the circuit of the electromagnet was closed, *B* was drawn away from *A* and an instantaneous arc was formed.

MANIPULATION

The grating was adjusted so that the principal image fell near the end of the plate farthest from the slit. The grating and plateholder were then adjusted to give a sharp focus at each end of the plate. Exposures to test the focus could be made in air. When the proper focus was secured, the brass plate was placed over the small end of the spectrograph and did not need to be removed so long as the grating was left in this position. The other end had to be opened after each exposure.

The arc or spark terminals were placed in position so that the cone of light filled the grating as nearly as possible, and the photographic plate was put in place. The front end-plate was then fastened on and the pumps started. Liquid air was kept on the trap to prevent mercury vapor from getting back into the spectrograph and also to freeze out any vapors present. In about an hour the pressure was usually reduced to about 0.0001 mm. The mercury trap was then closed and the apparatus allowed to stand for several hours. This served to remove the occluded gases from the walls of the tube, and also to give the plate time to dry and adjust itself to the conditions. When the pumps were again started, it was

often possible to obtain a vacuum of the order of 0.00001 mm. But as soon as an exposure was started the pressure would again increase slightly. Pressures were read on a very sensitive McLeod gauge.

Arc exposure.—The terminals A and B were connected through a rheostat to the 110-volt alternating potential with A and B separated. They were then allowed to make contact and immediately separated again by the electromagnet, forming a bright arc of very short duration. With a direct current it frequently happened that sufficient gases were liberated to make the arc continuous for a few seconds, especially in the case of the carbon arc, but this caused fogging of the plates and the alternating current was generally used. Even with it quite a little gas was liberated and time had to be allowed between arcs for the pumps to remove the gases liberated. In the early stage of the exposure it was necessary to allow ten or fifteen seconds between arcs, but after a number of contacts had been made, less gas was liberated and the contacts could be made much more frequently. In general the frequency of the contacts was regulated so as to keep the pressure about 0.0001 mm, but frequently it increased to about 0.0005 mm or more. During the major part of the exposure, contacts were made at the rate of about one per second, but instead of being made at regular intervals, eight or ten were made in rapid succession because this heated the terminals up more and gave a more intense light. A mechanical interrupter was sometimes used, but never gave as good results as when the contacts were made by hand while the operator watched the color of the terminals.

The carbon arc was the least troublesome for several reasons. There was no difficulty in getting the arc to strike. It could be heated hotter without danger of melting. And the contact points did not stick. In the case of metallic terminals, it was necessary to be constantly on guard lest they fuse together until they could not be separated, or even melt off completely. Quite often it was necessary to open the spectrograph to remedy one of these causes, and then a new plate had to be inserted and the whole process of evacuation repeated. There was also difficulty in getting the arc to strike between metals, but if they were brightly polished just before being put in, and if the arc was not made to pass between

them until all the air had been removed so that oxidation of the surface was avoided, the arc could usually be struck without difficulty. The method of exposure had to be varied continually, and it was difficult to compare the times of two exposures. In general an exposure required from fifteen to thirty minutes, depending on the width of the slit, current, etc., but occasionally good plates were secured in two or three minutes. On account of the short duration of the arc the potential and current strength could not be determined accurately. The potential drop across the arc was probably considerably below 100 volts, and the current strength from three to five amperes. Exposures longer than thirty minutes did not help much, for with a fine slit the deposit from the arc usually completely or nearly closed it in that time. The slit had to be cleaned after each exposure.

Spark exposure.—The "hot spark in vacuo" described by Millikan and Sawyer,[1] and previously used in this laboratory for spark spectra in the extreme ultra-violet, was used in this investigation. The high potential was secured from an induction coil operated as a transformer, the primary of which was connected to the 110-volt alternating potential through a variable rheostat. An auxiliary spark gap in series with the spark gap in vacuo, and a large plate condenser in parallel with the spark gaps, served to produce an intense disruptive discharge. The potential was equivalent to that required for a spark of four or five centimeters in air. A motor-driven contact-maker closed the primary and produced a series of sparks about three times per minute. From two to five hours were required for an exposure. Longer exposures could not be made on account of the variation in the length of the plate during exposure, producing either double lines or broad, hazy ones. This method of exposure required much less attention than the arc, but required a much longer time and the lines were not so sharp.

CALCULATION OF WAVE-LENGTHS

When light of any wave-length from slit S strikes the grating G at an angle i, it forms an image of the slit at a point P such that the angle $NGP = i$. P is called the principal image. The wave-

[1] *Physical Review*, 12, 168, 1918.

length corresponding to any bright line, say at A, may be measured in terms of its distance d from the principal image.

Let us designate the angles NGA and AGP by θ and α, the wave-lengths corresponding to A and N by λ and λ_n, the grating space by σ, and the radius of curvature by R. GN is the normal. Now the concave grating formula for the first order is:

$$\lambda = \sigma(\sin i - \sin \theta) \tag{1}$$

or

$$\lambda = \lambda_n - \sigma \sin \theta \tag{2}$$

When θ is small, $\sin \theta$ is approximately equal to θ and $(\lambda - \lambda_n)$ is proportional to θ, but in the present measurements θ is too large to

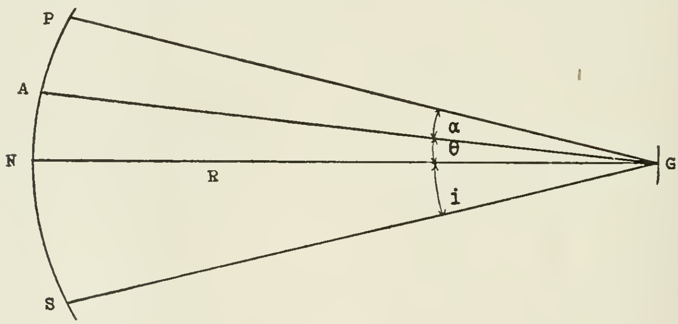


FIG. 2

make this assumption. Hence it was necessary to plot a correction curve. The correction to be applied to wave-lengths calculated on the assumption of the above proportionality is found as follows:

Expanding $\sin i$ and $\sin \theta$ and substituting in (1) we get

$$\frac{\lambda}{\sigma} = (i - \theta) \cdot \left(\frac{i^3 - \theta^3}{3!} - \frac{i^5 - \theta^5}{5!} \right). \tag{3}$$

Taking out $(i - \theta)$ as a factor and substituting $\theta = i - \alpha$, equation (3) reduces to

$$\lambda = \alpha\sigma \left(1 - \frac{3i^2 - 3i\alpha + \alpha^2}{6} + \ldots \right). \tag{4}$$

The terms omitted are of the fourth and higher degrees in i and α, and are negligible. This may also be written in the form

$$\lambda = \alpha\sigma(1 - b) \tag{5}$$

where

$$b = \tfrac{1}{6}(3i^2 - 3ia + a^2). \tag{6}$$

Now the correction b is a function of a and therefore of λ. If b_1 is the correction in the case of some line λ_1 taken as a standard

$$\frac{\lambda}{\lambda_1} = \frac{a}{a_1}\,\frac{1-b}{1-b_1} = \frac{d}{d_1}\,\frac{1-b}{1-b_1}$$

or

$$\lambda = d \cdot \frac{\lambda_1}{d_1}\left(1 + \frac{b_1 - b}{1 - b_1}\right) = \lambda' + C \tag{7}$$

where

$$\lambda' = d \cdot \frac{\lambda_1}{d_1} \tag{8}$$

and

$$C = \lambda' \cdot \frac{b_1 - b}{1 - b_1}.$$

But since b_1 is negligible in comparison with unity, as will be seen presently, the last expression reduces to

$$C = \lambda_1(b_1 - b). \tag{9}$$

From (5) and (7)

$$a = \frac{\lambda}{\sigma(1 - b_1)}, \qquad \text{and} \qquad \lambda = \lambda'\left(\frac{1-b}{1-b_1}\right).$$

Hence,

$$a = \frac{\lambda'}{\sigma}(1 - b_1)^{-1}$$

Again we may neglect b_1 in comparison with unity and we get

$$a = \lambda'/\sigma, \qquad \text{and} \qquad i = \lambda'_n/\sigma. \tag{10}$$

Substituting these values in (6) we now get

$$b = \frac{1}{6\sigma^2}(3\lambda'^2_n - 3\lambda'_n\lambda' + \lambda'^2) \tag{11}$$

and

$$C = (b_1 - b) = \frac{\lambda'}{6\sigma^2}[(\lambda_1^2 - \lambda'^2) - 3\lambda'_n(\lambda_1 - \lambda')]. \tag{12}$$

This gives the correction C as a function of the grating constant, the standard wave-length, the wave-length at the normal, and the

unknown wave-length. λ_n was kept at approximately 2000 A most of the time and was known with sufficient accuracy for this correction. The correction is zero at the principal image and at the standard line, negative between them, and positive for all values of λ greater than λ_1. C was plotted against λ' for several standard lines and by interpolation the correction, corresponding to any λ measured in terms of any standard line in the neighborhood of

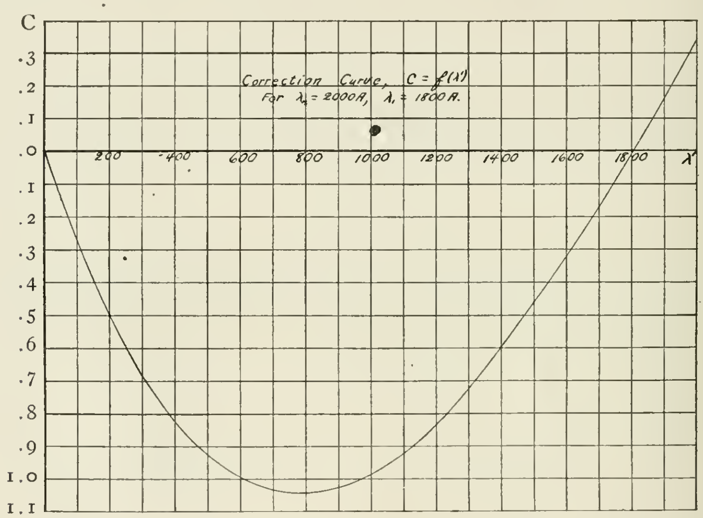


Fig. 3

2000 A, could be read to .01 A. The magnitude of the correction involved may be seen from the accompanying correction curve for $\lambda_1 = 1800$ and $\lambda_n = 2000$.

The plates were measured on a Gaertner 8-cm comparator. Since the distance d_1 involved in equation (8) was about 8 cm, it was very necessary that the screw have a constant pitch throughout the whole distance. This was found not to be true, and a calibration curve was plotted for it in terms of a standard decimeter, certified by the Bureau of Standards. The use of lines of longer wave-length than λ 2150 as standards was impossible because they were more than 8 cm from the principal image.

On account of the variation of the length of the plate with humidity and temperature, especially the former, the dispersion

constant (λ_1/d_1) could not be assumed the same for all plates, or even for the same plate on different days. At first considerable difficulty was encountered in the variation of the length of the plate during the measurement of it. For if a setting was made on the principal image, then on a line to be measured, and finally on the standard line, and if the plate contracted between the second and third readings, equation (8) would give too large a value for λ', but if the readings were taken in the reverse order, it would give too small a value. This was found to be the case if readings were taken immediately after the plate was adjusted on the comparator, but after it was left for about thirty minutes a condition of equilibrium was reached, and readings thereafter in both orders were quite consistent. To still further lessen the error due to contraction, which might occur during the measurement of all the lines on a plate, certain sharp lines which were found in both the arc and spark spectra were measured many times on each of several plates and their wave-lengths served as secondary standards for correcting the wave-lengths of other lines on each plate.

There were no carbon lines within 8 cm of the principal image which could be used as standards, but certain very prominent lines occurred in the spectra of both aluminum and carbon. These were measured in terms of the aluminum line at λ 1862.70, and were used as standards in measuring the other carbon lines. These measurements were then checked in two distinct ways: (1) using carbon as one electrode and aluminum as the other, plates were secured giving the standard aluminum line and most of the carbon lines, which could then be measured directly; (2) by resetting the plate during a series of measurements the carbon line at λ 2297.60 could be used as a standard. This second check was of least value because it meant handling the plate and a consequent change in length.

The standards for zinc were taken from Paschen's measurements[1] which were reduced from the Rowland System to the International by the correction curve given by Kayser,[2] and then to vacuum by the Bureau of Standards' reduction tables.[3]

[1] Kayser, *Handbuch der Spectroscopie*, 6, 860. [2] *Ibid.*, p. 890.

[3] *Bureau of Standards Bulletin*, 14, 731, 1919.

Wherever more than one standard line was available on a plate, all were used in determining the dispersion constant to avoid errors due to the peculiarities which might exist in any one line.

RESULTS

Tables I, II, and III give the wave-lengths in International units of the arc and spark spectra of aluminum, zinc, and carbon. The arc lines were sharp in every case and the spark lines of carbon and

TABLE I

ALUMINUM

λ	Intensity		λ	Intensity		λ	Intensity	
	Arc	Spark		Arc	Spark		Arc	Spark
678.0	......	5	1328.7	......	2	1622.6	......	3
696.2	2	......	1334.8 }	7†	3	1629.2	......	4
718.9	3	......	1336.2 }		3	1639.4	......	4
833.5 }	6†	2	1343.8	......	1	1644.9	......	4
834.9 }		2	1353.3	0	......	1651.9	......	2
881.2	......	2	1360.0	......	2	1670.9	10	4
904.5	5	4	1380.0	5	1	1673.2	......	4
977.4	3	4	1384.5	6	3	1688.7	......	2
1010.7	2	0	1437.4	1	......	1706.9	......	1
1037.2	4	2	1456.7	......	1	1719.5	6	0
1110.4	1	......	1464.3	......	0	1721.3	6	0
1113.5	1	......	1473.6	......	3	1725.0	8	1
1152.3	1	......	1486.2	......	1	1728.5	0	......
1176.0	2	4	1491.1	......	1	1749.0	2	......
1190.1	4	......	1499.8	......	3†	1749.7	......	2
1192.1	4	......	1506.0	......	2	1760.0	5	......
1206.9	6	......	1515.8	......	2	1761.8	4	......
1215.9	6	4	1540.1	3	......	1763.8	5	1
1253.6	......	0	1548.5	......	2‡	1764.4	4	......
1260.9	0	......	1552.6	......	3	1767.6	5	0
1262.8	......	0	1560.7	......	1	1777.2	1	......
1265.2	1	......	1561.3	1	......	1808.1	0	......
1266.1	......	0	1562.4	......	0	1818.8	0§	......
1292.4	......	2	1581.7	......	6	1854.7	10	8
1299.6	0	......	1598.6	......	2	1858.0	4	......
1302.4	1	......	1601.0	......	2	1862.70*	10	8
1305.5	0	......	1605.8	8	1	1930.99*	6	......
1307.4	......	1†	1611.9	8	3	1935.87*	6	2
1319.3	......	1	1619.7	......	3	1990.49*	8	1

* Used as standards.
† Broad.
‡ Probably double.
§ Possibly not a line.

aluminum were good, but the zinc spark lines were always hazy and in certain portions of the plate there was so much continuous background and the lines were so close together that measuring was very difficult. There was some disagreement in the wave-

lengths calculated from measurements on different plates, but when the averages for several plates and for at least four measurements of each plate were taken, the agreement between the wave-lengths

TABLE II

CARBON

λ	Intensity		λ	Intensity		λ	Intensity	
	Arc	Spark		Arc	Spark		Arc	Spark
460.1	......	1	1077.0	......	0	1374.8	0	3†
539.8	......	2	1085.9	......	1†	1432.3	1†	2
544.0	......	0	1093.3	......	1	1463.7	2	3
550.1	......	0	1139.8	0	3	1482.2	1	1
562.0	......	1	1142.3	1	5	1548.5	......	3
565.2	......	1	1146.4	0	......	1551.1	......	3
574.9	......	2	1176.0	8	8	1561.2	10	6
595.5	1	5	1190.1	1†	2†	1599.9	......	1
600.9	......	0	1194.1	2†	2†	1613.6	1	3
636.8	......	2	1206.9	......	1	1619.8	......	1
642.3	......	4	1215.9	3	5	1624.1	......	0
651.9	2	6	1230.7	......	0	1657.4	10	6
687.1	6	8	1247.9	......	5	1716.8	1†	4
699.3	......	0	1261.8	5	4†	1721.7	......	3
704.6	......	0	1265.2	......	0§	1752.1	1	1
711.6	......	0	1268.6	......	0§	1760.7	2	4
800.2	0	4	1272.9	......	0§	1808.1	10	7†
807.1	6	6	1277.9	5	4†	1891.1	......	0§
810.2	0	3	1280.9	4	3†	1892.7	......	0§
858.9	8	7	1284.2	......	2†	1930.8	8	6
904.5	10†	8	1303.1	......	3	1954.1	2	5
946.1	1	2	1321.8	......	0§	2020.2	4†	4†
977.4	6	8	1324.3	6	5	2072.5	5	4†
1010.7	8	7	1327.3	......	0§	2074.2	5	
1037.2	10	8	1329.7	6	4	2131.9	1	3
1064.0	......	0	1335.5	10†	8†	2297.60	2	5
1066.5	3	5	1362.8	1	2			

† Broad. § Possibly not a line.

for the arc and the spark were such that there was no justification for publishing the wave-lengths separately. But the relative intensities varied considerably. It is believed that the wave-lengths given are in most cases accurate to within one- or two-tenths of an angstrom, except in the case of hazy zinc lines.

The present investigation does not represent any extension of the spark spectrum over that of Millikan, Sawyer, and Bowen, but it is hoped that, by the use of curved plates, sharper lines may have been secured and more accurate measurements made. So far as the author is aware, no other investigator has secured the arc spectra for metals below λ 1376. The present investigation

TABLE III

ZINC

λ	Intensity		λ	Intensity		λ	Intensity	
	Arc	Spark		Arc	Spark		Arc	Spark
467.5.....	......	0	1204.5.....	1	2	1420.6.....	3	2‖
473.0.....	......	1	1212.9.....	......	1	1426.0.....	......	2‖†
479.1.....	......	1†	1216.2.....	......	1	1430.7.....	......	1
568.5.....	......	0§	1223.7.....	......	4	1432.5.....	......	2
573.2.....	......	0	1229.0.....	......	3	1434.8.....	......	0§
577.9.....	......	0	1234.4.....	......	0	1439.5.....	4	1
583.5.....	......	0	1240.2.....	......	0‖	1443.0.....	......	2
678.6.....	......	5‡	1243.7.....	......	0	1445.6.....	5	4
714.5.....	......	2‡	1250.5.....	......	1	1451.3.....	5	4
756.0.....	......	0	1253.9.....	......	4	1457.4.....	4	4
768.1.....	0	1	1263.1.....	......	4	1460.3.....	......	1
780.4.....	0	1‖	1266.1.....	......	3	1464.4.....	......	3
786.6.....	......	0§	1268.8.....	......	3	1466.3.....	......	1
825.4.....	......	1‖	1273.4.....	......	1	1473.7.....	......	5
830.4.....	......	0‖	1278.3.....	1	0	1477.5	4	1‖†
848.6.....	......	0§	1280.8.....	......	1§	1478.7	3	
853.0.....	......	1‖	1284.5.....	......	1	1486.6.....	6	3
881.9.....	2	4‡	1292.8.....	......	3‖†	1491.2.....	......	3
889.5.....	1	2	1295.7.....	......	3‖†	1493.6.....	1	......
893.6.....	0	0	1297.3.....	......	0	1499.8.....	......	5†
898.0.....	......	0	1301.6.....	......	1‖	1506.2.....	......	5
900.8.....	0	......	1304.4.....	......	1‖	1510.8.....	2	......
946.0.....	......	0§	1307.1.....	......	2	1516.1.....	0	5
950.1.....	......	0‖†	1319.3.....	......	2‖†	1524.4.....	4	......
954.5.....	......	0§	1319.6.....	......	1	1528.2.....	0§	......
958.6.....	......	1†	1322.2.....	......	2‖†	1534.4.....	2	......
968.0.....	......	0	1324.2.....	......	0§	1535.4.....	2	......
973.5.....	......	1‖	1327.3.....	......	0§	1541.8.....	0§	......
977.7.....	......	2‖	1328.7.....	......	2‖†	1547.6.....	0†	......
991.6.....	......	1‖	1336.0.....	0§	......	1552.7.....	0	5
1001.9.....	......	1‖	1340.0.....	......	1‖	1556.6.....	0	......
1008.0.....	......	2‖	1344.0.....	......	2‖	1561.3.....	0	4‡
1010.7.....	......	2‖	1349.3.....	......	1	1573.7.....	0	......
1018.4.....	......	2‖	1353.0.....	......	1	1581.9.....	4	6
1029.9.....	......	2‖	1360.2.....	......	2	1590.0.....	6	......
1037.4.....	......	2‖	1362.9.....	......	1	1595.9.....	0	......
1049.8.....	......	3	1364.6.....	......	1	1598.5.....	......	4
1133.6.....	......	1‖	1366.0.....	......	1	1601.1.....	......	4
1136.0.....	......	1‖	1374.8.....	......	2‖	1619.8.....	0	5
1145.5.....	......	2	1378.0.....	......	2‖	1622.8.....	0	5
1155.9.....	......	2‖	1387.8.....	......	3	1629.3.....	2	6
1166.2.....	......	2‖	1388.8.....	......	1	1639.6.....	1	6
1170.6.....	......	1‖	1395.7.....	4	3	1645.2.....	1	6
1176.0.....	......	3‖†	1402.0.....	......	2	1652.1.....	1	4
1179.8.....	......	1‖	1406.6.....	......	2	1671.9	4	7
1182.4.....	......	1‖	1409.1.....	......	1	1673.2	3	
1195.8.....	......	2	1411.6.....	......	1	1682.5.....	6	......
1200.7.....	......	2‖	1414.4.....	......	0	1688.7.....	......	4

* Used as standards.
‖ Hazy.
† Broad.
‡ Probably double.
§ Possibly not a line.

TABLE III—*Continued*

λ	Intensity		λ	Intensity		λ	Intensity	
	Arc	Spark		Arc	Spark		Arc	Spark
1695.3	......	2	1844.7	......	1‖	2058.4	4	......
1706.6	......	4	1856.7	......	1‖	2062.58*	9	10
1727.1	2	......	1864.6	......	1‖†	2064.9	5	3
1749.7	......	4	1871.8	......	1	2067.1	2	......
1753.7	......	2	1874.3	......	1	2070.4	1†	......
1762.2	......	2‖	1877.9	......	1	2079.8	4	1‖
1768.5	......	4	1881.8	......	1	2087.7	4†	1‖
1777.4	......	1	1907.3	......	1	2096.4	2	......
1797.0	3	......	1919.3	0	2	2097.7	2	......
1808.4	......	0‖	1929.2	1	......	2100.55*	8	8
1811.3	......	1‖	1942.2	......	1	2102.9	4	4
1814.2	......	0§	1946.0	......	1	2105.1	4	......
1816.5	......	0‖	1954.4	......	1§	2134.9	1	......
1825.3	7	1‖	1969.5	......	1	2139.29*	8	8
1825.4	......	1‖	1982.0	......	1§	2145.14*	4	4
1833.7	1	1	2026.16*	10	10			
1839.1	......	2	2054.2	0	......			

demonstrates the practicability of the vacuum arc spectrograph for metals to about λ 600, and it is believed that it can be extended even farther. Its advantages over the spark are: (1) That it gives sharper lines; and (2) that it requires less time for an exposure. The method can be extended to find the spectra of salts by using a vertical arc and cored carbons. The carbon arc was used first because it is the most easily operated. Then, too, it was desired to know the carbon spectrum so that carbon could be used as one terminal in securing the spectra of other substances By this arrangement the sticking of the terminals does not cause so much trouble. But for the purpose of identifying lines at least one exposure was made for each metal, using both terminals of the same metal.

On account of the variation of the length of the celluloid plates with moisture conditions, glass is much to be preferred if it can be bent to the proper curvature. However, when compared with straight glass plates, the advantage derived from a sharp focus more than offsets the disadvantage of the variable length.

In conclusion the author wishes to acknowledge his indebtedness to Professor H. G. Gale, Dr. A. J. Dempster, and Dr. H. B. Lemon for helpful suggestions.

Ryerson Physical Laboratory
University of Chicago
September 1, 1923

AN INTERCOMPARISON OF TEMPERATURE SCALES

By W. E. FORSYTHE

ABSTRACT

List of laboratories of which comparison was made.—This paper deals with a comparison of the high-temperature scales in use at the Bureau of Standards, the General Electric Company's Research Laboratories at Schenectady, the Physical Laboratory at the University of Wisconsin, the National Physical Laboratory of England, and the Nela Research Laboratory at Cleveland.

Method of comparison.—Several tungsten lamps that had been carefully aged and calibrated were sent to each of the foregoing laboratories and the temperatures of the lamps were measured for specified currents with a disappearing filament optical pyrometer using a plate of red glass as the monochromatic screen. The reductions necessary to make the readings comparable because a non-black body was used are pointed out.

Results of the comparison are given in Table II. The agreement found is very good, the maximum difference being only a few degrees for any point of the entire range from 1400° K to 2700° K.

In all radiation work general agreement upon a uniform standard of high temperatures is of the first importance and next in order is the standardization of some short and convenient method for accurate temperature measurements. The present paper deals with the intercomparison of the high-temperature standards now used in several research laboratories in this country and abroad by the method of brightness temperatures as measured by the disappearing filament optical pyrometer.

Day[1] and his co-workers of the Geophysical Laboratory some ten years ago did some very laborious, accurate, and important work in extending earlier temperature measurements with the constant volume nitrogen thermometer up to the melting-point of palladium. In this work the temperatures of several of the standard melting-points were determined on the gas thermometer scale. Buckingham[2] has calculated the corrections to be applied to temperatures measured on the constant volume nitrogen scale to reduce them to the absolute scale. These corrections are given directly to temperatures of about 1500° K and by extrapolation can be

[1] *American Journal of Science*, 26, 405, 1905.

[2] *Bulletin of Bureau of Standards*, 3, 237.

extended to the palladium point. According to Buckingham's extrapolated values the correction to be applied to Day and Sosman's value for the melting-point of palladium is about 0°6 C. The two points investigated by Day and Sosman most used for reference in the field of high-temperature radiation are the gold point and the palladium point. Practically all the work that has been done in the high-temperature region has been based upon one or the other of these values, i.e., gold point, 1336° K, or palladium point 1823° K.

Some work done with the disappearing filament pyrometer in the Nela Research Laboratory[1] and at the Reichsanstalt[2] showed a possible error in the melting-point of palladium when compared with the foregoing value for the melting-point of gold. This later data has been obtained by measurements on the relative brightness for a particular wave-length interval of the black body at the gold point and at the palladium point. From the ratio thus determined and Wien's equation c_2 taken as 14350 μ deg., a value of 1828° K is found for the palladium point in terms of 1336° K for the gold point. This is but 5° higher than the value set by Day and Sosman and is almost within their assigned limit of error.

Almost all of the temperature measurements above the melting-point of gold, or at most above 200° or 300° higher than this temperature, are determined by means of an optical pyrometer calibrated from a black body held at one or more standard temperatures.

The accuracy of the determinations in this high-temperature region depends upon two things: first, upon how well the black-body conditions have been reproduced, and second upon the accuracy with which the standard melting-points have been determined. When gold or palladium is used as the standard metal, the error due to impurities may in general be neglected if high purity samples are obtained from Heraeus or the mint. If the temperature scale is extrapolated above that of the standard temperature, i.e., gold or palladium point, by the use of an absorbing screen or a sector disk in terms of one of the radiation laws, there is a possibility of a third source of error. This last error may be caused by

[1] *Astrophysical Journal*, **42**, 300, 1915.

[2] *Zeitschrift für Instrumentenkunde*, **33**, 95, 1913.

an error in the transmission of the sector or absorbing glass, by the improper location of the sector,[1] or by an error in wave-length[2] used, if a colored glass is used as the monochromatic screen. These three sources of error may lead to very great uncertainties in the final results.

The sources of error just mentioned apply to measurements on a black body. If the temperature of a non-black body is determined by measuring the brightness for a given wave-length λ with the pyrometer, the temperature thus obtained is called the brightness temperature of the body and is the temperature to which a black body must be raised to give the same brightness for the same wave-length. As a non-black body may have a different brightness temperature for every different wave-length examined, the brightness temprature of a hot body is indefinite unless the wave-length employed in the measurement is specified. Thus if we say the brightness temperature of a body is 1500° K for $\lambda = 0.665\ \mu$, we mean that for this wave-length the body has the same brightness as a black body at 1500° K for this same wave-length.

To discover the extent of agreement among the high temperature scales in use by a number of research laboratories at home and abroad, several tungsten lamps that had been carefully aged and then calibrated in the Nela Research Laboratory were sent to different laboratories with the request that the temperature of the lamps be measured for specified currents and returned to this laboratory. This intercomparison has been made possible by the kind cooperation of Dr. Stratton, director of the Bureau of Standards, Sir J. E. Peteval, director of the National Physical Laboratory of Great Britain, Dr. C. E. Mendenhall, of the University of Wisconsin, and Dr. Langmuir, of the Research Laboratory of the General Electric Company. In each of the laboratories the temperatures were measured with a disappearing filament optical pyrometer using a plate of red glass as the monochromatic screen.[3]

[1] *Journal of the Optical Society*, **4**, 317, 1920.

[2] *Astrophysical Journal*, **42**, 294, 1915.

[3] The optical pyrometer can be used to measure the apparent brightness or brightness temperatures of the planets (*Nature*, **8**, 533, 1923) for different parts of the visible spectrum by the use of suitable color glass screens.

Although this intercomparison has been carried out by measurements on tungsten filaments, the results give not only a comparison of the brightness temperature of tungsten but also a valid comparison of the high-temperature scale. As was mentioned above, when we say that a non-black body has a particular brightness temperature for a definite wave-length interval, we mean that it has the same brightness for this wave-length interval as the black body for the same temperature. If strictly monochromatic radiations were used there would be no question about the comparison. By considering the "effective wave-length" of a color glass having a wide transmission band, as will appear later, the results obtained are just as definite as they would be if a screen were used that had a very narrow transmission band. Thus the tungsten filament is used as a convenient source with which to compare the temperature of the different standard black bodies.

In Figure 1 is shown a picture of one of each type of lamp used in the intercomparison and in Table I a description of the different individual lamps is given.

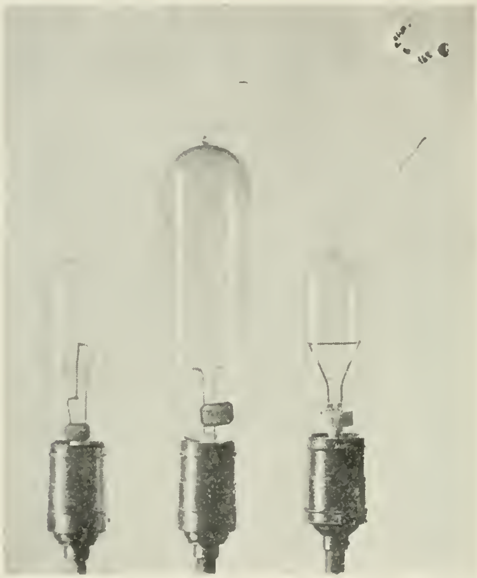
The exact point at which the temperature of each of the ribbon lamps was to be measured was indicated either by a pointer, a notch in the supporting lead, or a small notch in the ribbon itself. The wire filament of the lamp T-30-C was in the form of a hairpin loop with a rather sharp bend. The temperature of this filament was measured at the center of the loop.

Different practices have been followed at the different laboratories in establishing high-temperature scales. The Bureau of Standards bases its high-temperature scale on the melting-point of gold (1336° K) and extrapolation by means of Wien's[1] equation using for c_2 14350 μ deg.[2] At the National Physical Laboratory the high-temperature scale is based on the melting-point of palladium taken as 1828° K and extrapolation made by Wien's equation using 14350 μ deg. for c_2. At the physical laboratory of the University of Wisconsin high-temperature measurement is based upon

[1] Wien's equation has been used here rather than Planck's because of greater convenience in computation. Both laws give practically the same energy distribution in the visible spectrum for temperatures below 3000° K.

[2] Since this intercomparison was made the Bureau of Standards have decided to use 14330 μ deg. for c_2.

the melting-point of palladium taken as 1822° K. The scale is extrapolated by Wien's equation with c_2 as 14350 μ deg. At the Research Laboratory at Schenectady the high-temperature scale is



1 2 3

Fig. 1.—Picture of one of each of the different types of lamps used in the intercomparison. Numbers 1 and 3 have ribbon filaments; number 2 has a 20-mil. (0.5 mm) diameter wire filament.

TABLE I

Description of the Lamps Used in the Intercomparison

Lamp No.	Kind of Filament
T-25-C	Horizontal ribbon, $2\frac{1}{2}$ mm wide, gas-filled
T-16-B	Vertical ribbon, $2\frac{1}{2}$ mm wide, vacuum
T-30-C	Wire, 20 mil. (0.5 mm) diameter—gas-filled
T-17-C	Horizontal ribbon, 3 mm wide, gas-filled
T-18-B	Vertical ribbon, $2\frac{1}{2}$ mm wide, vacuum
T-77-B	Same as T-16-B
T-78-C	Same as T-30-C

based on the melting-point of gold (1336° K) and the scale extrapolated by Wien's equation with c_2 as 14350 μ deg. At Nela Research Laboratory the high-temperature scale is based upon the melting-point of gold (1336° K) and the scale extrapolated by Wien's

equation with c_2 taken as 14350 μ deg. For convenience in calibration of optical pyrometers the melting-point of palladium is used as the point of reference. Using the best Heraeus palladium obtainable, the value of this point in terms of the scale employed has been found to be 1828° K.

For a strict intercomparison of the results obtained in the different laboratories certain factors must be taken into account to reduce all measurements to the same basis. The common basis assumed for this purpose is the one adopted by the General Electric Company for use in its laboratories. This temperature scale is based upon the assumption of Wien's equation with c_2 taken as 14350 μ deg. and upon the melting-point of gold taken as 1336° K, i.e., degrees Centigrade+273 degrees. On this scale the melting-point of palladium is 1828° K. For convenience in calibration of optical pyrometers, a black body held at the melting-point of palladium is used as a point of reference.

As it was found that the red glasses used as monochromatic screens in the various laboratories did not have the same spectral transmission, the brightness temperatures obtained were not all to be ascribed to the same wave-length. If the brightness temperature of a particular source is measured with an optical pyrometer using a colored glass having a wide transmission band as a monochromatic screen, there has been some question as to just what wave-length this brightness temperature is to be ascribed. The "effective wave-length" of a monochromatic screen has been defined[1] as the wave-length such that for any definite temperature interval for a black body the ratio of the radiation intensities for this wave-length shall equal the ratio of the integral luminosities through the screen used.

The color temperature of a particular source has been defined as the temperature of a black body which has the same distribution of energy in the visible spectrum as the source under consideration. It has been found experimentally that most metals when heated radiate in such a manner that they can be color matched against a black body. These color matches are very easily and accurately made with an ordinary contrast photometer. It is to be noted

[1] *Astrophysical Journal*, 42, 295, 1915.

that, when two bodies have the same color temperature, it is not necessary that they shall have the same brightness for any particular wave-length interval.

From a consideration of the effective wave-length and the color temperature, it has been shown[1] that the brightness temperature, obtained with a colored glass having a wide transmission band as the monochromatic screen, is to be ascribed to the effective wave-length of the monochromatic screen for the temperature interval between the brightness temperature obtained and the corresponding color temperature of the source being studied.

First, the reduction due to a difference in the standard temperature will be considered. If a temperature T_2 is obtained by extrapolation by means of Wien's equation from a standard temperature T_1 the following is the relation between the value of T_2 and another T_2' obtained from another T_1' of the standard temperature:

$$1/T_2 - 1/T_1 = 1/T_2' - 1/T_1' \qquad (1)$$

The second reduction is made necessary by the use of a different value of the constant c_2. If different values of c_2 are used in extrapolating temperatures from a standard temperature T_1 by means of Wien's equation the following is the relation between the two c_2's and the resulting temperature:

$$c_2(1/T_1 - 1/T_2) = c_2'(1/T_1 - 1/T_2') \qquad (2)$$

The third reduction was to bring the various brightness temperatures reported to the same wave-length. From the conditions that hold at color match and Wien's equation the following relation can be shown to hold between two brightness temperatures (S_1 and S_2) and the wave-length (λ_1 and λ_2) to which they correspond:

$$1/S_2 = \lambda_2/\lambda_1(1/S_1 - 1/T_c) + 1/T_c \qquad (3)$$

where T_c is the color temperature corresponding to the brightness temperature S.

By means of the three relations just given the different temperatures reported have all been reduced to a common basis and are given in Table II. From the values given it can be seen that the

[1] *General Electric Review*, **120**, 752, 1917; *Trans. Faraday Soc.*, **15**, 21, 1920.

agreement is very good. The variation in the brightness temperatures of lamp T-25-C between the two measurements in this Laboratory is 2°, which is the same as the maximum difference between

TABLE II

RESULTS OF INTERCOMPARISON OF TEMPERATURE SCALES

$c_2 = 14350\ \mu$ deg.: $\lambda = 0.665\ \mu$: melting-point of Au = 1330° K (pd. = 1828° K).

Lamp	Current	N.R.L. Oct. 1916	G.E. Lab. Schenectady	N.R.L. Nov. 1916	Bureau of Standards	N.R.L. April 1917
T-25-C......	10.9	1826° K	1828° K	1826° K	....	....
	14.6	2214	2214	2215	....	....
	18.0	2518	2518	2516	....	....
T-16-B......	5.3	....	....	1429	1431° K	1427° K
	6.8	1618	1613	1617	1619	1614
	8.6	1816	1811	1811	1813	1812
	11.8	2128	2116	2122	2122	2121
T-30-C......	15.3	1813	1813	1813	1814	1813
	21.0	....	2307	2304	2302	2303
	27.5	2756	2752	2752	2762	2752

Lamp	Current	N.R.L. March 1916	University of Wisconsin C.E.M.		University of Wisconsin G.R G.	N.R.L. July 1917
T-17-C......	11.0	1810	1813	....	1816	1810
	14.6	2193	2197	....	2202	2196
	18.0	2499	2506	....	2516	2497
T-18-B......	6.7	1599	1602	....	1605	1597
	8.8	1806	1816	....	1819	1807
	12.0	2105	2119	....	2123	2107

Lamp	Current	N.R.L. April 1920	N.P.L. April 1922	N.R.L. June 1922	Bureau of Standards Jan. 1923	N.R.L. Jan. 1923
T-77-B......	5.2	1410° K	1403	1406	1401	1401
	6.7	1599	1596	1595	1596	1591
	8.5	1796	1794	1794	1792	1791
	11.7	2106	2106	2104	2105	2104
T-78-C......	15.4	1825	1825	1826	1824	1828
	20.4	2262	2265	2266	2265	2269
	27.4	2746	2757	2753	2755	2755

the values of this Laboratory and the Research Laboratory at Schenectady. The agreement for T-16-B is about the same between the first two measurements in this Laboratory and between this Laboratory and Schenectady. The lamp apparently held much

more constant during its trip to the Bureau of Standards, and here the agreement is even better. The results given by T-30-C are all that can reasonably be expected in the circumstances. The agreement of the results shown in the second part of the table on the two lamps sent to the University of Wisconsin is not so good as that of the results obtained by the other two laboratories. The lamps held very constant, as shown by the two measurements made in Nela Research Laboratory.

Three lamps were sent to the National Physical Laboratory, but unfortunately one of them was broken in transit. After the remaining two lamps had been measured again in the Nela Research Laboratory the Bureau of Standards made a measurement on the temperature of these lamps. The results of this part of the intercomparison are given in the third part of Table II.[1] The lamps were burned many hours at the National Physical Laboratory, and the measurements reported were obtained just before the lamps were returned. For this reason but little weight is to be given to the earlier values obtained in this laboratory.

From the data given in the table it appears that the characteristics of both lamps were slowly changing. However, the agreement is a very satisfactory one.

It is gratifying to know that the high-temperature scales in use in different laboratories in this country show so small a divergence and that this scale and the one used at the National Physical Laboratory of Great Britain are in such close agreement over the important range between 1400° K to 2750° K.

It is expected that this intercomparison will be carried farther and results obtained from more of the industrial laboratories of this country and elsewhere.

Nela Research Laboratory
National Lamp Works
Cleveland, Ohio
August 1923

[1] *Report of the National Physical Laboratory*, p. 54, 1922.

NOTE ON THE FORMULATION OF ABSORPTION BANDS IN THE NEAR INFRA-RED

By W. F. COLBY

ABSTRACT

The two well-known absorption bands of *HCl* have been reformulated with half-parameter numbers. From these formulae the combination band corresponding to a vibrational quantum change $1 \rightarrow 2$ is calculated and found to be in agreement with the faint absorption band recently observed.

In a recent number of this *Journal*[1] the writer was joint author in a report on a new faint absorption band of hydrogen chloride which lies in the same region as the well-known fundamental band with center at $3.4\,\mu$. The new band appeared only when the absorbing gas was heated to 300° C. It had all the characteristics of the band predicted by Kratzer[2] as due to a vibrational quantum spring from 1 to 2, although displaced with respect to his calculated band by an almost constant amount of about 80 units. None of the observational data which entered into the computation could have accounted for so large a discrepancy. Thanks to a suggestion of Dr. Pauli, the formulae for the strong absorption bands of hydrogen chloride are here restated with half parameter numbers and, with the help of the new molecular constants thus obtained, a much more satisfactory expression for the weak band results. Kratzer[3] has pointed out in recent papers how advantageous the half parameter numbers are in accounting for the fine structure and other line characteristics in the visible bands and in explaining the missing center in the near infra-red bands. Kramers and Pauli[4] have likewise used them successfully in a discussion of the infra-red bands. More recently Curtis[5] has used half numbers in stating the helium-band spectrum, and again Kiuti[6] in the band spectrum of hydrogen.

[1] Colby, Meyer, and Bronk, *Astrophysical Journal*, **57**, 7, 1923.

[2] A. Kratzer, *Zeitschrift für Physik*, **3**, 289, 1920.

[3] A. Kratzer, *Annalen der Physik*, **71**, 72, 1923.

[4] H. A. Kramers, *Zeitschrift für Physik*, **13**, 343, 1923. Kramers and Pauli, *ibid.*, **13**, 351, 1923.

[5] W. E. Curtis, *Proceedings of the Royal Society*, **103**, 315, 1923.

[6] Masazo Kiuti, *Proceedings of Phys.-Math. Society of Japan*, 3d Series, **5**, No. 2.

Aside from providing a more satisfactory explanation of the missing center, this restatement of the formula for the fundamental absorption band of HCl yields nothing new nor decisive. The original formula given for this band was of the form

$$\nu = A + Bm + Cm^2 + Dm^3.$$

A transformation $m = n + \frac{1}{2}$ yields a similar formula in n. When, however, one considers the relation of several bands through the combination principle, the choice of parameter numbers becomes very important.

With the notation of Kratzer, the energy term for these bands may be written.

$$W_n^m = W_n^0 + hB_n m^2 - h\beta_n m^4$$

or with half parameter numbers

$$W_n^m = W_n^0 + hB_n(m+\tfrac{1}{2})^2 - h\beta_n(m+\tfrac{1}{2})^4.$$

The parameter number m in former paper corresponds here to a change in rotation from

$m - \frac{1}{2}$ to $m + \frac{1}{2}$ and in vibration from $n = n_1$ to $n = n_2$.

We have then for the frequency

$$\nu_m = \frac{W_{n_2}^0 - W_{n_1}^0}{h} + B_{n_2}(m+\tfrac{1}{2})^2 - B_{n_1}(m-\tfrac{1}{2})^2 - \beta_{n_2}(m+\tfrac{1}{2})^4 + \beta_{n_1}(m-\tfrac{1}{2})^4,$$

or in powers of m and neglecting terms with $(\beta_{n_1} - \beta_{n_2})$

$$\nu_m = \nu_{n_1 n_2} + \frac{B_{n_2} - B_{n_1}}{4} + (B_{n_1} + B_{n_2} - \beta)m + (B_{n_2} - B_{n_1})m^2 - 4\beta m^3.$$

Comparing this notation with that used by us in previous papers,

$$B_n = \frac{h}{8\pi^2 I_0} - a_n, \qquad \beta = \frac{hu^2}{8\pi^2 I_0}.$$

For the fundamental band

$$n_1 = 0, \qquad n_2 = 1$$

and

$$\nu = \left(\nu_{01} + \frac{B_1 - B_0}{4}\right) + (B_0 + B_1 - \beta)m + (B_1 - B_0)m^2 - 4\beta m^3.$$

The four constants here involved may be determined from the empirical equation for this band, viz.,

$$\nu = 28860.7 + 205.983\, m - 3.01023\, m^2 - .0205658\, m^3$$

giving

$$\begin{aligned} \nu_{0_1} &= 28861.45 \\ B_1 &= 101.489 \\ B_0 &= 104.499 \\ \beta &= .00514 \end{aligned}$$

For the band at 1.7 μ,

$$n_1 = 0, \qquad n_2 = 2,$$

and

$$\nu = \left(\nu_{02} + \frac{B_2 - B_0}{4}\right) + (B_0 + B_2 - \beta)m + (B_2 - B_0)m^2 - 4\beta m^3 .$$

This band has been measured by Imes[1] but not with the same precision as the fundamental band, partly because it is so faint and partly because the wave-length determinations are complicated by the partial resolution of each line into two components due to the chlorine isotopes. The new constants B_2 and ν_{02} were nevertheless determined from these measurements. This was done most conveniently by adding wave numbers corresponding to m and $-m$. β is thus eliminated and one obtains by least squares

$$\begin{aligned} \nu_{02} &= 56671.2 \\ B_2 &= 98.618 \end{aligned}$$

The weak band corresponds to a vibration change from 1 to 2 and the formula reads

$$\nu = \nu_{02} + \frac{B_2 - B_1}{4} + (B_1 + B_2 - \beta)m + (B_2 - B_1)m^2 - 4\beta m^3 .$$

This contains only constants obtained from the other bands and may be written numerically

$$\nu = 27809.1 + 200.102\, m - 2.871\, m^2 - .02056\, m^3.$$

[1] E. S. Imes, *Astrophysical Journal*, **50**, 251, 1919.

The following table shows computed and observed values.

m	ν_{comp}	obs	Diff.
−9	25790.6	25797	6.4
−10	25541.5	25547	5.5
−11	25287.9	25288	0.1
−12	25030.0	25021	−9.0
−13	24767.7	24760	−7.7

The precision of measurement of the faint band is by no means so great as in the case of the other bands and the probable error may in the weakest lines be nearly as great as the deviations here indicated. In addition to this, however, comes the uncertainty in the two constants determined from the band at 1.7 μ, which may account for the apparently systematic error.

In addition to the above computation we have another test in the determination of the constant u which was used in our former paper and which now enters in the new constant

$$\beta=\left(\frac{hu^2}{8\pi^2 I_0}\right).$$

A direct computation of this constant from the constants of the fundamental yields

$$u=0.00701.$$

If, however, one makes use of a relation, developed by Kratzer in his earlier paper, which involves also the band at 1.7 μ, viz.,

$$\frac{W_2^\circ}{h}=\frac{4W_1^\circ}{h}-\frac{h}{2\pi^2 I_0 u}$$

one obtains

$$u=0.00711,$$

here, again, an agreement within the errors of observation.

On the whole one may feel that this is satisfactory evidence of the necessity for introducing half parameter numbers into the formulation of these bands. Such evidence has been made available by the measurement of this faint band which allows so straightforward an application of the combination principle.

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ON THE COSMIC ORIGIN OF THE RADIOACTIVE SUBSTANCES IN THE ATMOSPHERE

By HERMANN BONGARDS

ABSTRACT

The decomposition products of radium, actinium, and thorium in the atmosphere.—The presence of the emanations of the radioactive elements in the earth's atmosphere has been attributed hitherto to the *exhalations of the soil* and the rocks. There have been very few careful measurements for the free upper atmosphere, but it was known that the disintegration products from the soil were sufficient to account for the ionization observed near the ground.

Comparison of observations made at different places.—Observations made by the author in 1913 at Lindenberg, by means of steel wires from 5 to 15 meters in length, carried to altitudes of 4000 meters by kites, showed an unmistakable dependence of the quantity of emanation present upon the barometer at the surface and the temperature of the air strata measured. A period of twenty-seven or twenty-eight days in the variations observed suggested the sun as the source, since this is approximately the period of rotation of the sun. This suggestion was corroborated by a study of the calcium clouds shown by spectroheliograms taken between September 25 and October 12, 1913. Observations made at Manila, Philippine Islands, during the same period of time, by I. R. Wright and O. F. Smith, show that the fluctuations at the two places were the same. The author concludes therefore that *the sun is the source of the emanations,* and not the soil as hitherto believed.

The dissemination of the decomposition products of the emanations of radium, actinium, and thorium in the atmosphere was first established by J. Elster and H. Geitel.[1] As a consequence of this discovery, variations are observed in the emanation content of the atmosphere in many parts of the earth. The especially large quantities of emanation in the air in the upper layers of the earth's surface, and also in caves and cellars, suggest that their origin should be sought in the soil. The variations of the emanation content in the atmosphere were explained by the change of exhalation from the soil; and the connection that had already been established between the emanation content and such meteorological phenomena as insolation, precipitation, and variations of air pressure leads one back to the influence which these phenomena would exert on the exhalation from the ground. The observed fact that the soil and stones themselves are radioactive appears to support this hypothesis. But it seems that, in such investigations

[1] *Physikalische Zeitschrift,* 2, 590, 1901; 3, 305, 1902; 4, 96, 522, 1903.

of the soil and its stony contents, methods of measurement are not always used which exclude the absorption of atmospheric emanation before and during the process of the experiment. Only isolated observations of the emanation content of the atmosphere at high altitudes are available at present, and often these are contradictory.

Starting with the hypothesis that the emanation comes from the earth, investigations upon mountain peaks would not be suitable for drawing conclusions as to the emanation content of the free atmosphere. Measurements in flying bodies would offer the only mode of attack. Such measurements have been undertaken by H. Brandes[1] in kites, and by H. Fleming[2] in balloons. But the methods that they used are not suitable for yielding quantitative results. Consequently, today we do not have any estimate of the quantity of emanation that is mixed with the free air at high altitudes.

This question is, furthermore, of the greatest significance in the explanation of the ionized state of the atmosphere, and of the theory of atmospheric electricity. The fact that the quantities of radium and its products of disintegration contained in the earth's atmosphere are sufficient to produce the total ionization observed near the ground follows from the ionization balance, described by E. von Schweidler and K. W. F. Kohlrausch.[3]

In 1913, during the period from July 29 to December 2, in Lindenberg (lat., 52° 12′.5; long., 14° 7′.5 west), the author attempted to observe the variations with the time in the radioactive content of the free atmosphere. Steel wires from 5 to 15 meters in length were carried up by kites to heights of 4000 meters. Since they were grounded and accordingly charged negatively with respect to the surrounding air, under the influence of the earth's field, they received a deposit of positively charged disintegration products of emanation. The strength of this deposit was measured, after landing, in the ionization chamber of an electrometer. A description of the method and a preliminary statement of the results were published soon after the completion of the measurements.[4] The final compilation of

[1] Dissertation, Kiel, 1905. [2] *Physikalische Zeitschrift*, **9**, 801, 1908.

[3] L. Graetz, *Handbuch d. Electr. und d. Magn.*, Leipzig, **3**, 234, 1915.

[4] *Die Arbeiten d. Kgl. Preuss. Aeron. Obs. Lindenberg*, **9**, 414, 1913.

the results followed after the close of the war, and was published with a detailed exposition of the methods of calculation.[1] A graphical summary of all the single observations (ninety-eight in number) was also given, and showed an unmistakable dependence of the emanation content upon the change of air pressure at the ground and the potential temperature of the stratum of air measured. At that time, valid arguments were made against the prevalent conception that the relation, so frequently observed, between air pressure and variations of amount of emanation, could be traced back to an exhalation from the ground modified by pressure changes.

The relation of the emanation content to temperature (which is a little less evident in the lower layers of the atmosphere) might give rise to the question as to whether the heat liberated in the disintegration of the radioactive substances entered into the phenomenon, and, therefore, directly acted upon the air pressure. A rough calculation having shown that such an effect was not probable, it will be assumed that a corpuscular radiation penetrating into the atmosphere brings in atoms of emanation, on the one hand, and also energy absorbed by the air in the form of heat, on the other hand. A clearly perceptible periodicity, at times, of a period of from twenty-seven to twenty-eight days, suggested the sun as a possible source of this radiation, since this was approximately the period of rotation of the sun. A further support of this hypothesis comes from a parallel consideration[2] of these results and a series of photographic exposures on the sun in the light of a calcium line, which were kindly put at our disposal by the Mount Wilson Observatory of the Carnegie Institution and the Yerkes Observatory of the University of Chicago. In the period from September 25 to October 2, 1913, an obvious relation was observed between the movement of the calcium clouds over the disk of the sun and the quantity of radioactive disintegration products in the earth's atmosphere. In order to obtain a further proof of this relation, measurement of the emanation content by a flying machine, with simultaneous spectroheliographic exposures, was proposed.

[1] *Physikalische Zeitschrift*, 21, 141, 1920.

[2] H. Bongards, *Physikalische Zeitschrift*, 24, 16, 1923.

While attempting to carry out this plan in spite of the unfavorable economic situation, the author discovered a paper by I. R. Wright and O. F. Smith,[1] which previously had not been accessible to him because it appeared during the war. These observers carried out measurements upon the emanation content of the atmosphere from July, 1913, to July, 1914, at Manila, Philippine Islands. Their method consisted in separating the emanation from the air by cocoanut charcoal, and gave quantitative results, instead of relative results such as were obtained by the wire method used in Lindenberg. During the common period of observation, of about four months, the observations in Manila are not so numerous as those in Lindenberg. The observations in Manila took place on thirteen different days during this period. On nine of these dates, observations were also made in Lindenberg. In three other cases, the observations in Lindenberg occurred on the day succeeding those in Manila. The measurements in Manila gave average values over a period of twenty hours, the air current being drawn through the charcoal from 1 P.M. of one day to 9 A.M. of the succeeding day. In Lindenberg, an observation lasted only two hours. Accordingly, a comparison of the two sets of observations would be more reliable if, for Lindenberg, the average of the readings on two successive days were taken. Such averages were made in the eight cases in which there were values for comparison. In three other cases, only the values on the succeeding day were used. Up to the end of September 8, which fell on a gap in the Lindenberg measurements, there were comparison observations in Lindenberg for all of those in Manila. These observations are given in Table I, and are represented graphically in Figure 1. The times, in the units indicated, are plotted as abscissae, and as ordinates the relative or absolute values of the emanation content. It is seen that the curves change together. This is all the more extraordinary and convincing since it has to do with a comparison of results of two entirely different modes of observation. That the agreement is accidental appears to be excluded, for the number of twelve observations distributed arbitrarily over a space of time of more than three months is too great. Furthermore, the position of the two places on opposite sides

[1] *Physical Review*, (2) **5**, 459, 1915.

of the earth, and under wholly different climatic conditions, with a difference in latitude of 37,° does not allow the conclusion that simultaneous meteorological conditions could have produced these simultaneous results.

TABLE I

Date	Manila	Lindenberg		
1913	Cr $\times 10^{12}$ Radioactive Equivalent per Cu. Meter (Curie $\times 10^{12}$)	Saturation Current in Static Units $\times 10^{6}$ per Sq. Cm of the Surface of the Active Wire		
	Mean	Single Obs.	Sum	Mean
July 28......	14.5	....	....	0.66
29......	....	0.66	....	....
Aug. 4......	23.6	1.91	3.36	1.68
5......	....	1.45		
18......	39.5	10.65	17.61	8.80
19......	....	6.96		
25......	19.6	1.50	3.58	1.79
26......	....	2.08		
Sept. 1......	49.7	13.00	....	13.00
2......	....	....	....	....
8......	17.3	....	....	....
9......	....	....	....	....
22......	53.4	....	....	....
23......	....	6.07	....	6.07
29......	52.0	15.24	37.54	18.77
30......	....	22.30		
Oct. 13......	17.1	1.83	5.87	2.93
14......	....	4.04		
20......	77.1	3.48	12.11	2.93
21......	....	8.63		
27......	92.2	9.09	19.54	9.77
28......	....	10.45		
Nov. 3......	49.9	3.86	5.64	2.82
4......	....	1.78		
10......	75.4	....	....	3.90
11......	....	3.90	....	

Therefore, it appears to be absolutely necessary to abandon the hypothesis that the variation of the emanation content of the atmosphere can be ascribed to changes of the exhalation from the soil. Accordingly, we must also give up the hypothesis that most of the emanation in the atmosphere comes from the earth's surface. Inasmuch as the source of emanation cannot be wholly in the atmosphere itself, we cannot avoid the assumption of a cosmic

origin for it. The sun would naturally be considered as the most probable source, and it would be assumed that corpuscular radiations are emitted from the sun which are carriers of emanation atoms. Since these atoms presumably possess positive charges, we accordingly have the foundations of a theory for the vertical current from the air to the ground, the drop of potential through the atmosphere, and the ionization of the atmosphere.

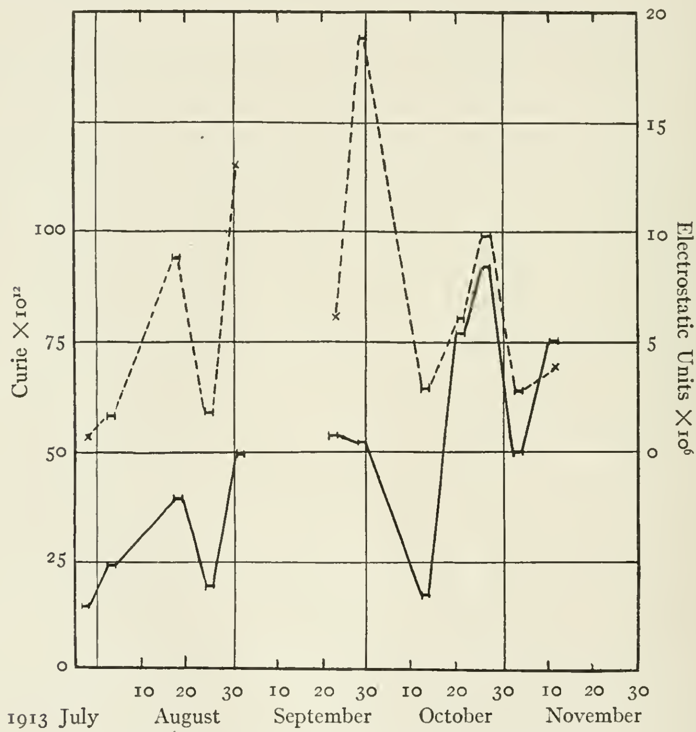

Fig. 1.—Lower curve, Manila; upper curve, Lindenberg

A further proof of this hypothesis would offer no insuperable difficulties. It would only be necessary to carry out simultaneous measurements of the emanation content near the ground and in the free atmosphere at numerous places suitably chosen. In addition, spectroheliograms of the sun should be made. The compilation of the information thus obtained would be sufficient to complete

the chain of proof for the solar origin of the emanation in the air. The significance of this knowledge in its bearing upon atmospheric electricity, terrestrial magnetism, meteorological and astrophysical problems is so great as to make the necessary expense of securing it appear small.

HAMBURG-GROSSBORSTEL
June 1923

ON THE DOUBLE STAR 9 ARGUS. A CORRECTION

In Vol. 58, October 1923, page 144, *for* $\left(\frac{m_1}{m}\right)$ *read* $\left(\frac{m_1}{m_2}-\frac{\Delta m}{2m_2}\right)$ where $\Delta m = m_1 - m_2$ and $m_1 > m_2$, m_2 here designating the mass of the smaller star, which was called m in the original paper.

The term due to Δm is small only when the mass ratio is close to one. In the case of 9 Argus, this term equals -0.2 and the mass ratio, m_2/m_1, is therefore not 0.6 but 0.4 (page 145).

OTTO STRUVE

ERRATUM

Vol. 58, July, 1923, in the article on Edward E. Barnard by Edwin B. Frost:

Page 31, third line, *for* Padang Padang *read* Solok.

REVIEWS

Newcomb-Engelmann's Populäre Astronomie, Sechste Auflage. Edited by H. LUDENDORFF, with the collaboration of G. EBERHARD, E. FREUNDLICH, and A. KOHLSCHÜTTER. Leipzig: Wilhelm Engelmann, 1921. Large Octavo, 24×18 cm. Pp. xii+889. Figs. 240.

We welcome this new addition of what we regard as the best general descriptive work on astronomy. It is based on Professor Simon Newcomb's classical *Popular Astronomy* which appeared in 1878 and of which, unfortunately, no revision has been made since 1882. The masterly presentation by Newcomb made a fine impression throughout the world, and the German translation was undertaken by Rudolph Engelmann, previously an astronomer at the Leipzig Observatory. It was published in 1881. Eleven years later a second edition was prepared by Engelmann's friend, H. C. Vogel, then director of the Potsdam Observatory, who included in the work the important progress made in astrophysics in the interim. In 1905 the third edition appeared, also edited by Vogel. The fourth edition (1911) was edited by Paul Kempf, with the assistance of G. Eberhard, H. Ludendorff, and K. Schwarzschild. This edition was exhausted in a space of three years, and the fifth appeared in 1914, also under the editorship of Kempf. In its various editions the work has thus had the benefit of the careful scrutiny of the above array of able astronomers at the Potsdam Observatory who have in each edition brought the book thoroughly up to date. In spite of the many additions made necessary by the progress of the science, the editors have endeavored to prevent the volume from reaching undue size. Thus the fifth edition exceeded the third by 88 pages; and the present volume is only 53 pages larger than its predecessor, the fifth edition. This has involved great discrimination in adding new material and leaving out that which is obsolete. In spite of the number of times the book has been worked over, the editors have lived up to their intention of retaining, just as far as they are valid today, the original statements of the American author, and a person who is sufficiently acquainted with the English language can read Newcomb's words in many places in this edition.

The tables are inserted in the text at the places where the particular subject is being described, and, while not exhaustive, represent adequately our present knowledge. Thus, on page 599 we find a table of radial velocities of 206 stars of third magnitude and brighter; and on

page 203 the parallaxes of 57 of the nearest stars, with their distances in light years and their magnitudes. Among other useful tables we may name the list of 42 stars of second magnitude and brighter, and those giving the computed orbits of 56 binary stars having periods of less than 120 years, the orbits of 67 spectroscopic binaries, and the mass functions for 51 such orbits. The tabular data and the numerical values given in the work can be particularly depended upon, by reason of the repeated revisions. We have noted a slip on page 713, where the linear velocity of rotation of the Andromeda Nebula, N.G.C. 224, at a distance of 2′ from the center, is given as 88, where it should be 58, kilometers per second. On the same page there seems to be also a misunderstanding about the rotation of the Andromeda Nebula, which was actually found to vary in a linear manner from the center outward, and not just as stated in this book. The discussions of star streaming, of the structure of the universe, and of cosmogony in general, are sane, clear, and logical. The statement of the problems as to the character of the spiral nebulae appears to us reasonable. The brief biographical sketches of deceased astronomers who have made important contributions to our science have been retained and constitute a useful section of the work.

The book is well bound in moleskin.

Since the above was written we have learned that the edition above referred to, of which 6000 copies were printed, was exhausted at the end of 1922. This, in itself, is sufficient and surprising evidence of the popularity of this work among readers of the German language. A seventh edition was published at the beginning of the present year. Aside from the correction of errors and the revision of some tables, the seventh edition differs from the sixth chiefly in the inclusion of an appendix (Nachtrag) of 20 pages, intended to cover the main items of astronomical progress in the last two years. This brings the total number of pages of the seventh edition to 902. The reviewer has read this supplement, but has not seen the main body of the seventh edition. The price has been set at 23 Swiss francs, or about $4.50, for the bound volume; and 20 Swiss francs for the work in paper binding.

We have also received definite information that a translation of this work, based on the seventh edition, is at last to appear in English. It is being undertaken by Dr. Henry Meier of Centre College, Danville, Kentucky, who was employed in the Nautical Almanac office under Professor Newcomb soon after his arrival from Switzerland, and who was also Professor Newcomb's assistant from 1885 to 1896. He is thus qualified for preparing this American edition. It is certain that this edition in English will be warmly welcomed and widely used.

E. B. F.

GENERAL INDEX TO *ASTROPHYSICAL JOURNAL*, VOLS. XXVI–L (1907–1919)

The Editors announce with pleasure that a General Index to Volumes XXVI–L (1907–1919) of the *Astrophysical Journal* has been prepared by Professor Storrs B. Barrett, of the Yerkes Observatory, and is now ready for distribution.

This General Index is uniform in style with the General Index to Volumes I–XXV (1895–1907), which was also compiled by Mr. Barrett and appeared in 1908. It is arranged by subjects and by authors, forming a volume of 116 pages, and is of the same format as this *Journal*.

From our own experience in the use of the first General Index, we can speak with assurance of the value of this second General Index to all users of the *Journal*.

The University of Chicago Press has fixed the price of the new index at $2.50, in paper covers, and makes a special offer, until April 1, 1924, of the two General Indices for $3.25.

To subscribers whose set of the *Journal* does not include the early volumes, the first General Index will serve as a bibliography of the papers appearing in those volumes, many of which are now out of print.

INDEX TO VOLUME LVIII

SUBJECTS

INDEX TO VOLUME LVIII

AUTHORS